Routing Metric Designs for Greedy, Face and Combined-Greedy-Face Routing

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***Abstract*—Different geographic routing protocols have different requirements on routing metric designs to ensure proper operation. Combining a wrong type of routing metric with a geographic routing protocol may produce unexpected results, such as geographic routing loops and unreachable nodes. In this paper, we propose a novel routing algebra system to investigate the compatibilities between routing metrics and three geographic routing protocols including greedy, face and combined-greedy-face routing. Four important algebraic properties, respectively named odd symmetry, transitivity, source independence and local minimum freeness, are defined in this algebra system. Based on these algebraic properties, the necessary and sufficient conditions for loop-free and delivery guaranteed routing are derived when greedy, face and combined-greedy-face routing serve as packet forwarding schemes or as path discovery algorithms respectively. Our work provides essential criterions for evaluating and designing geographic routing protocols.**

***Index Terms*—geographic routing, routing algebra, loop-freeness, delivery guarantee.**

I. Introduction

Geographic routing, also called position-based routing, has received significant attention since it was originally proposed in [1] [2] in the 1980s. Compared with topology-based routing [3]–[7], geographic routing has two unique benefits, especially for large-scale, highly dynamic and energy-constraint wireless networks. First, in geographic routing, a forwarding node transmits a packet only based on the positions of the destination node and its one-hop neighbors. The size of routing tables and the overhead of routing control messages are minimal. Second, the convergence time for geographic routing can be neglected. This is very attractive for highly dynamic wireless networks since it is extremely difficult to design fast-convergent topology-based routing protocols. These two benefits coupled with the progress on Global Positioning System (GPS) and self-configuring localization mechanisms [8]–[10] have promoted geographic routing as a promising solution for many wireless networks.

The dominant design of geographic routing, which is also the focus of this paper, is Combined-Greedy-Face (CGF) routing, which is a combination of two routing schemes: greedy routing and face routing. Greedy routing tries to bring the message closer to the destination in each step using only local information. Thus, each node forwards the message to the neighbor that is most suitable from a local point of view. Face routing helps to recover from dead end situation, where greedy routing hits a void and cannot find a suitable neighbor to the destination. Various methods can be used to define the most suitable neighbor for greedy routing, the time to switch between greedy routing and face routing, and the faces to be traversed by face routing. These methods are named *routing metrics* in this paper. Besides location information, many performance-affecting characteristics have been taken into account in the design of routing metrics for CGF routing recently. The performance of wireless networks can improve a lot due to these more complicated routing metrics [11]–[14]. However, the design of routing metrics cannot be arbitrary due to potential incompatibility with CGF routing, which may greatly degrade network performance and even create routing loops. For example, it has been pointed out in [15] that a projected-distance-based routing metric can create routing loops between two intermediate connected nodes in CGF routing. Unfortunately, despite the potential serious impact of routing metric designs on greedy, face and CGF routing, systematic analysis of such impact is still lacking in the current literature.

The goal of this paper, hence, is to fill in this void and present an in-depth analysis about greedy, face and CGF routing's compatibility constraints on the design of routing metrics. This paper will focus on two aspects of the compatibility: loop-freeness and delivery guarantee. Loop-freeness refers that a routing protocol should never create any forwarding loop in any network topology at its stable state, and a forwarding loop indicates that a packet will be endlessly forwarded among several nodes which form a circle until the packet will be forcedly dropped due to lifetime is expired. Delivery guarantee refers that a packet initiated from any source node must be relayed to its destination by a routing protocol as long as there does exist a path between the source and the destination.

To realize this goal, this paper proposes a novel routing algebra for greedy, face and CGF routing analysis. Using this algebra, four important routing algebraic properties, including odd symmetry, transitivity, source independence and local minimum freeness, are identified. These algebraic properties determine the compatibility between routing metrics and greedy, face and CGF routing.

The remainder of this paper is organized as follows. Section II briefly introduces greedy, face and CGF routing. Section III establishes network model and proposes a routing algebra. Section IV systematically analyzes the requirements for realizing compatibilities when greedy, face and CGF routing are used as packet forwarding schemes. If greedy, face and CGF routing are regarded as path discovery schemes, the requirements for realizing compatibilities when combined with source routing and hop-by-hop routing are discussed in section V and VI respectively. Section VII illustrates the application of our theoretical resluts. Section VIII concludes our work and discusses the future directions.

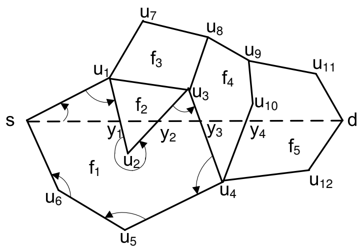

Fig. 1. Most Forward Face Routing

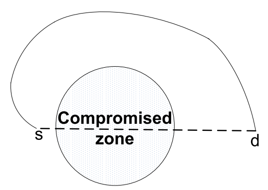

Fig. 2. Baseline for MFF Routing

II. Overview of Greedy, Face and CGF Routing

In this section, we first briefly discuss the design of greedy, face and CGF routing, and then discuss their roles in a routing system.

A. The Design of of Greedy, Face and CGF Routing

At each relaying node, greedy routing selects the locally optimal next hop from a set of candidate neighboring nodes, in the hope that it can approximate global optimal path. Although greedy routing is simple, it does not provide delivery guarantee.

Different from greedy routing, face routing itself can be delivery guaranteed in a planar graph constructed using various existing methods (see a survey in [16]). There are several different variants of face routing which mainly differ in the decision about when and which face to traverse next. Due to space limitation, we just discuss a simple face routing scheme in this paper. Others can be analyzed in a similar way. In this routing scheme, the "best" neighboring face to the destination will be the next to traverse after the current face is completely explored. We called this routing scheme as **M**ost **F**orwarding **F**ace (MFF) routing. The face switching process of MFF can be illustrated by the following example.

Assume there is a flow $f_{\langle s,d\rangle}$ that needs to be delivered in a wireless network $G(V,E)$. The reduced planar graph from $G(V,E)$ is illustrated as Fig. 1. As a complete routing scheme, MFF routing starts at node s, and ends at node d. The dashed line $\widetilde{sd}$ is the baseline which decides the faces to be traversed by MFF routing. Face f_1 is the first intersected face by $\widetilde{sd}$. There are 3 intersection points y_1, y_2 and y_3 between $\widetilde{sd}$ and the edges of face f_1. According to the well-known right hand rule, MFF routing first traverses all the edges of face f_1 in clockwise order $\overrightarrow{su_1}$, $\overrightarrow{u_1u_2}$, ..., $\overrightarrow{u_5u_6}$ and $\overrightarrow{u_6s}$, and then finds y_3 is the best intersection point to the destination d based on a routing metric definition. Hence, f_4 becomes the next explored face by MFF routing, and $\overrightarrow{u_3u_8}$ is the first traversed edge in f_4.

