Greedy Routing and Its Algebra in Geographic Routing

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***Abstract*—In this paper, we propose a novel routing algebra system to investigate greedy routing in geographic routing. Four important algebraic properties, respectively named mutex, transition, source independence and strict preference, are defined in this algebra system. Based on these algebraic properties, the necessary and sufficient conditions for the loop-free and consistent compatibilities of greedy routing based routing protocols are derived. The application of this proposed algebra is illustrated with existing different greedy routings in geographic routing. In theory, our work provides essential criterions for evaluating and designing greedy routing based routing protocols.**

***Index Terms*—geographic routing, greedy routing, routing algebra, loop-freeness, convergence, consistency.**

I. Introduction

Geographic routing, also called position-based routing, has been receiving significant attention since it was originally proposed in [**?**] [**?**] in the 1980s. Compared with topology-based routing [**?**], [**?**], [**?**], [**?**], [**?**], geographic routing has two unique benefits, especially for large-scale, highly dynamic and energy-constraint wireless networks. First, in geographic routing, a forwarding node transmits a packet only based on the positions of the destination node and the one-hop neighbors. The size of routing tables and the overhead of routing control messages are minimal. Second, the convergence time for geographic routing can be neglected. This is very attractive for highly dynamic wireless networks since it is extremely difficult to design fast-convergent topology-based routing protocols. These above benefits coupled with the progress on Global Positioning System (GPS) and self-configuring localization mechanisms [**?**], [**?**], [**?**] have promoted geographic routing as a promising solution for many wireless networks.

Usually, a geographic routing protocol is composed of two important parts: greedy routing and face routing. Greedy routing tries to bring the message closer to the destination in each step using only local information. Thus, each node forwards the message to the neighbor that is most suitable from a local point of view. Face routing helps to recover from dead end situation, where greedy routing hits a void and cannot find a suitable neighbor to the destination. Various methods can be used to define the most suitable neighbor for greedy routing and these methods are named *routing metrics* in this paper. Besides location information, recently, many performance-affecting characteristics have also been taken into account in the design of routing metrics for greedy routing metrics. The performance of wireless networks can improve a lot due to these more complicated routing metrics [**?**], [**?**], [**?**], [**?**]. However, the design of routing metrics cannot be arbitrary due to potential incompatibility with greedy routing, which may greatly degrade network performance and even create routing loops. For example, it has been pointed out in [**?**] that a projected-distance-based greedy routing metric can create routing loops between two intermediate connected nodes in greedy routing. Unfortunately, despite the potential serious impact of routing metric designs on the greedy routing, systematic analysis of such impact is still lacking in the current literature.

The goal of this paper, hence, is to fill in this void and present an in-depth analysis about the relationship between greedy routing and their constraints on the design of routing metrics. To realize this goal, this paper proposes a novel routing algebra for greedy routing analysis. Using this algebra, four important routing algebraic properties, including mutex, transition, source independence and strict preference, are identified. These algebraic properties determine the compatibility among greedy routing algorithm, routing metrics and other components of routing system.

The remainder of this paper is organized as follows. Section II introduces the conception of compatibility of greedy routing based routing protocols. Section III discusses the role of geographic routing in routing system. Section IV establishes network model and proposes a routing algebra. Section V systematically analyzes the requirements for realizing compatibilities of greedy routing based routing protocols. Section VI illustrates the application of the proposed algebra. Section VII concludes and discusses the future directions.

II. Compatibility of greedy routing Based Routing Protocols

Greedy routing is an algorithm that makes the locally optimum choice at each stage with the hope of approximating the global optimum. For geographic routing, the greedy routing chooses the lightest weight out-going link at each forwarding node, and the weight of each link is decided by the chosen routing metric that captures location information and various performance affecting factors.

While it is important to capture performance affecting factors in greedy routing, the compatibility between greedy routing and routing metric must be strictly analyzed to avoid potential routing problems. In this paper, we will focus on analyzing two basic aspects of compatibility, including loop-freeness and consistency.

Loop-freeness requires that a routing protocol should never create any forwarding loops in any network topology at its stable state. Loop-freeness is the most important requirement for a routing system and imposes the minimum constraints on compatibility. It is obvious that a protocol lack of loop-free compatibility is unusable for any network.

Consistency represents the requirement that the decisions of packet forwarding made by all nodes along the path should be consistent with each other. If a node v_1 decides its traffic to v_n should follow the path $p(v_1, v_n) = \langle v_1, v_2, ..., v_n \rangle$, then all the nodes along p should make the same decision; i.e., v_2 should forward the traffic to v_3, and v_3 should forward the trafiic to v_4, and so forth. Consistent compatibility guarantees available knowledge and easy control of packet delivering paths, which is critical for network management and security.

III. The Role of Geographic Routing in a Routing System

The compatibility between greedy routing and routing metrics depends on how geographic routing is used in a routing system. There are two fundamentally different ways of applying geographic routing.

