On the fundamentals of optical scanning holography

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Optical scanning holography is a real-time holographic recording technique in which holographic information about a three-dimensional (3D) object is acquired using a single two-dimensional active optical scan. Applications of the technique include optical scanning microscopy, pattern recognition, 3D holographic display, and optical remote sensing. This paper introduces the basics of optical scanning holography.
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I. INTRODUCTION

Holography[1–4] is a method of storing three-dimensional (3D) optical information and is an important tool for scientific and engineering studies.[5–7] The purpose of this paper is to review the basic principles of holography using the concept of Fresnel zone plates and to introduce an unconventional holographic technique known as optical scanning holography. The two holographic techniques will then be compared.

II. BASICS OF OPTICAL HOLOGRAPHY

Consider a planar object located at $z=0$ and described by a transparency $t(x,y)$. The object is illuminated by a monochromatic plane wave of wavelength λ as shown in Fig. 1. If we describe the amplitude and phase of a light field at $z=0$ by the complex function $\boldsymbol{\Psi}(x,y;z=0)$, we can obtain the complex light field a distance z away, according to Fresnel diffraction[8,9]

$$\boldsymbol{\Psi}(x,y;z) = \boldsymbol{\Psi}(x,y;z=0) \otimes h(x,y;z). \tag{1a}$$

The symbol $\otimes$ denotes two-dimensional (2D) convolution defined by[8]

$$f(x,y) \otimes g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')g(x-x',y-y')dx'dy', \tag{1b}$$

and the free-space impulse response function $h(x,y;z)$ is given by[8]

$$h(x,y;z) = \exp(-ik_0 z)\frac{ik_0}{2\boldsymbol{\pi} z}\exp\left[\frac{-ik_0}{2z}(x^2+y^2)\right]. \tag{2}$$

In Eq. (2) $k_0=2\boldsymbol{\pi}/\boldsymbol{\lambda}$ is the wave number of the light field. If we refer to the situation shown in Fig. 1, we can identify $\boldsymbol{\Psi}(x,y;z=0)$, which is given by $\boldsymbol{\Psi}(x,y;z=0)=1\times t(x,y)$, where the factor of unity in front of $t(x,y)$ assumes plane wave illumination with amplitude unity and with zero initial phase at $z=0$. Hence, according to Eq. (1), the complex amplitude a distance z from the object is given by

$$\boldsymbol{\Psi}(x,y;z) = t(x,y) \otimes h(x,y;z) = t(x,y) \otimes \exp(-ik_0 z)\frac{ik_0}{2\boldsymbol{\pi} z} \times\exp\left[\frac{-ik_0}{2z}(x^2+y^2)\right]. \tag{3}$$

If we can record or store this original complex amplitude (amplitude and phase), and at a later time we are able to restore the exact complex amplitude, then we do not lose any information. Because our eyes observe the intensity generated by the same complex field, it makes no difference to observe at time t_1, the time the original complex field is recorded, or t_2, the time when the exact complex field is restored. The restored complex field preserves the entire parallax and depth cue much like the original complex field and is interpreted by our brain as the same 3D object. Recording media such as photographic films and CCD cameras respond only to light intensity; that is, they record $I(x,y) =|\boldsymbol{\Psi}(x,y;z)|^2$, and hence the phase information of the complex field is lost. However, if $\boldsymbol{\Psi}(x,y;z)$ interferes with, say, a normally incident plane wave of amplitude $\boldsymbol{\Psi}_r$ at the recording medium, the resulting recorded intensity becomes

$$I(x,y) = |\boldsymbol{\Psi}_r + \boldsymbol{\Psi}(x,y;z)|^2 = \boldsymbol{\Psi}_r^2 + |\boldsymbol{\Psi}|^2 + \boldsymbol{\Psi}_r\boldsymbol{\Psi}^* + \boldsymbol{\Psi}_r\boldsymbol{\Psi}, \tag{4}$$

where $\boldsymbol{\Psi}_r$ has been assumed real for simplicity. Note that $\boldsymbol{\Psi}$ appears in the last term of Eq. (4), even though there are other undesirable terms introduced due to the nonlinear operation, which is the square of the absolute value of the total field $\boldsymbol{\Psi}_r+\boldsymbol{\Psi}(x,y;z)$. If the recorded film transmittance is linearly proportional to the exposure, which is proportional to the intensity shown in Eq. (4), the resulting developed transparency is the hologram $H(x,y)$ of the object, $t(x,y)$, that is, $H(x,y)\propto I(x,y)$. Interfering the original wave field $\boldsymbol{\Psi}$ with $\boldsymbol{\Psi}_r$ is the original idea of Gabor[10,11] for holographic recording.