MFF routing is similar to compass routing II [17]. The only difference is that the baseline joining MFF routing's starting node and destination may not be a straight line segment. For instance, consider a wireless sensor network with a compromised zone as illustrated in Fig. 2. It is undesirable to route flows into the compromised zone. Hence, the solid curve is much better than the dashed straight line as a baseline for MFF routing, since the chance of passing nodes located in the compromised zone is much higher if the dashed straight line is used as a baseline. How to choose between all the possible baselines is the design issue of face routing's metrics.

As a packet forwarding scheme, greedy routing is simple but provides no packet delivery guarantee. On the other hand, MFF routing provides packet delivery guarantee but is complicated and may create very inefficient paths. Hence, greedy routing and MFF routing often are combined together to form CGF routing where MFF routing is used as a recovery scheme for greedy routing and starts at dead ends for greedy routing. Note that MFF routing's routing metric may be different from greedy routing.

B. The Roles of Greedy, Face and CGF Routing

The compatibility between greedy, face and CGF routing schemes and routing metrics depends on how these geographic routing schemes are used in a routing system. There are two fundamentally different ways of applying these routing schemes.

The first approach is to treat these routing schemes as packet forwarding schemes so that for every arriving packet, a relaying node locally decides and forwards the packet to this next-hop neighbor. The benefit of this approach is that a node does not need to keep any routing state regarding destinations or flows. On the other hand, since no routing state is kept in each node, techniques [18] that can shorten pure CGF routing's paths cannot be used.

In the second approach, these geographic routing schemes can be used as on-demand route discovery mechanisms and be combined with different traditional packet forwarding schemes to form various routing systems. By maintaining a small amount of routing states, various techniques [18] can be used to optimize pure geographic routing paths.

Two widely used packet forwarding schemes can be combined with geographic routing. The first scheme is source routing, where a source node caches paths discovered by geographic routing and includes path information in its packet headers. Intermediate nodes then relay the flow based on the path information carried in forwarded packets. The second scheme is hop-by-hop routing, where soft states about the next hop is set along the path discovered by geographic routing. A source node only appends the destination address of a flow in its packet headers, and an intermediate node relays the flow based on its local soft states about the next-hop to reach the destination node. For both packet forwarding schemes, while a small number of soft states of discovered paths have to be maintained in either source nodes or relaying nodes, the benefit of reduced per-packet computation and message overhead as well as the enabling of path optimization techniques can be significant.

III. Network Model and Routing Algebra

Greedy, face and CGF routing schemes have different compatibility requirements on routing metric design and their requirements also depend on their roles in a routing system. In this section, we introduce our network model and a novel routing algebra. They will be used to systematically analyze these compatibility requirements in the remainder of this paper.

A. Network Model

A wireless network is modeled as a connected directed graph $G(V,E)$ with cardinalities $|V|$ and $|E|$. V is the set of vertices, and every vertex in the graph represents a node in the wireless network. Every link in the wireless network is mapped to a directed edge between the corresponding vertices.

If nodes u and v can communicate directly, i.e. one is in another's physical radio range, there are two links with inverse direction between nodes u and v. The link $\overrightarrow{uv}$ is responsible for transmitting packets from node u to v, and the link $\overrightarrow{vu}$ is responsible for the inverse packets transmission. The link $\overrightarrow{uv}$ is called node u's out-going link and the link $\overrightarrow{vu}$ is called node u's in-coming link. In this paper, we assume edge-symmetrical graph where there must be an edge $\overrightarrow{uv}$ from vertex u to v if there is an edge $\overrightarrow{vu}$ from vertex v to u.

Vertex v is named a neighbor of vertex u if there is an edge $\overrightarrow{uv}$ from the vertex u to v. The neighbor set $N(u)$ of vertex u is the collection of its neighbor vertices.

A path from the source vertex u_1 to the destination u_n can be represented by a sequence of vertices $u_1u_2...u_n$ where there is an directed edge from vertex u_k to u_{k+1} for $1 \le k \le n-1$. A path is simple if all vertices are distinct. A cycle is a path where all vertices are distinct except for the source vertex and the destination vertex.

B. Routing Algebra

The routing algebra, based on the connected directed graph, is a 4-tuple $\langle L, F, w, \preceq \rangle$, where

L : a set of labels,

F : a set of traffic flows,

w : a link weight function,

$\preceq$: an order relationship.

A label $l_{\overrightarrow{uv}}$ in L describes the characteristics of the link $\overrightarrow{uv}$, such as the geographical positions of nodes u and v, and packet loss ratio, energy consumption and the channel frequency. Each link in the wireless network is coupled with a label. Hence, the set L captures all the communication characteristics of the network.

Furthermore, for face routing, to facilitate the representation of the conditions for face switching and greedy routing resuming, virtual links between locations that do not have nodes are introduced in the algebra. A virtual link $\overrightarrow{xy}$ between any points x and y in the network area is associated with a label $l_{\overrightarrow{xy}}$ which captures the characteristics of its end points, such as their distance to the destination or compromised zones. The label of any virtual link is also included in the set L.

A flow $f_{\langle s,d \rangle}$ in F represents that there is traffic that needs to be delivered from node s to node d. The cardinality of F is less than or equal to $|V| \times (|V-1|)$.

$w(\cdot)$ is a function that calculates the weight of a link for different flows. $w(\cdot)$ has two variables. One is the label of a link and another is the flow. For example, for a flow $f_{\langle s,d \rangle}$, the weight of node u's out-going link $\overrightarrow{uv}$ is $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle})$, where $l_{\overrightarrow{uv}} \in L$ denotes the label of link $\overrightarrow{uv}$, and $f_{\langle s,d \rangle} \in F$ indicates the existence of traffic demand between the source node s and the destination node d.

$\preceq$ is used to compare all the out-going links of a forwarding node based on $w(\cdot)$, so that the forwarded packets can be relayed through the lightest weight (i.e. best) link. If $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) \preceq w(l_{\overrightarrow{ux}}, f_{\langle s,d \rangle})$, we say that link $\overrightarrow{uv}$ is lighter or equal to (i.e. not worse than) link $\overrightarrow{ux}$ for flow $f_{\langle s,d \rangle}$.

Note that for greedy routing, not all the out-going links can be regarded as candidates for packet relaying. It is only those links who are strictly lighter than a threshold value ϕ can forward packets. ϕ is also tightly related to the switch point between greedy routing and face routing.

Using the above novel routing algebra, we can discuss compatible routing metric design based on the following properties.

Definition 1: ***Odd symmetry:*** $\forall\ l_{\overrightarrow{uv}}, l_{\overrightarrow{vu}} \in L$, $f_{\langle s,d \rangle} \in F$, $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) \prec \phi$ implies $w(l_{\overrightarrow{vu}}, f_{\langle s,d \rangle}) \succeq \phi$.