The first approach is to treat geographic routing as a packet forwarding scheme so that for every arriving packet, a relaying node calculates the locally optimal next-hop neighbor and forwards the packet to this next-hop neighbor. The benefit of this approach is that a node does not need to keep any routing state regarding destinations or flows. On the other hand, this approach also brings a lot of per-packet computation and message overhead to a wireless network. For every arrival packet, a node must calculate the most preferred next-hop node based on the information carried in the packet. The per-packet computation overhead can be much higher than traditional routing-table-based packet forwarding schemes. Such high computation overhead can potentially slow down a node's packet forwarding speed and drain the node's battery. In addition, the lack of routing states in all relaying nodes indicates that every packet must carry extra information for the next-hop computation, including source and destination positions and other performance affecting factors. Given the fact that packet size in wireless networks usually is small to cope with the unreliability of wireless links, the per-packet overhead for carrying such extra information for next-hop computation is very high. Finally, since the next-hop node of each packet is dynamically decided by each relaying node and no routing state is kept in the network, it is very difficult to track the paths taken by the packets of a flow and diagnose potential network connectivity or security problems.

In the second approach, geographic routing can be used as a on-demand route discovery mechanism and be combined with different traditional packet forwarding schemes to form various routing systems. Two widely used pack forwarding schemes can be combined with geographic routing. The first scheme is source routing, where a source node caches paths discovered by geographic routing and includes path information in its packet headers. Intermediate node then relay the flow based on the path information carried in forwarded packets. The second scheme is hop-by-hop routing, where soft states about the next hop is set along the path discovered by geographic routing. A source node only appends the destination address of a flow in its packet headers, and an intermediate node relays the flow based on its local soft states about the next-hop to reach the destination node. For both packet forwarding schemes, while a small number of soft states of discovered paths have to be maintained in either source nodes or relaying nodes, the benefit of reduced per-packet computation overhead and message overhead can be significant. The soft states also provide crucial support for network security and management since these soft states clearly defines the forwarding path of packets and facilitate easy monitoring and diagnosis of routing abnormality. In fact, combination of various path discovery methods, such as the flooding-based path discovery scheme, Dijkstra's algorithm and the Bellman-Ford algorithm, with various packet forwarding schemes have been widely accepted in wireless routing protocols, including DSR [?], AODV [?], DLAR [?] and LBAR [?] as a great tradeoff for balancing both the storage overhead for maintaining routing states and the computation and message overhead for packet forwarding.

IV. Network Model and Routing Algebra

As discussed in the previous two sections, different aspects of compatibility have different requirements on routing metrics design for greedy routing and different approaches for using geographic routing also causes the greedy routing to have different requirements on its routing metric designs. To systematically analyze these compatibility requirements (See Section V for details), in this section, we define our network model and propose a new routing algebra based on this network model.

A. Network Model

A wireless network is modeled as a connected directed graph $G(V, E)$ with cardinalities $|V|$ and $|E|$. V is the set of vertices, and every vertex in the graph represents a node in the wireless network. Every link in the wireless network is mapped to a directed edge between the corresponding vertices.

If nodes u and v can communicate directly, i.e. one is in another's physical radio range, there are two links with inverse direction between nodes u and v. The link $\overrightarrow{uv}$ is responsible for transmitting packets from node u to v, and the link $\overrightarrow{vu}$ is responsible for the inverse packets transmission. The link $\overrightarrow{uv}$ is called node u's out-going link and the link $\overrightarrow{vu}$ is called node u's in-coming link. In this paper, we assume edge-symmetrical graph where there must be an edge $\overrightarrow{uv}$ from vertex u to v if there is an edge $\overrightarrow{vu}$ from the vertex v to u.

The vertex v is named a neighbor of the vertex u if there is an edge $\overrightarrow{uv}$ from the vertex u to v. The neighbor set $N(u)$ of the vertex u is the collection of its neighbor vertices, which can be defined as:

$$N(u) = \{v|\overrightarrow{uv} \in E\} \tag{1}$$

A path from the source vertex u_1 to the destination u_n can be represented by a sequence of vertices $u_1u_2...u_n$ where there is an directed edge from vertex u_k to u_{k+1} for $1 \le k \le n-1$. A path is simple where all vertices are distinct. A circle is a path where all vertices are distinct except for the source vertex and the destination vertex.

B. Routing Algebra

The routing algebra, based on the connected directed graph, is a 4-tuple $\langle L, F, w, \preceq \rangle$, where

L : a set of labels,

F : a set of traffic flows,

w : a link weight function,

$\preceq$: an order relationship.

A label $l_{\overrightarrow{uv}}$ in L describes the characteristics of the link $\overrightarrow{uv}$, such as the geographical positions of nodes u and v, the packet loss ratio, energy consumption per bit transmission, the channel frequency and so on. Each link in the wireless network is coupled with a label. Hence, the set L captures all the communication characteristics of the network.

A flow $f_{\langle s,d \rangle}$ in F represents that there is traffic that needs to be delivered from s to d. The cardinality of F is less than or equal to $|V| \times (|V-1|)$.

$w(.)$ is a function that calculates the weight of a link for different flows. $w(.)$ has two variables. One is the label of a link and the other is the flow. For example, for a flow $f_{\langle s,d \rangle}$, the weight of node u's out-going link $\overrightarrow{uv}$ is $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle})$, where $l_{\overrightarrow{uv}} \in L$ denotes the label of link $\overrightarrow{uv}$, and $f_{\langle s,d \rangle} \in F$ denotes the traffic demand between the source node s and the destination node d.

$\preceq$ is used to compare all the out-going links of a forwarding node based on $w(.)$, so that the forwarded packets can be relayed through the lightest weight link. If $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) \preceq w(l_{\overrightarrow{ux}}, f_{\langle s,d \rangle})$, we say that link $\overrightarrow{uv}$ is not worse than (i.e. lighter or equal to) link $\overrightarrow{ux}$ for flow $f_{\langle s,d \rangle}$.