In the terminology of holography the complex field $\boldsymbol{\Psi}(x,y;z)$ generated from the object is called the object wave and $\boldsymbol{\Psi}_r$ is called the reference wave. Once we have the hologram, we can retrieve $\boldsymbol{\Psi}(x,y;z)$, the original light field by illuminating the hologram, say, by an incident plane wave of amplitude $\boldsymbol{\Psi}_{\text{rec}}$. We then have

$$\boldsymbol{\Psi}_{\text{rec}}H(x,y) = \boldsymbol{\Psi}_{\text{rec}}[\boldsymbol{\Psi}_r^2 + |\boldsymbol{\Psi}|^2 + \boldsymbol{\Psi}_r\boldsymbol{\Psi}^* + \boldsymbol{\Psi}_r\boldsymbol{\Psi}], \tag{5}$$

which is the light field immediately after the hologram. For

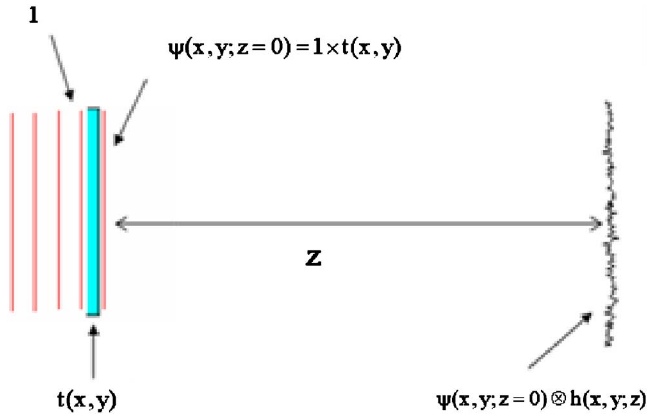

Fig. 1. Fresnel diffraction of $\psi(x,y;z=0)$ at a distance z. The complex field ψ is obtained by the illumination of a transparency $t(x,y)$ by a plane wave.

simplicity, if $\Psi_{\rm rec}$ is a constant such as is the case for normal illumination by a plane wave, we have retrieved the original light field (the last term in the bracket), but with some unwanted contributions (the first three terms inside the bracket).

III. HOLOGRAM AS A COLLECTION OF FRESNEL ZONE PLATES

We first give an illustrative example of holographic recording of a single point object. In Fig. 2 we show a setup for holographic recording. The point source is the object, which is given by the illumination of a pinhole aperture. The reference wave is provided by the collimated beam directed by beam splitters BS1, BS2, and mirror M1 in Fig. 2.

The object wave Ψ_0 reaching the recording medium z_0 away from the pinhole aperture is a diverging spherical wave given by [see Eq. (3)]

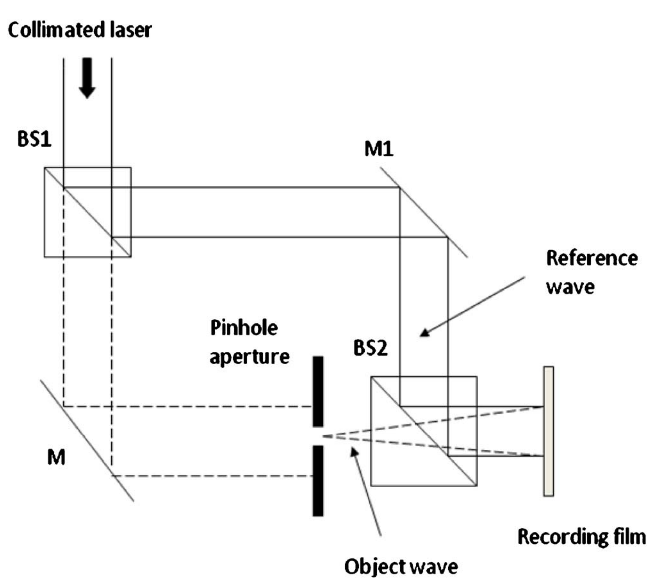

Fig. 2. Holographic recording of a point source object (M and M1 are mirrors). The point source is created by the illumination of a pinhole aperture by a plane wave. The film records the reference wave and the object wave simultaneously.

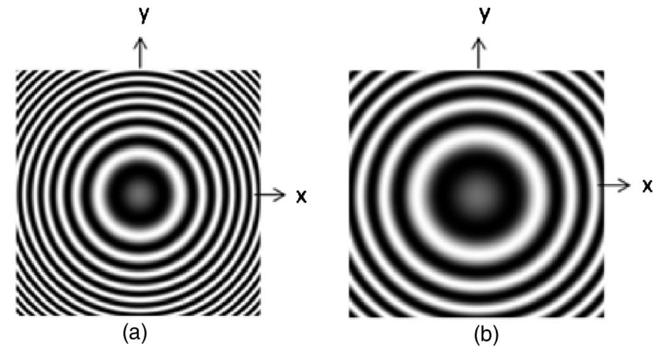

Fig. 3. Plots for a Fresnel zone plate based on Eq. (9) for (a) $z_0=4$ and (b) $z_0=8$.