Definition 2: ***Transitivity:*** $\forall\ l_{\overrightarrow{u_1u_2}}, l_{\overrightarrow{u_2u_3}}, ..., l_{\overrightarrow{u_{k-1}u_k}} \in L$, $f_{\langle s,d \rangle} \in F$, $w(l_{\overrightarrow{u_1u_2}}, f_{\langle s,d \rangle}) \prec \phi$, $w(l_{\overrightarrow{u_2u_3}}, f_{\langle s,d \rangle}) \prec \phi$, ..., and $w(l_{\overrightarrow{u_{k-1}u_k}}, f_{\langle s,d \rangle}) \prec \phi$ imply $w(l_{\overrightarrow{u_1u_k}}, f_{\langle s,d \rangle}) \prec \phi$.

Definition 3: ***Source independence:*** $\forall\ f_{\langle s_1,d \rangle}, f_{\langle s_2,d \rangle} \in F$, $l_{\overrightarrow{uv}} \in L$, $w(l_{\overrightarrow{uv}}, f_{\langle s_1,d \rangle}) = w(l_{\overrightarrow{uv}}, f_{\langle s_2,d \rangle})$ is always satisfied.

Definition 4: ***Local minimum free:*** $\forall\ f_{\langle s,d \rangle} \in F$, $l_{\overrightarrow{ud}} \in L$, there does not exist a local minimal point for $w(\cdot)$ except the global minimal point d. Here node u may or may not be the node s. Note that x is a local minimum point of $w(\cdot)$ if there exists some $\epsilon > 0$ such that $\forall x'$ in $|x - x'| < \epsilon$, we have $w(l_{\overrightarrow{ux}}, f_{\langle s,d \rangle}) \preceq w(l_{\overrightarrow{ux'}}, f_{\langle s,d \rangle})$.

The odd symmetry property states that for two links with opposite directions, only one of them can be in the candidate set of greedy routing. The transitivity property ensures that the progress to the destination is monotonic. The source independence property ensures that the selection of the next-hop for a packet is irrelevant to its source. The local minimum free property ensures the existence of monotonic progress between any two nodes along a baseline.

It is worth noting that our routing algebra is different from Sobriho's routing algebra [19] [20], which focuses on analyzing the compatibility issues for link-state, distance vector and path vector protocols [19]–[23]. Geographic routing are substantially different from these routing protocols and, hence, cannot be captured by Sobrinho's routing algebra.

IV. Requirements When Greedy, Face or CGF Routing Used as a Packet Forwarding Scheme

Odd symmetry, transitivity, source independence and local minimum freeness are important algebraic properties that determine compatibilities between geographic routing and routing metrics. In this section, we systematically analyze the requirements for realizing compatibilities when greedy, face or CGF routing is used as a packet forwarding scheme. The requirements for realizing compatibilities when these routing schemes are used as path discovery algorithms will be discussed in sections V and VI.

Without loss of generality, our discussion of face routing is based on the representative MFF routing, whose design is introduced in section II-A.

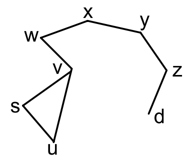

Fig. 3. Greedy Routing

A. Greedy Routing's Compatibility Requirement

Theorem 1: As a packet forwarding scheme, a greedy routing algorithm GR is loop-free if and only if the routing metric has two properties: odd symmetry and transitivity.

Proof:

Sufficient conditions:

We prove the sufficient conditions by contradiction. Assume that when a flow $f_{\langle s,d \rangle}$ initiated by node s is routed to its destination node d by GR, a loop consisting of links $\overrightarrow{u_1u_2}, \overrightarrow{u_2u_3}, ..., \overrightarrow{u_{k-1}u_k}$ and $\overrightarrow{u_ku_1}$ $(k \geq 2)$ is created in the packet forwarding process.

Based on the principle of greedy routing, for flow $f_{\langle s,d \rangle}$, the weight of each link in the loop is strictly less than ϕ, i.e. $w^g(l_{\overrightarrow{u_1u_2}}, f_{\langle s,d \rangle}) \prec \phi$, $w^g(l_{\overrightarrow{u_2u_3}}, f_{\langle s,d \rangle}) \prec \phi$, ..., $w^g(l_{\overrightarrow{u_{k-1}u_k}}, f_{\langle s,d \rangle}) \prec \phi$ and $w^g(l_{\overrightarrow{u_ku_1}}, f_{\langle s,d \rangle}) \prec \phi$. Based on the transitivity property, we can conclude that $w^g(l_{\overrightarrow{u_1u_k}}, f_{\langle s,d \rangle}) \prec \phi$ is satisfied. Then, both $w^g(l_{\overrightarrow{u_1u_k}}, f_{\langle s,d \rangle}) \prec \phi$ and $w^g(l_{\overrightarrow{u_ku_1}}, f_{\langle s,d \rangle}) \prec \phi$ are satisfied, which contradicts the fact that the routing metric has the odd symmetry property. Hence, the loop cannot be created by greedy routing.

Necessary conditions:

We prove the necessary conditions by simple examples.

If the routing metric does not have the odd symmetry property, a routing loop may be created between two relaying nodes by GR. Consider a simple network in Fig. 3, Due to the lack of the odd symmetry property, for a flow $f_{\langle s,d \rangle}$, both $w^g(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) \prec \phi$ and $w^g(l_{\overrightarrow{vu}}, f_{\langle s,d \rangle}) \prec \phi$ are satisfied. In addition, $\overrightarrow{uv}$ and $\overrightarrow{vu}$ are respectively the lightest out-going links for nodes u and v based on this routing metric. According to greedy routing, once a packet of flow $f_{\langle s,d \rangle}$ reaches node u, node u will send it to node v since $\overrightarrow{uv}$ is the local optimal and $w^g(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) \prec \phi$. For the same reason, each packet of $f_{\langle s,d \rangle}$ at node v will be immediately forwarded to node u. Hence, a routing loop is formed between nodes u and v for flow $f_{\langle s,d \rangle}$.

If the routing metric has the odd symmetry property, but lacks the transitivity property, routing loops consisting multiple nodes can also be created by GR. Still considering Fig. 3, assume that the links satisfy the following inequalities: $w^g(l_{\overrightarrow{su}}, f_{\langle s,d \rangle}) \prec \phi \prec w^g(l_{\overrightarrow{sv}}, f_{\langle s,d \rangle})$, $w^g(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) \prec \phi \prec w^g(l_{\overrightarrow{us}}, f_{\langle s,d \rangle})$ and $w^g(l_{\overrightarrow{vs}}, f_{\langle s,d \rangle}) \prec w^g(l_{\overrightarrow{vw}}, f_{\langle s,d \rangle}) \prec \phi \prec w^g(l_{\overrightarrow{vu}}, f_{\langle s,d \rangle})$. Obviously, the odd symmetry property is satisfied but there is a lack of the transitivity property. Hence, for flow $f_{\langle s,d \rangle}$, GR creates a routing loop comprised of nodes s, u and v. ∎

B. MFF Routing's Compatibility Requirement

In order to discuss MFF routing with our algebra, we need formally define the baseline of MFF routing, and then build a relationship between the baseline and the corresponding algebra property.