Note that for greedy routing, not all the out-going links can be regarded as candidates for packet relaying. It is only those links, whose weight is strictly less than ϕ, can forward packets, where ϕ is a threshold value specified in a greedy routing based routing protocol.

Using the above novel routing algebra, we can discuss the compatibilities of greedy routing based on the following algebraic properties.

Definition 1: ***Mutex:*** $\forall\ l_{\overrightarrow{uv}}, l_{\overrightarrow{vu}} \in L$, $f_{\langle s,d \rangle} \in F$, $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) \prec \phi$ implies $w(l_{\overrightarrow{vu}}, f_{\langle s,d \rangle}) \succeq \phi$.

Definition 2: ***Transition:*** $\forall\ l_{\overrightarrow{u_1u_2}}, l_{\overrightarrow{u_2u_3}}, ..., l_{\overrightarrow{u_{k-1}u_k}}, l_{\overrightarrow{u_1u_k}} \in L$, $f_{\langle s,d \rangle} \in F$, $w(l_{\overrightarrow{u_1u_2}}, f_{\langle s,d \rangle}) \prec \phi$, $w(l_{\overrightarrow{u_2u_3}}, f_{\langle s,d \rangle}) \prec \phi$, ..., and $w(l_{\overrightarrow{u_{k-1}u_k}}, f_{\langle s,d \rangle}) \prec \phi$ implies $w(l_{\overrightarrow{u_1u_k}}, f_{\langle s,d \rangle}) \prec \phi$.

Definition 3: ***source independence:*** $\forall\ f_{\langle s_1,d \rangle}, f_{\langle s_2,d \rangle} \in F$, $l_{\overrightarrow{uv}} \in L$, $w(l_{\overrightarrow{uv}}, f_{\langle s_1,d \rangle}) = w(l_{\overrightarrow{uv}}, f_{\langle s_2,d \rangle})$ is always satisfied.

Definition 4: ***strict preference:***$\forall\ f_{\langle s,d \rangle} \in F$, there does not exist cases where two links $\overrightarrow{uv}$ and $\overrightarrow{uw}$ from node u satisfy $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) = w(l_{\overrightarrow{uw}}, f_{\langle s,d \rangle})$.

In geographic routing, a greedy algorithm relays a flow "closer" to the destination step by step, and the "closer" is decided by the routing metric specified in the greedy routing. Euclidean distance based metrics are not necessary, and projected distance based metrics illustrated in section VI also are applicable. In physical meaning, mutex indicates that there is a determined node between two nodes who is "closer" to the destination than another node for a specific flow. Transition represents that the "distance" between the relayed node and the destination is strictly decreased in the transmission process of a specific flow. Source independence refers that the "distance" between a relayed node and the destination has nothing to do with the information of the source node for a specific flow. Strict preference infers that there is only one node among all one nodes neighbors who is the "closest" node to the destination, which is used for "tie break".

It is worth noting that our routing algebra is different from the routing algebra proposed by Sobrinho in [**?**], [**?**]. Sobriho's routing algebra focuses on analyzing the compatibility issues for link-state, distance vector and path vector protocols [?], [?], [**?**], [**?**], [**?**]. This algebra is based on the assumption that the weight of a link is irrelevant to different flows. In greedy routing, however, the positions of both the source and the destination of a flow affect the path selection. Hence, Sobriho's assumption of flow-independent link weight is invalid. Moreover, greedy routing is substantially different from the Dijkstra's algorithm and the Bellman-Ford algorithm analylized in Sobrinho's routing algebra. Hence, Sobrinho's routing algebra is not applicable to analyze greedy routing. In this paper, we develope our novel algebra and its related algebraic properties to address the modeling and analyzing of greedy routing.

V. Requirements for Realizing Compatibilities

Mutex, transition, source independence and strict preference are important algebraic properties that determine compatibilities between greedy routing and routing metrics. In this section, we will systematically discuss the requirements for realizing compatibilities corresponding to different ways of using geographic routing, including geographic routing as a packet forwarding scheme, geographic routing combined with source routing and geographic routing combined with hop-by-hop routing.

A. Greedy Routing as a Packet Forwarding Scheme

When greedy routing is used as a packet forwarding scheme, each relaying node dynamically computes the next-hop node for each in-coming packet. Since no state about flow paths is maintained, no node has the knowledge about the entire path. Hence, the issue of consistent forwarding does not exist in this

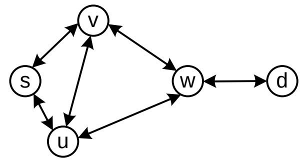

Fig. 1. a simple wireless network

case. Therefore, only the loop-free compatibility are discussed in this section.

Theorem 1: As a packet forwarding scheme, a greedy routing algorithm R is loop-free if and only if the routing metric has two properties: mutex and transition.

Proof:

Sufficient conditions:

We prove the sufficient conditions by contradiction. Assuming that a communication flow $f_{\langle s,d\rangle}$ initiating by node s is routed to its destination node d by the protocol R in a wireless network $G(V,E)$, a loop is created in the packet forwarding process.

According to the number of nodes in the loop, there are two kinds of loops, i.e, a two-node loop and a loop comprised of at least three nodes.