Definition 5: ***baseline:*** $\forall$ $f_{\langle s,d \rangle} \in F$, a directed curve $\widetilde{sd}$ from s to d is called a baseline if and only if it has the following property: for any two segments $\widetilde{su}$ and $\widetilde{sv}$ of $\widetilde{sd}$ (i.e. $\forall$ $\widetilde{su}, \widetilde{sv} \subseteq \widetilde{sd}$), $w^f(l_{\overrightarrow{sv}}, f_{\langle s,d \rangle}) \prec w^f(l_{\overrightarrow{su}}, f_{\langle s,d \rangle})$ is satisfied if and only if $\widetilde{su} \subset \widetilde{sv}$. Essentially, a baseline is a monotonically decreasing curve from node s to node d.

Lemma 1: A baseline exists if and only if the routing metric has the local minimum free property.

Proof:

Sufficient condition:

The gradient of routing metric $w^f(\cdot)$ essentially constructs a vector field in the plane. If the routing metric $w^f(\cdot)$ is local minimum free with a global minimum at d, for any node s in the plane, its field line can serve as the baseline to node d and is monotonically decreasing.

Necessary condition:

If the routing metric has a local minimum point $y \neq d$ for a flow $f_{\langle s,d \rangle}$, then there exists a disk around y such that for any node y' in the disk, $w^f(l_{\overrightarrow{sy'}}, f_{\langle s,d \rangle}) \succeq w^f(l_{\overrightarrow{sy}}, f_{\langle s,d \rangle})$. Hence, any directed curve that traverses y and goes to d cannot be monotonically decreasing in terms of $w^f(\cdot)$. ∎

To prove the sufficient condition for the delivery guarantee of MFF routing, we introduce the following lemma which has been proved in [16].

Lemma 2: Let f_i be a face in a planar graph $G(V, E)$ and let d be a node located in the interior of f_i. There exists no path in $G(V, E)$ which connects d with a node on the face boundary.

In order to ensure loop-freeness and guaranteed delivery for MFF routing, the routing metric must have an algebra property described in the following theorem.

Theorem 2: As a packet forwarding scheme, a MFF routing algorithm FR is loop-free and delivery guaranteed if and only if the routing metric has the local minimum free property.

Proof:

Sufficient conditions:

Consider that a flow $f_{\langle s,d \rangle}$ initiated by node s is routed to its destination node d by FR in a planar graph $G(V, E)$, and there exists a path from s to d in $G(V, E)$.

Assume FR starts to traverse a face f_i from an edge e that intersects with $\widetilde{sd}$ at point t. It is obvious that there is a path from t to d since s has a path to both t and d.

We next prove that FR can always find an edge that intersects $\widetilde{sd}$ in a point $p \neq t$ to switch to the next face after FR fully explores face f_i. Note that based on the right hand rule, f_i must contain a segment of $\widetilde{td} \subset \widetilde{sd}$, which implies that $\widetilde{sd}$ traverses into face f_i at point t. Suppose that $\widetilde{td} \subset \widetilde{sd}$ does not intersect f_i's boundary at any other point. Then, d must be located in the interior of f_i. By lemma 2, t and d cannot reach each other and, hence, contradicts the fact that there exists a path from t to d. Therefore, $\widetilde{td}$ must intersect with f_i on other points besides t. Denote the lightest of these points based on $w^f(\cdot)$ as p. it is obvious that $\widetilde{st} \subset \widetilde{sp}$. Due to the monotonically decreasing nature of the baseline, we have $w^f(l_{\overrightarrow{sp}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{st}}, f_{\langle s,d\rangle})$, and p is the intersection point that triggers FR to switch to the next face.

Last, we prove all the explored face will never be traversed again. Suppose a face f_i is explored for a second time after FR exploring a series of faces f_i, f_{i+1}, ..., f_{i+m}, f_i. Denote the intersection points between $\widetilde{sd}$ and these faces that trigger the switch from one face to the next face as $x_0, x_1, x_2, ..., x_m$. The edges corresponding to these intersection points are e_0, e_1, ..., e_m. Based on our second step in proof, we have $w^f(l_{\overrightarrow{sx_1}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sx_0}}, f_{\langle s,d\rangle})$, $w^f(l_{\overrightarrow{sx_2}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sx_1}}, f_{\langle s,d\rangle})$, ..., $w^f(l_{\overrightarrow{sx_m}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sx_{m-1}}}, f_{\langle s,d\rangle})$. Due to the definition of baseline, $w^f(l_{\overrightarrow{sv}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{su}}, f_{\langle s,d\rangle})$ implies $\widetilde{su} \subset \widetilde{sv} \subseteq \widetilde{sd}$. Hence, we have $\widetilde{sx_0} \subset \widetilde{sx_1} \subset ... \subset \widetilde{sx_m}$, which indicates $w^f(l_{\overrightarrow{sx_m}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sx_0}}, f_{\langle s,d\rangle})$. Note that both e_m and e_0 are the edges of f_i that intersect $\widetilde{sd}$. Since FR switches from f_i to f_{i+1} at e_0, we have $w^f(l_{\overrightarrow{sx_0}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sx_m}}, f_{\langle s,d\rangle})$, which contradicts the fact $w^f(l_{\overrightarrow{sx_m}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sx_0}}, f_{\langle s,d\rangle})$. Hence, a face is only explored once in MFF.

Since we have a finite number faces, face exploration will eventually reach an edge e which intersects $\widetilde{sd}$ at point d.

Necessary conditions:

We prove the necessary conditions through simple examples.

Assume the routing metric lacks the local minimum free property. That is to say, for any curve $\widetilde{sd}$, either there exists a local minimum point $x \in \widetilde{uv}$ ($\widetilde{uv} \subseteq \widetilde{sd}, x \neq u \neq v$) such that both $w^f(l_{\overrightarrow{sx}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{su}}, f_{\langle s,d\rangle})$ and $w^f(l_{\overrightarrow{sx}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sv}}, f_{\langle s,d\rangle})$ are satisfied, or $\widetilde{sd}$ is a non-decreasing curve.

In the first case, consider a simple network in Fig. 1. where point y_2 satisfies $w^f(l_{\overrightarrow{sy_2}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sy_1}}, f_{\langle s,d\rangle})$, $w^f(l_{\overrightarrow{sy_2}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sy_3}}, f_{\langle s,d\rangle})$. Based on the principle of MFF routing algorithm, face f_2 is the next traversed face after f_1 is fully explored. However, after face f_2 is fully traversed, face f_1 will be traversed again. Hence, all the packets are endlessly relayed among the edges of face f_1 and f_2 and a routing loop is created.

In the second case, still consider Fig. 1 as an example, $w^f(l_{\overrightarrow{ss}}, f_{\langle s,d\rangle}) \prec w^f(l_{\overrightarrow{sx}}, f_{\langle s,d\rangle})$ is always satisfied for any point $x \in \widetilde{sd}$ ($x \neq s$). FR algorithm cannot find the next face to traverse after f_1 is fully explored. Hence it has to incorrectly end although the path does exist. ∎

C. CGF Routing's Compatibility Requirement

In CGF routing, MFF routing will be adopted when greedy routing hits a dead end node. MFF routing phase can ends at the destination node or any node that is closer to the destination than the dead end node based on greedy routing metric. That is to say, starting from the dead end node u of greedy routing, MFF routing ends at finding a node v that the virtual link $l_{\overrightarrow{uv}}$ satisfies $w^g(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \prec \phi$, where $w^g(\cdot)$ is the used greedy routing metric. When CGF routing is used as a packet forwarding scheme algorithm, we have:

Theorem 3: As a packet forwarding scheme, a CGF routing algorithm R is delivery guaranteed if and only if the routing metric for its greedy algorithm GR has odd symmetry and transitivity properties, and the routing metric for its MFF routing algorithm FR has the local minimum free property.

Proof:

Sufficient conditions:

Consider a flow $f_{\langle s,d\rangle}$ initiated by node s is routed to its destination node d by the CGF routing algorithm R. At the source node s, R's greedy algorithm GR is adopted to find the next relaying node u_1. Due to the odd symmetry and transitivity properties of routing metric for greedy routing, no routing loops is created in the greedy routing process. Hence, greedy routing either ends at the destination node d or ends at some node u_m which is a dead end after passing nodes s, u_1, u_2,..., u_{m-1}. If the former happens, CGF finds the loop-free path to d. Hence, we just need to discuss how to ensure packet delivery guarantee under the latter case.

For easy description, let $u_0 = s$. Since node u_m is the first dead end node, $w^g(l_{\overrightarrow{u_i u_{i+1}}}, f_{\langle s,d\rangle}) \prec \phi$ $(0 \leq i \leq m-1)$ is satisfied. MFF routing algorithm FR will be adopted at node u_m. Due to the local minimum free property of the routing metric for MFF routing, there must exist at least a curve $\widetilde{u_m d}$ which has no local minimum points and d is the global minimum point. Hence, if greedy routing algorithm GR is not adopted again, MFF routing FR routes packets to the destination node d from u_m based on theorem 2. In the other case, when MFF routing FR arrives at the first node v_n such that $w^g(l_{\overrightarrow{u_m v_n}}, f_{\langle s,d\rangle}) \prec \phi$, greedy routing algorithm GR is adopted again at node v_n.

Now we will prove that node v_n is different than anyone of u_0, u_1, u_2, ..., u_{m-1} and u_m by contradiction. Assuming u_i and v_n is the same node where $0 \leq i \leq m$. Based on the principle of greedy algorithm, $w^g(l_{\overrightarrow{u_i u_{i+1}}}, f_{\langle s,d\rangle}) \prec \phi$, $w^g(l_{\overrightarrow{u_{i+1} u_{i+2}}}, f_{\langle s,d\rangle}) \prec \phi$, ..., $w^g(l_{\overrightarrow{u_{m-1} u_m}}, f_{\langle s,d\rangle}) \prec \phi$ are satisfied. Hence, $w^g(l_{\overrightarrow{u_i u_m}}, f_{\langle s,d\rangle}) \prec \phi$ is satisfied due to the transitivity property. Based on the principle for switching MFF to greedy routing, $w^g(l_{\overrightarrow{u_m v_n}}, f_{\langle s,d\rangle}) \prec \phi$ is satisfied. Because node u_i is the same node with v_n, both $w^g(l_{\overrightarrow{v_n u_m}}, f_{\langle s,d\rangle}) \prec \phi$ and $w^g(l_{\overrightarrow{u_m v_n}}, f_{\langle s,d\rangle}) \prec \phi$ are satisfied, which contradicts the fact that routing metric for greedy routing algorithm has the odd symmetry property. Hence, node v_n is different from u_0, u_1, u_2, ..., u_{m-1} and u_m.

Hence, a routing loop cannot be created by the alternation between greedy routing and MFF routing. Since the number of nodes in the wireless network $G(V, E)$ is finite, eventually CGF routing algorithm R can delivery packets to d.

Necessary conditions:

The necessary conditions is obvious. On the one hand, if the routing metric for greedy routing lacks odd symmetry or transitivity property, a loop may be created in greedy routing process as shown by the proof of theorem 1. On the other hand, if the source node s is a dead end for greedy routing algorithm GR, MFF routing algorithm will be adopted immediately at the source node. Hence, if the routing metric for MFF routing lacks the local minimal free property, MFF routing may create a routing loop or incorrectly end as illustrated in the proof of theorem 2 before finding a node to enter the greedy routing algorithm GR. ■

V. Requirements When Greedy, Face or CGF Routing Combined with Source Routing

Greedy routing, MFF routing or CGF routing can act as a reactive path discovery algorithm and be combined with source routing as follows. A source node initiates one of these three routing algorithms on-demand to deliver a route request message to a destination node. When the destination receives the route request message that records the path it traverses, the destination can return the path information to the source node. Optimization techniques can be used at the source node to convert the returned path to simple path by eliminating cycles in the returned path. As long as greedy routing, MFF routing and CGF routing can correctly forward the route request message to the destination, their combination with source routing can properly operate. Hence, when greedy, MFF or CGF routing is combined with source routing, their compatibility requirements are the same as when they are used as packet forwarding schemes. These compatibility requirements can be formally expressed as:

Theorem 4: As an on-demand path discovery scheme that is combined with source routing, a greedy routing algorithm GR is loop-free if and only if the routing metric has odd symmetry and transitivity properties.

Theorem 5: As an on-demand path discovery scheme that is combined with source routing, a MFF routing algorithm FR is loop-free and delivery guaranteed if and only if the routing metric has the local minimum free property.

Theorem 6: As an on-demand path discovery scheme that is combined with source routing, a CGF routing algorithm R is loop-free and packet delivery guaranteed if and only if the routing metric for its greedy algorithm GR has odd symmetry and transitivity properties, and the routing metric for its MFF routing algorithm FR has the local minimum free property.

VI. Requirements When Greedy, Face or CGF Routing Combined with Hop-by-hop Scheme

Greedy routing, MFF routing and CGF routing can also be used as on-demand path discovery algorithms and be combined with hop-by-hop routing. When a new flow arrives at a source node, the source firstly initiates a route discovery phase to set up the routing states along the path to the destination. Then, in the subsequent data packets' transmission phase, all the data packets can follow the routing states left by the route discovery phase to reach the destination.

There are two different schemes to build routing states which are named backward building and forward building respectively. These two schemes can affect greedy, face and CGF routings' compatibility requirements.

In both methods, a source sends a route request message that records the entire path it traverses. Once the destination receives the route request message, it can optimize the path by eliminating all its cycles. Then, the destination returns a route reply message to the source along the optimized path. In the backward building scheme, the reply message sets up

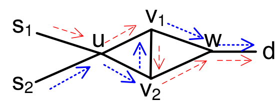

Fig. 4. Paths Discovered by Greedy Routing

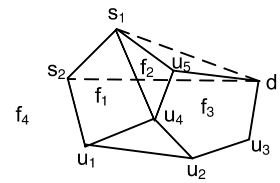

Fig. 5. Face Routing Loop

the routing tables of all the nodes it traverses similar as in AODV [4].