If a two-node loop $c(m,n)$ is created by the greedy routing algorithm R when relaying flow $f_{\langle s,d\rangle}$'s traffic, all packets of $f_{\langle s,d\rangle}$ are routed back and forth between node m and n until these packets are forced to be dropped due to TTL's expiration. Obviously, there are two links ($l_{\overrightarrow{mn}}$ and $l_{\overrightarrow{nm}} \in L$) between nodes m and n in this case. Based on the principle of greedy routing, a packet of $f_{\langle s,d\rangle}$ can be transmitted from node u to node v if and only if $w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \prec \phi$ is satisfied. Hence, if $c(m,n)$ is created, both $w(l_{\overrightarrow{mn}}, f_{\langle s,d\rangle}) \prec \phi$ and $w(l_{\overrightarrow{nm}}, f_{\langle s,d\rangle}) \prec \phi$ will be satisfied. This is a contradiction to the fact that the routing metric has the mutex property, so the two-node loop cannot be created in $f_{\langle s,d\rangle}$ transmission process.

Assume that the greedy routing algorithm creates a loop consisting more than two nodes, denoted as $c(u_1,u_2,...,u_k)$, $k \geq 3$, while forwarding flow $f_{\langle s,d\rangle}$'s traffic. All the packets of $f_{\langle s,d\rangle}$ that are relayed by u_1 will be forwarded back to u_1 through the following links $\overrightarrow{u_1u_2}, \overrightarrow{u_2u_3}, ..., \overrightarrow{u_{k-1}u_k}, \overrightarrow{u_ku_1}$. Furthermore, $l_{\overrightarrow{u_1u_k}} \in L$ is also satisfied due to the symmetry of links. Based on the principle of greedy routing, for the flow $f_{\langle s,d\rangle}$, the weight of each link in the circle $c(u_1,u_2,...,u_k)$ is strictly less than ϕ, i.e. $w(l_{\overrightarrow{u_1u_2}}, f_{\langle s,d\rangle}) \prec \phi$, $w(l_{\overrightarrow{u_2u_3}}, f_{\langle s,d\rangle}) \prec \phi$, ..., $w(l_{\overrightarrow{u_{k-1}u_k}}, f_{\langle s,d\rangle}) \prec \phi$ and $w(l_{\overrightarrow{u_ku_1}}$, and $f_{\langle s,d\rangle}) \prec \phi$. Based on the transition property, we can conclude that $w(l_{\overrightarrow{u_1u_k}}, f_{\langle s,d\rangle}) \prec \phi$ is satisfied. Then, both $w(l_{\overrightarrow{u_1u_k}}, f_{\langle s,d\rangle}) \prec \phi$ and $w(l_{\overrightarrow{u_ku_1}}, f_{\langle s,d\rangle}) \prec \phi$ are satisfied, which contradicts the fact that the routing metric has the mutex property. Hence, the loops that consist more than two nodes cannot be created by greedy routing.

Necessary conditions:

We prove the necessary conditions through simple examples.

If the routing metric does not have the mutex property, a routing loop may be created between two relaying nodes by the greedy routing scheme R. Consider a simple network in Fig. 1, where for flow $f_{\langle s,d\rangle}$, $w(l_{\overrightarrow{su}}, f_{\langle s,d\rangle}) \prec w(l_{\overrightarrow{sv}}, f_{\langle s,d\rangle}) \prec \phi$, $w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \prec w(l_{\overrightarrow{uw}}, f_{\langle s,d\rangle}) \prec \phi \prec w(l_{\overrightarrow{us}}, f_{\langle s,d\rangle})$ and $w(l_{\overrightarrow{vu}}, f_{\langle s,d\rangle}) \prec w(l_{\overrightarrow{vw}}, f_{\langle s,d\rangle}) \prec w(l_{\overrightarrow{vs}}, f_{\langle s,d\rangle})$. Due to the lack of the mutex property, we also have $w(l_{\overrightarrow{vu}}, f_{\langle s,d\rangle}) \prec \phi$. According to greedy routing, all packets of flow $f_{\langle s,d\rangle}$ will be forwarded to node u by the source node s and then be relayed to node v by node u. Since $\overrightarrow{vu}$ is the local optimal and $w(l_{\overrightarrow{vu}}, f_{\langle s,d\rangle}) \prec \phi$, all the packets of $f_{\langle s,d\rangle}$ coming from node u will be immediately forwarded back to node u again, forming a routing loop between nodes u and v.

If the routing metric has the mutex property, but lacks of the transition property, routing loops consisting multiple nodes can also be created by greedy routing. Still considering Fig. 1, assume that the links satisfy the following inequalities: $w(l_{\overrightarrow{su}}, f_{\langle s,d\rangle}) \prec \phi \prec w(l_{\overrightarrow{sv}}, f_{\langle s,d\rangle})$, $w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \prec w(l_{\overrightarrow{uw}}, f_{\langle s,d\rangle}) \prec \phi \prec w(l_{\overrightarrow{us}}, f_{\langle s,d\rangle})$ and $w(l_{\overrightarrow{vs}}, f_{\langle s,d\rangle}) \prec w(l_{\overrightarrow{vw}}, f_{\langle s,d\rangle}) \prec \phi \prec w(l_{\overrightarrow{vu}}, f_{\langle s,d\rangle})$. Obviously, the mutex property is satisfied but there is a lack of the transition property. Hence, for flow $f_{\langle s,d\rangle}$, greedy routing creates a routing loop comprised of nodes s, u, and v. ■

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B. Greedy Routing Combined with Source Routing

When greedy routing is used as an on-demand path discovery algorithm and combined with source routing, the relaying nodes do not need to keep any flow state. When a new flow arrives at a source node, the source initates a route discovery phase by sending a route request message using greedy routing. The route request message records all the nodes it traverses. The destination returns a route reply message to the source after it receives the route request message. The route reply message carries the entire source to destination path recorded in the route request message. After learning the path from the route reply message, the source node can send its data packets using source routing and enclose the entire path in the header of each out-going packet. All relaying nodes only forward packets according to the path in these packet, automatically guarantees the consistency of routing. In addtion, as long as the greedy-routing-based path discovery does not create loops while forwaridng the route request messages, loop-freeness is automatically guaranteed by source routing. Hence, based on theorem 1, we have:

Theorem 2: As a on-demand path discovery scheme that is combined with source routing, a greedy routing algorithm R is loop-free and consistent if and only if the routing metric has two properties: mutex and transition.

C. Greedy Routing Combined with Hop-by-hop Routing

Greedy routing can also be used as an on-demand path discovery algorithm and be combined with hop-by-hop routing. When a new flow arrives at a source node, the source firstly initiates a route discovery phase, and all the routing tables in all the necessarily relayed nodes are set up. Then, in the subsequent data packets' transmission phase, all the data packets can follow the state left by the route discovery phase to reach the destination.

There are two different ways to build routing tables according to when they are set up in the route discovery phase. One is that the routing tables are built by the route reply message, named backward building hop-by-hop routing in this paper. Another is that the route request message is responsible for building routing tables, called forward building hop-by-hop routing in terms of this paper.

In backward building hop-by-hop routing, a source sends a route request message using greedy routing before transmitting data packets. The route request message records all the nodes it traverses. The destination returns a route reply message after it receives the route request message. The route reply message carries the entire source to destination path recorded in the route request message, and is relayed along the inverse path discovered by the route request message. In the reply message transmission, all the routing tables of nodes along the path are set up in this process. This process is similar to the building of routing tables in **??**.

Different to flooding based route discovery, the routing tables also can be built by the route request message in greedy routing, which is called forward building hop-by-hop routing in this paper. In this scheme, when a new flow arrives at a source node, the source sends a route request message using greedy routing. The route request message sets up the routing tables in all the nodes it traverses. A reply message coming from the destination is also needed before the source node begin to send data packets to ensure the existence of the path.

While this combination of greedy routing and hop-by-hop routing is similar to the combination of greedy routing and source routing, their requirement for achieving loop-freeness is different, and is also relevant to the ways of building routing tables.

Theorem 3: A greedy routing based backward building hop-by-hop routing protocol R is loop-free if and only if the corresponding routing metric has three properties: mutex, transition and source independence.

Proof:

Sufficient conditions:

We prove the sufficient conditions by contradiction. Assume that the greedy routing algorithm creates a loop $c(u_1, u_2, ..., u_k)$ while forwarding a flow $f_{\langle s_x,d\rangle}$'s traffic based on the routing tables built by the greedy algorithm. All the packets of $f_{\langle s_x,d\rangle}$ that are relayed by u_1 will be forwarded back to u_1 through the following links $\overrightarrow{u_1u_2}$, $\overrightarrow{u_2u_3}$, ..., $\overrightarrow{u_{k-1}u_k}$, $\overrightarrow{u_ku_1}$. Furthermore, $\overrightarrow{u_1u_k} \in E$ is also satisfied due to the symmetry of edges. Based on the principle of greedy routing, for each link in the circle, there must be a flow $f_{\langle s_y,d\rangle}$ satisfy $w(l_{\overrightarrow{u_iu_{i+1}}}, f_{\langle s_y,d\rangle}) \prec \phi$ where $1 \le i < k-1$. Due to the source independence property, for a flow $f_{\langle *,d\rangle}$ denotes any flow destined for node d in F, $w(l_{\overrightarrow{u_iu_{i+1}}}, f_{\langle *,d\rangle}) = w(l_{\overrightarrow{u_iu_{i+1}}}, f_{\langle s_y,d\rangle})$ is always satisfied. Hence, $w(l_{\overrightarrow{u_1u_2}}, f_{\langle *,d\rangle}) \prec \phi$, $w(l_{\overrightarrow{u_2u_3}}, f_{\langle *,d\rangle}) \prec \phi$, ..., $w(l_{\overrightarrow{u_{k-1}u_k}}, f_{\langle *,d\rangle}) \prec \phi$ and $w(l_{\overrightarrow{u_ku_1}}, f_{\langle *,d\rangle}) \prec \phi$ are satisfied. Based on the transition property, we can conclude that $w(l_{\overrightarrow{u_1u_k}}, f_{\langle *,d\rangle}) \prec \phi$ is satisfied. Then, both $w(l_{\overrightarrow{u_1u_k}}, f_{\langle *,d\rangle}) \prec \phi$ and $w(l_{\overrightarrow{u_ku_1}}, f_{\langle *,d\rangle}) \prec \phi$ are satisfied, which contradicts the fact that the routing metric has the mutex property.