In the forward building scheme, the route reply message does not setup the routing table. Instead, the routing table is setup by the the route confirmation message, which is sent by the source node along the optimized path after the source node receives the route reply message.

In the following, we analyze the compatibility requirements of greedy, face and CGF routing when they are respectively combined with backward building and forward building hop-by-hop routing.

Theorem 7: A greedy routing based backward building hop-by-hop routing protocol GR is loop-free if and only if the routing metric has three properties: odd symmetry, transitivity and source independence.

Proof:

Sufficient conditions:

We prove the sufficient conditions by contradiction. Assume that the greedy routing algorithm creates a loop $c(u_1, u_2, ..., u_k)$ while forwarding a flow $f_{\langle s_x,d\rangle}$'s traffic based on the routing tables built by the greedy algorithm GR. All the packets of $f_{\langle s_x,d\rangle}$ that are relayed by u_1 will be forwarded back to u_1 through the following links $\overrightarrow{u_1u_2}$, $\overrightarrow{u_2u_3}$, ..., $\overrightarrow{u_{k-1}u_k}$, $\overrightarrow{u_ku_1}$. Furthermore, $\overrightarrow{u_1u_k} \in E$ is also satisfied due to the symmetry of edges. Based on the principle of greedy routing, for each link $\overrightarrow{u_iu_{i+1}}$ $(1 \leq i < k-1)$ in the circle, there must be a flow $f_{\langle s_y,d\rangle}$ that satisfies $w^g(l_{\overrightarrow{u_iu_{i+1}}}, f_{\langle s_y,d\rangle}) \prec \phi$. Due to the source independence property, we can denote $f_{\langle *,d\rangle}$ as any flow destined for node d in F such that $w^g(l_{\overrightarrow{u_iu_{i+1}}}, f_{\langle *,d\rangle}) = w^g(l_{\overrightarrow{u_iu_{i+1}}}, f_{\langle s_y,d\rangle})$ is always satisfied. Hence, $w^g(l_{\overrightarrow{u_1u_2}}, f_{\langle *,d\rangle}) \prec \phi$, $w^g(l_{\overrightarrow{u_2u_3}}, f_{\langle *,d\rangle}) \prec \phi$, ..., $w^g(l_{\overrightarrow{u_{k-1}u_k}}, f_{\langle *,d\rangle}) \prec \phi$ and $w^g(l_{\overrightarrow{u_ku_1}}, f_{\langle *,d\rangle}) \prec \phi$ are satisfied. Based on the transitivity property, we can conclude that $w^g(l_{\overrightarrow{u_1u_k}}, f_{\langle *,d\rangle}) \prec \phi$ is satisfied. Then, both $w^g(l_{\overrightarrow{u_1u_k}}, f_{\langle *,d\rangle}) \prec \phi$ and $w^g(l_{\overrightarrow{u_ku_1}}, f_{\langle *,d\rangle}) \prec \phi$ are satisfied, which contradicts the fact that the routing metric has the odd symmetry property. Hence, loop-freeness is guaranteed.

Necessary conditions:

Obviously, if the routing metric lacks odd symmetry or transitivity property, a loop may be created in a route discovery phase as shown by the proof of theorem 1. Hence, we only need to prove the necessitate of the source independence property using a simple example. Consider the example in Fig. 4. Assume that due to lack of source-independence, for flow $f_{\langle s_1,d\rangle}$, the links satisfy $w^g(l_{\overrightarrow{s_1u}}, f_{\langle s_1,d\rangle}) \prec \phi$, $w^g(l_{\overrightarrow{uv_1}}, f_{\langle s_1,d\rangle}) \prec w^g(l_{\overrightarrow{uv_2}}, f_{\langle s_1,d\rangle}) \prec \phi \prec w^g(l_{\overrightarrow{us_1}}, f_{\langle s_1,d\rangle})$, $w^g(l_{\overrightarrow{v_1v_2}}, f_{\langle s_1,d\rangle}) \prec w^g(l_{\overrightarrow{v_1w}}, f_{\langle s_1,d\rangle}) \prec \phi \prec w^g(l_{\overrightarrow{v_1u}}, f_{\langle s_1,d\rangle})$, $w^g(l_{\overrightarrow{v_2w}}, f_{\langle s_1,d\rangle}) \prec \phi \prec$

$w^g(l_{\overrightarrow{v_2v_1}}, f_{\langle s_1,d\rangle}) \prec w^g(l_{\overrightarrow{v_2u}}, f_{\langle s_1,d\rangle})$, $w^g(l_{\overrightarrow{wd}}, f_{\langle s_1,d\rangle}) \prec \phi \prec w^g(l_{\overrightarrow{wv_1}}, f_{\langle s_1,d\rangle}) \prec w^g(l_{\overrightarrow{wv_2}}, f_{\langle s_1,d\rangle})$. Hence, the path $s_1uv_1v_2wd$ is discovered by the greedy routing algorithm for flow $f_{\langle s_1,d\rangle}$, while the path $s_2uv_2v_1wd$ is discovered for flow $f_{\langle s_2,d\rangle}$ due to the lack of source independence property.

Assume flows $f_{\langle s_1,d\rangle}$ and $f_{\langle s_2,d\rangle}$ occur simultaneously. The route reply messages for $f_{\langle s_1,d\rangle}$ and $f_{\langle s_2,d\rangle}$ are respectively M_{s_1d} and M_{s_2d}. After setting up the routing table in node v_1 as $\langle destination = d, next\_hop = w\rangle$, M_{s_2d} leaves for node v_2. At the same time, M_{s_1d} sets up the routing table in node v_2 as $\langle destination = d, next\_hop = w\rangle$, and leaves for node v_1. After M_{s_2d} reaches node v_2, the routing table in node v_2 is renewed as $\langle destination = d, next\_hop = v_1\rangle$. The same thing happens when M_{s_1d} arrives at node v_1, and the routing table in node v_1 is updated as $\langle destination = d, next\_hop = v_2\rangle$. After the route discovery phases of these two flows are finished, a loop between nodes v_1 and v_2 is created for both $f_{\langle s_1,d\rangle}$ and $f_{\langle s_2,d\rangle}$. Considering the complications of traffic flows, this case is possible to happen in real networks. ■

Theorem 8: A greedy routing based forward building hop-by-hop routing protocol GR is loop-free if and only if the routing metric has two properties: odd symmetry and transitivity.

Proof:

Necessary conditions:

The proof for necessary condition is straightforward since without odd symmetry and transitivity, a loop may be created in the route discovery phase as illustrated in the proof of theorem 1.

Sufficient conditions:

To prove that a loop cannot exist, we next show that any forwarding path created by nodes' routing entries must be in the increasing order of these entries' latest setup times.