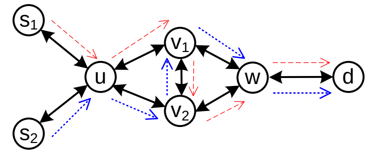

Fig. 2. a simple wireless network

Hence, the loop-freeness compatibility is guaranteed.

Necessary conditions:

Obviously, if the routing metric lacks mutex or transition property, a loop may be create in the process of packets transmission as shown by the proof of theorem 1. Hence, we only need to prove the necessitate of the source independence property using a simple example. Consider the example in Fig. 2. Assume that due to lack of source-independence, for flow $f_{\langle s_1,d\rangle}$, the links satisfy $w(l_{\overrightarrow{s_1u}}, f_{\langle s_1,d\rangle}) \prec \phi$, $w(l_{\overrightarrow{uv_1}}, f_{\langle s_1,d\rangle}) \prec w(l_{\overrightarrow{uv_2}}, f_{\langle s_1,d\rangle}) \prec \phi \prec w(l_{\overrightarrow{us_1}}, f_{\langle s_1,d\rangle})$, $w(l_{\overrightarrow{v_1v_2}}, f_{\langle s_1,d\rangle}) \prec w(l_{\overrightarrow{v_1w}}, f_{\langle s_1,d\rangle}) \prec \phi \prec w(l_{\overrightarrow{v_1u}}, f_{\langle s_1,d\rangle})$, $w(l_{\overrightarrow{v_2w}}, f_{\langle s_1,d\rangle}) \prec \phi \prec w(l_{\overrightarrow{v_2v_1}}, f_{\langle s_1,d\rangle}) \prec w(l_{\overrightarrow{v_2u}}, f_{\langle s_1,d\rangle})$, $w(l_{\overrightarrow{wd}}, f_{\langle s_1,d\rangle}) \prec \phi \prec w(l_{\overrightarrow{wv_1}}, f_{\langle s_1,d\rangle}) \prec w(l_{\overrightarrow{wv_2}}, f_{\langle s_1,d\rangle})$. Hence, the path $s_1uv_1v_2wd$ is discovered by the greedy routing algorithm for flow $f_{\langle s_1,d\rangle}$. While the path $s_2uv_2v_1wd$ is discovered by the same greedy routing algorithm for flow $f_{\langle s_2,d\rangle}$ due to the lack of source independence. Obviously, neither mutex nor transition is disobeyed in this case.

Assume flows $f_{\langle s_1,d\rangle}$ and $f_{\langle s_2,d\rangle}$ occur simultaneously, and the route reply message packet for $f_{\langle s_1,d\rangle}$, $f_{\langle s_2,d\rangle}$ is respectively MP_{s_1d}, MP_{s_2d}. After setting up the routing tables in node v_1 based on packet MP_{s_2d}, this packet is leaving for node v_2, the routing tables in node v_1 is $\langle destination = d, next\_hop = w\rangle$. At the same time, similar process occurs in node v_2 based on packet M_{s_1d}, the routing table in node v_2 is set up as $\langle destination = d, next\_hop = w\rangle$, and packet M_{s_1d} is leaving for node v_1. After packet M_{s_2d} reaches node v_2, the routing table in node v_2 will be renewed as $\langle destination = d, next\_hop = v_1\rangle$, the same thing happens when packet M_{s_1d} arrives at noce v_1, and the routing table in node v_1 is updated as $\langle destination = d, next\_hop = v_2\rangle$. After the route discovery phases of these two flows finished, a loop between nodes v_1 and v_2 is created for both $f_{\langle s_1,d\rangle}$ and $f_{\langle s_2,d\rangle}$. Considering the complications of traffic flows, this case is possible to happen in real networks. ■

However, the requirement for achieving loop-freeness is different if the routing tables is built by the route request message.

Theorem 4: A greedy routing based forward building hop-by-hop routing protocol R is loop-free if and only if the corresponding routing metric has two properties: mutex and transition.

Proof:

We just need to prove the sufficient conditions due to the necessary conditions can be proved with the same examples in theorem 1.

We prove the sufficient conditions by contradiction. Assume that the greedy routing algorithm creates a loop $c(u_1, u_2, ..., u_k)$ while forwarding a flow $f_{\langle s_x,d\rangle}$'s traffic based on the routing tables built by the greedy algorithm in a forward building way.

Without loss of generality, we assume the routing table on node u_i is built by the route request message packet $MP_{s_i d}$ for a traffic flow $f_{\langle s_i,d\rangle}$, and the building time is $t(u_i, MP_{s_i,d})$, where $1 \leq i \leq k$. If $s_1 = s_2 = ... = s_k$, i.e., all the routing tables in the relayed nodes are built by a same traffic flow, the loop of $c(u_1, u_2, ..., u_k)$ cannot be created based on theorem 1. In the following, we will discuss the case where at least one traffic flow has a different source node from others.

For easy description, let $k + 1 = 1$. Considering the overwrite characteristic of route discovery scheme, $t(u_i, MP_{s_i,d}) < t(u_{i+1}, MP_{s_{i+1},d})$ must be satisfied where $1 \leq i \leq k$, or the link $\overrightarrow{u_i u_{i+1}}$ cannot be a component of the path from nodes u_x to d. Hence, both $t(u_1, MP_{s_1,d}) < t(u_k, MP_{s_k,d})$ and $t(u_k, MP_{s_k,d}) < t(u_1, MP_{s_1,d})$ are satisfied, which is a contradiction.