Take any routing entry $E_{i,d}$ in node u_i as an example, where $E_{i,d}$ specifies node u_i's next hop denoted as u_{i+1}, to destination d. Assume that the latest setup time of $E_{i,d}$ is t_i and the latest setup time for u_{i+1}'s entry $E_{i+1,d}$ is t_{i+1}. This implies that at t_i, a route confirmation message M_{s_xd} caused by flow $f_{\langle s_x,d\rangle}$ visited u_i and set up $E_{i,d}$. Message M_{s_xd} then would be forwarded to the next hop u_{i+1} and set the routing entry $E_{i+1,d}$ at a time t'_{i+1}. We always have $t'_{i+1} > t_i$ since a message M_{s_xd} itself cannot create a routing loop with the odd symmetry and transitivity properties. The latest setup time of $E_{i+1,d}$, possibly done by another confirmation message M_{s_yd} caused by a different flow $f_{\langle s_y,d\rangle}$, hence is lower bounded by t'_{i+1}. Hence, we have $t_{i+1} \geq t'_{i+1} > t_i$.

Given a cycle of forwarding path u_1, u_2, ..., u_k, u_1, their corresponding forwarding entries' latest set up times must satisfy $t_1 < t_2$, $t_2 < t_3$, ..., $t_{k-1} < t_k$ and $t_k < t_1$. We can then desire the contradiction that $t_1 < t_k$ and $t_1 > t_k$. Hence a cycle does not exist. ■

Theorem 9: A MFF routing based backward building hop-by-hop routing protocol FR may create routing loops even if the routing metric has local minimum free property.

Proof: We prove it through a simple example.

Consider the example in Fig. 5 which is a planar graph. Assume flows $f_{\langle s_1,d\rangle}$ and $f_{\langle s_2,d\rangle}$ occur simultaneously. The curves $\widetilde{s_1d}$ and $\widetilde{s_2d}$ are the baselines for MFF routing, and they are local minimum free with d as the global minimum point. The curve $\widetilde{s_1d}$ only intersects one face, i.e., the exterior face f_4. According to the right hand rule, the path $s_1s_2u_1u_2u_3d$ is discovered for flow $f_{\langle s_1,d\rangle}$. The curve $\widetilde{s_2d}$ intersects 3 faces, i.e., f_1, f_2 and f_3. Based on the right hand rule, the simple path $s_2s_1u_5d$ is discovered for flow $f_{\langle s_2,d\rangle}$. Hence, the route reply message M_{s_1d} to the source node s_1 carries the path $s_1s_2u_1u_2u_3d$, and the route reply message M_{s_2d} to the source node s_2 carries the path $s_2s_1u_5d$. In the routing tables building process, consider the possibility that M_{s_1d} arrives at node s_2 and M_{s_2d} reaches node s_1 at the same time. Hence, the routing table in node s_2 is set to $\langle destination = d, next\_hop = u_1\rangle$ and then M_{s_1d} leaves for node s_1. At the same time, the routing table in node s_1 is set to $\langle destination = d, next\_hop = u_5\rangle$ and then M_{s_2d} leaves for node s_2. Once M_{s_1d} reaches node s_1, the routing table in node s_1 is renewed as $\langle destination = d, next\_hop = s_2\rangle$. The same thing happens when M_{s_2d} arrives at node s_2, and the routing tables in node s_2 is updated as $\langle destination = d, next\_hop = s_1\rangle$. After the route discovery phase of these two flows finishes, a loop between nodes s_1 and s_2 is created for both $f_{\langle s_1,d\rangle}$ and $f_{\langle s_2,d\rangle}$. Considering the complications of traffic flows, this case is possible to happen in real networks. ■

Theorem 10: A MFF routing based forward building hop-by-hop routing protocol FR is loop-free and packet delivery guaranteed if and only if the routing metric has the local minimum free property.

Proof:

Obviously, if the routing metric lacks the local minimum free property, a loop may be created or MFF routing will incorrectly end in route discovery phase. Hence, we only need to prove the sufficient condition.

Based on the theorem 2, a path can be discovered by MFF routing FR between any two nodes in a wireless network if the routing metric has the local minimum free property. Furthermore, packet delivery guarantee can be proved in a similar way as the proof in theorem 8. ■

In a similar discussion, we can draw the following conclusion.

Theorem 11: A CGF routing based forward building hop-by-hop routing protocol R is loop-free and packet delivery guaranteed if and only if the routing metric for its greedy routing algorithm GR has the odd symmetry and transitivity properties, and the routing metric for its MFF routing algorithm FR has the local minimum free property.

VII. Case Studies

In this section, we show how to use our theoretical results to analyze routing metrics by using four routing metrics as examples.

A. Most Forward within Radius (MFR)

MFR metric is originally designed for greedy routing where a packet is forwarded to the closest node to the destination among all the neighbors of the forwarding node that are closer to the destination than the forwarding node [1].

For MFR metric, the label of a link $l_{\overrightarrow{uv}}$ is defined as $l_{\overrightarrow{uv}} = \langle (x_u, y_u), (x_v, y_v) \rangle$, where (x_u, y_u) and (x_v, y_v) are the geographic position of nodes u and v respectively. The $w(\cdot)$ function of MFR metric is defined as:

$$\begin{aligned} &w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) = ||v,d||_2 - ||u,d||_2 \\ &= \sqrt{(x_v - x_d)^2 + (y_v - y_d)^2} - \sqrt{(x_u - x_d)^2 + (y_u - y_d)^2} \end{aligned} \quad (1)$$

where $||v,d||_2$ is the Euclidean distance between nodes v and d.

The preference operator $\preceq$ is the common less than or equal to ($\leq$) operator for real numbers.

The threshold ϕ for greedy routing is 0, i.e., it is only when $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) < 0$ is satisfied that a packet of $f_{\langle s,d \rangle}$ can be relayed through link $\overrightarrow{uv}$. The baseline for face routing is the straight line segment joining the MFF routing's starting node and the destination, which is local minimum free and the global minimal point is the destination.

It is easy to know that MFR metric has odd symmetry, transitivity, source independence and local minimum freeness properties. Hence, we can conclude as follows:

i: As packet forwarding schemes, greedy routing with MFR metric is loop-free. MFF routing and CGF routing with MFR metric are loop-free and packet delivery guaranteed.

ii: Treating greedy routing as the path discovery algorithm, its combination with source routing and MRF metric is loop-free. MFF routing or CGF routing coupled with MFR metric and source routing is loop-free and packet delivery guaranteed.

iii: Treating greedy routing as the path discovery algorithm, its combination with backward building hop-by-hop routing and MRF metric is loop-free. The combination of either MFF routing or CGF routing with MFR metric and backward building hop-by-hop forwarding scheme may create routing loops.

iv: Treating greedy routing as the path discovery algorithm, its combination with forward building hop-by-hop routing and MRF metric is loop-free. The combination of either MFF routing or CGF routing with MFR metric and forward building hop-by-hop forwarding scheme is loop-free and packet delivery guaranteed.

Same conclusions can be drawn for NFR [2], GRS [24], GEAR [12], the best $PRR \times distance$ [11] and NADV [13] metrics through similar analysis.