Hence, the loop-freeness compatibility is guaranteed. ■

Although the requirement for achieving loop-freeness is different between backward building hop-by-hop routing and forwarding building hop-by-hop routing, the requirement for achieving consistency is same for both.

Theorem 5: A greedy routing based hop-by-hop routing (backward building or forwarding building) protocol R is consistent if and only if the corresponding routing metric has four properties: mutex, transition, source independence and strict preference.

Proof:

Sufficient conditions:

Due to the strict preference property, only one link is the lightest weight link among any node's out-links for a traffic flow. Hence, coupled with the property of mutex and transition, there is just one determinately loop-free path which can be found by the greedy routing for a traffic flow. Furthermore, the routing tables in the relayed nodes to this destination cannot be changed by other flows due to source independence. This has noting to do with the way of building routing tables. So the consistent compatibility is guaranteed.

Necessary conditions:

i. backward building hop-by-hop routing

Obviously, if the routing metric lacks of mutex or transition or source independence property, the protocol R is not loop-free, let alone consistent. Hence, we only need to prove the necessitate of the strict preference property in detail, we prove it through a simple example. Still considering Fig. 2. On the one hand, for a traffic flow $f_{\langle *,d\rangle}$, the links satisfy $w(l_{\overrightarrow{s_1 u}}, f_{\langle *,d\rangle}) \prec \phi$, $w(l_{\overrightarrow{s_2 u}}, f_{\langle *,d\rangle}) \prec \phi$, and $w(l_{\overrightarrow{uv_1}}, f_{\langle *,d\rangle}) = w(l_{\overrightarrow{uv_2}}, f_{\langle *,d\rangle}) \prec \phi$, where $*$ represents s_1 or s_2. Hence, when traffic flow $f_{\langle s_1,d\rangle}$ occurs, the routing table in node u may be set up as $\langle destination = d, next\_hop = v_1\rangle$. However, in the transmission of flow $f_{\langle s_1,d\rangle}$, flow $f_{\langle s_2,d\rangle}$ occurs, and the routing table in node u maybe be reset as $\langle destination = d, next\_hop = v_2\rangle$ by the greedy routing algorithm for flow $f_{\langle s_2,d\rangle}$, the consistent compatibility is destroyed.

ii. forward building hop-by-hop routing

Obviously, the proof of the necessitate of mutex, transition and the strict preference property is same to above proof. Hence, we only need to prove the necessitate of the source independence property in detail, we prove it using a simple example. Still considering the example in the proof of necessary conditions in theorem 3.

If the routing tables are orderly built for these two flows $f_{\langle s_1,d\rangle}$ and $f_{\langle s_2,d\rangle}$, the routing tables set up by the greedy routing for the former flow will be overwritten by the route discovery of the latter flow.

If flow $f_{\langle s_1,d\rangle}$ firstly occurs, the routing tables in the nodes u, v_1 and v_2 respectively are $\langle destination = d, next\_hop = v_1\rangle$, $\langle destination = d, next\_hop = v_2\rangle$ and $\langle destination = d, next\_hop = w\rangle$, all the data packets of flow $f_{\langle s_1,d\rangle}$ will be relayed along path $s_1uv_1v_2wd$. In flow $f_{\langle s_1,d\rangle}$'s transmission, flow $f_{\langle s_2,d\rangle}$ occurs, after its route discovery phase finished, the routing tables in the nodes u, v_1 and v_2 are respectively renewed as $\langle destination = d, next\_hop = v_2\rangle$, $\langle destination = d, next\_hop = w\rangle$ and $\langle destination = d, next\_hop = v_1\rangle$. In this case, the remainder packets of flow $f_{\langle s_1,d\rangle}$ will be relayed along a new path $s_1uv_2v_1wd$, consistency is destroyed.

■

VI. Illustrations

In this section, we will investigate the existing greedy routings using our analytical results in Section V.

A. Existing Routing Metrics

1) Most Forward within Radius (MFR): In WFR, the packet is forwarded to the neighbor whose progress is maximal among all the neighbors who are closer to the destination than the forwarding node, which tries to minimize the number of hops a packet has to traverse in order to reach the destination [?]. The design the routing algebra in MFR can be specified as a tuple $\langle L, F, w, \preceq\rangle$.

L describes all the links' geographic positions in $G(V, E)$ and $L = \{\langle(x_u, y_u), (x_v, y_v)\rangle | \overrightarrow{uv} \in E\}$, where (x_u, y_u) and (x_v, y_v) are the geographic position of nodes u and v respectively. F is comprised of all flows and $F = \{f_{\langle s,d\rangle} | s, d \in V\}$. The $w(.)$ function is defined as follows:

$$w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) = ||v, d||_2 - ||u, d||_2$$
$$= \sqrt{(x_v - x_d)^2 + (y_v - y_d)^2} - \sqrt{(x_u - x_d)^2 + (y_u - y_d)^2} \quad (2)$$

where $||v, d||_2$ is the Euclidean distance between nodes v and d and $||u, d||_2$ is the Euclidean distance between nodes u and d.

The preference operator $\preceq$ is defined as:

$$w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \preceq w(l_{\overrightarrow{uw}}, f_{\langle s,d\rangle}) \doteq w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \geq w(l_{\overrightarrow{uw}}, f_{\langle s,d\rangle}) \quad (3)$$

where $\geq$ is the greater than or equal to operator for real number.