B. Random Progress Forwarding (RPF)

RPF metric is a greedy routing metric [25]. In RPF, a packet is forwarded to the neighbor who has the maximum positive projected progress on the straight line segment joining the forwarding node and the destination. In another word, the $w(\cdot)$ in RPF is:

$$\begin{aligned} w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) &= -||u,v||_2 \cos \angle \overrightarrow{ud}, \overrightarrow{uv} \\ &= -\sqrt{(x_v - x_u)^2 + (y_v - y_u)^2} \cos \angle \overrightarrow{ud}, \overrightarrow{uv} \end{aligned} \quad (2)$$

where $\angle \overrightarrow{ud}, \overrightarrow{uv}$ is the angle from the edge $\overrightarrow{ud}$ to $\overrightarrow{uv}$, and the $\preceq$ is the "$\leq$" operator in real numbers.

The threshold ϕ for greedy routing is also 0. We can show that RPF metric lacks the odd symmetry property by

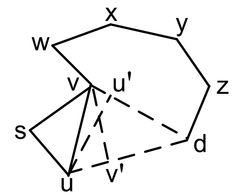

Fig. 6. Greedy Routing Loop Based On RPF Metric

Fig. 6 where both $w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) = -||u,v'||_2 < 0$ and $w(l_{\overrightarrow{vu}}, f_{\langle s,d \rangle}) = -||v,u'||_2 < 0$ are satisfied.

However, RPF metric has the local minimum free property. For any two nodes, the straight line segment joining these two nodes has no local minimum and the destination node is the global minimum point based on RPF metric. Hence, we can conclude as follows:

i: Either as packet forwarding schemes or as path discovery algorithms, greedy routing and CGF routing with RPF metric are not loop-free. That is to say, RPF is not a usable metric for greedy routing and CGF routing.

ii: Either as a packet forward scheme or as a path discovery algorithm combined with source routing or forward building hop-by-hop routing, MFF routing with RPF metric is loop-free and packet delivery guaranteed. However, if combined with backward building hop-by-hop routing, MFF routing with RPF metric may create routing loops.

C. Line Progress Forwarding (LPF)

LPF metric is also based on the projected distance, however, other than RPF metric, it is a usable metric for greedy routing in some cases. The only difference between LPF and RPF is that, for LPF metric, it is the straight line segment joining the source and the destination where the progress is projected. Hence, the $w(\cdot)$ function of LPF metric is:

$$\begin{aligned} w(l_{\overrightarrow{uv}}, f_{\langle s,d \rangle}) &= -||u,v||_2 \cos \angle \overrightarrow{sd}, \overrightarrow{uv} \\ &= -\sqrt{(x_v - x_u)^2 + (y_v - y_u)^2} \cos \angle \overrightarrow{sd}, \overrightarrow{uv} \end{aligned} \quad (3)$$

where $\angle \overrightarrow{sd}, \overrightarrow{uv}$ is the angle from the edge $\overrightarrow{sd}$ to $\overrightarrow{uv}$.

It is easy to know that LPF metric has odd symmetry, transitivity and the local minimum freeness properties, but lacks source independence property. Hence, we can conclude as follows:

i: As packet forwarding schemes, greedy routing with LPF metric is loop-free, and both MFF routing and CGF routing with LPF metric are loop-free and packet delivery guaranteed.

ii: As on-demand path discovery algorithms, greedy routing with LPF metric and source routing is loop-free, and both MFF routing and CGF routing with LPF metric and source routing are loop-free and packet delivery guaranteed.

iii: As on-demand path discovery algorithms combined with backward building hop-by-hop routing, greedy routing, MFF routing and CGF routing may create routing loops when they are coupled with LPF metric.

iv: As on-demand path discovery algorithms combined with forward-building hop-by-hop routing, greedy routing with LPF metric is loop-free, and both MFF routing and CGF routing with LPF metric are loop-free and packet delivery guaranteed.

D. Virtual Force Forwarding (VFF)

Consider the example in Fig 2, where there is a compromised zone that a routing protocol may want to avoid routing its packets through. A novel routing metric that reflects this consideration can be defined as:

$$
\begin{aligned}
& w(l_{\overrightarrow{uv}}, f_{\langle s,d\rangle}) \\
&= [\frac{\alpha}{(x_u - x_d)^2 + (y_u - y_d)^2} - \frac{1-\alpha}{(x_u - x_h)^2 + (y_u - y_h)^2}] \\
&-[\frac{\alpha}{(x_v - x_d)^2 + (y_v - y_d)^2} - \frac{1-\alpha}{(x_v - x_h)^2 + (y_v - y_h)^2}]
\end{aligned} \quad (4)
$$

where α is a tradeoff coefficient, (x_d, y_d) and (x_h, y_h) are the coordinations of the destination and the center of the compromised zone, respectively. The component related to (x_d, y_d) reflects the forwarding progress to the destination while the component related to (x_h, y_h) captures the potential damage from the compromised zone.

The preference operator $\preceq$ is again the "$\leq$" in real numbers.

The threshold ϕ for greedy routing is 0. The baseline for face routing does exist but may not be a straight line segment, and it is a gradient curve joining MFF routing's starting node and destination.

It is easy to know that VFF metric has odd symmetry, transitivity, source independence and the local minimum freeness properties. Hence, we can conclude as follows:

i: As packet forwarding schemes, greedy routing with VFF metric is loop-free, MFF routing and CGF routing with VFF metric are loop-free and packet delivery guaranteed.

ii: As on-demand path discovery algorithms combined with source routing, greedy routing with VFF metric is loop-free, MFF routing and CGF routing with VFF metric are loop-free and packet delivery guaranteed.

iii: As on-demand path discovery algorithms combined with backward building hop-by-hop routing, greedy routing with VFF metric is loop-free, MFF routing and CGF routing with VFF metric may create routing loops.

iv: As on-demand path discovery algorithms combined with forward building hop-by-hop routing, greedy routing with VFF metric is loop-free, MFF routing and CGF routing with VFF metric are loop-free and packet delivery guaranteed.

VIII. Conclusions

In this paper, we firstly discuss the necessity to analyze routing metric's impact on routing protocols, and then discuss the possible roles of greedy, face and CGF routing in a routing system. We propose a novel algebra and defined different algebraic properties to investigate compatibilities between routing metric and the three geographic routing protocols including greedy routing, MFF routing and CGF routing. The necessary and sufficient conditions for loop-free and delivery guaranteed routing are derived when greedy routing, face routing and CGF routing serve as different roles in a routing system. The applications of these conditions are illustrated by some different routing metric which are respectively based on Euclidean distance, projected distance and field of forces. Our work provides essential criterions for evaluating and designing geographic routing protocols. In our future work, we would like to analyze more variants of face routing with our algebra.

Acknowledgment

This work was supported in part by the US National Science Foundation (NSF) under grant CNS-0831865, by the Institute for Critical Technology and Applied Science (ICTAS) of Virginia Tech, and by China Scholarship Council.

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