The threshold ϕ is 0, that is to say, only when $w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) > 0$, can a packet of $f_{\langle s,d\rangle}$ be relayed through link $\overrightarrow{uv}$.

It is easy to check that the routing metric has mutex, transition and source independence properties, but lacks strict preference. Hence, we can conclude as follows:

i: MFR metric can combine with proactive hop-by-hop, reactive hop-by-hop or source routing packet forwarding scheme to produce a routing protocol, the protocol is loop-free in any networks, and also is convergent in a void-free network.

ii: If MFR metric combines with source routing packet forwarding scheme to produce a routing protocol, the protocol is consistent in a void-free network. However, if MFR metric combines with reactive hop-by-hop packet forwarding scheme to produce a routing protocol, the protocol is not consistent in a void-free network.

Same conclusions can be drawn for NFR [?], GRS [**?**], GEAR [?], the best $PRR \times distance$ [?] and NADV [?] metrics through similar analysis.

2) Random Progress Forwarding (RPF): In a wireless network $G(V, E)$, the packet is forwarded to the neighbor whose progress is maximal among all the neighbors who are closer to the destination than the forwarding node, but the progress is not based on Euclidean distance, but the projected distance [**?**].

The specifications of L, F, $\preceq$ and ϕ are same to MFR, the only difference is the $w(.)$ which is defined as follows:

$$w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) = ||u, v||_2 \cos\alpha = \sqrt{(x_v - x_u)^2 + (y_v - y_u)^2} \cos\alpha \quad (4)$$

where α is the angle from the edge $\overrightarrow{ud}$ to $\overrightarrow{uv}$.

It is very easy to illustrate an example to satisfy both $w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \prec \phi$ and $w(l_{\overrightarrow{vu}}, f_{\langle s,d\rangle}) \prec \phi$. So the algebra lacks of the mutex property, hence, we can conclude as follows:

If RPF metric combines with proactive hop-by-hop, reactive hop-by-hop or source routing packet forwarding scheme to produce a routing protocol, the protocol is not loop-free. That is to say, RPF is not a usable metric whatever the packet forwarding scheme is.

3) Line Progress Forwarding (LPF): LPF metric also is based on the projected distance, but it is a usable metric for some packet forwarding scheme. The only difference between LPF and RPF lies where a link is projected. For LPF metric, the definition of $w(.)$ is defined as follows:

$$w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) = ||u, v||_2 \cos\alpha = \sqrt{(x_v - x_u)^2 + (y_v - y_u)^2} \cos\alpha \quad (5)$$

where α is the angle from the edge $\overrightarrow{sd}$ to $\overrightarrow{uv}$.

It is easy to know that the algebra is of the properties of mutex and transition, but lacks of source independence and strict preference, hence, we can conclude as follows:

i: LPF metric can combine with proactive hop-by-hop or source routing packet forwarding scheme to produce a routing protocol, the protocol is loop-free in any networks, and also is convergent in a void-free network.

ii: if LPF metirc combines with reactive hop-by-hop packet forwarding scheme to produce a routing protocol, the protocol is not loop-free. Hence, this combination is not a usable protocol for any network.

iii: If LPF metric combines with source routing packet forwarding scheme to produce a routing protocol, the protocol is consistent in a void-free network.

B. Consistency for proactive hop-by-hop routing

The lack of strict preference also is called a tie phenomenon in terms of some paper. The tie must be broken to guarantee the consistency of proactive hop-by-hop routing.

In geographic routing, the tie can be broken by a deterministic rule based on the geographic positions of its neighbors . In detail, if $l_{\overrightarrow{uv}}, l_{\overrightarrow{uw}} \in L$ and $w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) = w(l_{\overrightarrow{uw}}, f_{\langle s,d\rangle})$ for a flow from the source node s to the destination node d, the preference order is defined as follows:

$$\begin{aligned} & w'(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \prec w'(l_{\overrightarrow{uw}}, f_{\langle s,d\rangle}) \\ \Leftrightarrow\ & w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \prec w(l_{\overrightarrow{uw}}, f_{\langle s,d\rangle}) \ \ || \\ & w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) = w(l_{\overrightarrow{uw}}, f\langle s,d\rangle) \\ & \qquad \&\& \ \ x_v < x_w \ \ || \\ & w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) = w(l_{\overrightarrow{uw}}, f_{\langle s,d\rangle}) \\ & \qquad \&\& \ \ x_v = x_w \ \ \&\& \ \ y_v < y_w \end{aligned} \quad (6)$$

VII. Conclusion

In this paper, we firstly discuss the necessity of building a route in on demand manner in geographic routing, then introduce the concept of loop-freeness, convergency and consistence to investigate the compatibility between routing metric, greedy routing and packet forwarding scheme. Based on our proposed algebra, the compatibilities are mathematically analyzed for all possible combinations of routing metric, greedy routing and packet forwarding scheme, and the necessary and sufficient conditions for the loop-free, convergent and consistent compatibilities of these combinations are derived. Furthermore, the applications of these conditions are illustrated by some concrete examples. Our work provides essential criterions for evaluating and designing routing system in geographic routing. In our future work, we would like to extend our algebra to cover the recovery schemes in geographic routing.