$$\begin{aligned}\Psi_0(x,y;z_0) &= \delta(x,y) \otimes h(x,y;z_0) = h(x,y;z_0)\\ &= \exp(-ik_0z_0)\frac{ik_0}{2\pi z_0}\exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right],\end{aligned} \tag{6}$$

where the delta function $\delta(x,y)$ has been used to model the transparency of the point source and the sampling property of the delta function; that is,[8]

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,\delta(x-x_0,y-y_0)\,dxdy = f(x_0,y_0) \tag{7}$$

has been used to evaluate the convolution in Eq. (6). For the reference plane wave we assume that the plane wave has the same initial phase as the point object at a distance z_0 from the film. Therefore, its field distribution on the film is $\Psi_r = a\exp(-ik_0z_0)$, where a is the amplitude of the plane wave on the film. The recorded intensity distribution on the film and hence the hologram is given by

$$\begin{aligned}H(x,y) \propto I(x,y) &= |\Psi_r + \Psi_0(x,y;z_0)|^2 = \Bigg| a\exp(-ik_0z_0)\\ &\quad + \exp(-ik_0z_0)\frac{ik_0}{2\pi z_0}\exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right]\Bigg|^2\end{aligned} \tag{8a}$$

or

$$\begin{aligned}H(x,y) &= a^2 + \left(\frac{k_0}{2\pi z_0}\right)^2 + a\frac{-ik_0}{2\pi z_0}\exp\left[\frac{ik_0}{2z_0}(x^2+y^2)\right]\\ &\quad + a\frac{ik_0}{2\pi z_0}\exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right],\end{aligned} \tag{8b}$$

where for brevity we have replaced the proportional sign by the equality sign. Note that the last term, which is the desired term in Eq. (8b), is the total complex field of the original object wave [see Eq. (6)] aside from the constant, $\exp(-ik_0z_0)$. Equation (8b) can be simplified to

$$H(x,y) = A + B\sin\left[\frac{k_0}{2z_0}(x^2+y^2)\right], \tag{9}$$

where $A=a^2+(k_0/2\pi z_0)^2$ and $B=ak_0/\pi z_0$. The expression in Eq. (9) is called the sinusoidal Fresnel zone plate, which is the hologram of the point source object.

A plot of the Fresnel zone plate is shown in Fig. 3(a), where we have set $A=B=1$, $k_0/2=1$ and $z_0=4$ for plotting purposes. Note that the spatial rate of change of the phase of the Fresnel zone plate, say, along the x-direction, is given by

$$f_{\text{local}} = \frac{1}{2\pi}\frac{d}{dx}\left(\frac{k_0}{2z_0}x^2\right) = \frac{k_0 x}{2\pi z_0}. \tag{10}$$

f_{local} has units of inverse length and is called the local fringe spatial frequency, which increases linearly with the spatial coordinate x. In other words, the farther from the zone's center, the higher the local spatial frequency.

In Fig. 3(b) we have doubled the value of z_0. The local fringe frequencies have become smaller than those of Fig. 3(a), as is evident from Eq. (10). Hence, the local frequency carries information about z_0; that is, from the local frequency we can deduce how far the object point source is away from the recording film—an important aspect of holography.

To reconstruct the original light field from $H(x,y)$, we can illuminate the hologram with a plane wave (called the reconstruction wave in holography), which gives, according to Fresnel diffraction or Eq. (1) and Eq. (8b),

$$\begin{aligned}\Psi_{\text{rec}}&H(x,y) \otimes h(x,y;z)\\ &= \Psi_{\text{rec}}\left\{A + a\frac{-ik_0}{2\pi z_0}\exp\left[\frac{ik_0}{2z_0}(x^2+y^2)\right]\right.\\ &\quad\left.+ a\frac{ik_0}{2\pi z_0}\exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right]\right\} \otimes h(x,y;z).\end{aligned} \tag{11}$$

The evaluation of Eq. (11) gives three light fields emerging from the hologram. The light field due to the first term in Eq. (11) is a plane wave because $\Psi_{\text{rec}}A \otimes h(x,y;z) \propto \Psi_{\text{rec}}A$, which is reasonable because a plane wave propagates without diffraction. This plane wave is called the zeroth-order beam. The field due to the second term is

$$\begin{aligned}\Psi_{\text{rec}}&\frac{-ik_0}{2\pi z_0}\exp\left[\frac{ik_0}{2z_0}(x^2+y^2)\right] \otimes h(x,y;z)\\ &\propto \frac{-ik_0}{2\pi z_0}\frac{ik_0}{2\pi z}\exp\left[\frac{ik_0}{2z_0}(x^2+y^2)\right] \otimes \exp\left[\frac{-ik_0}{2z}(x^2+y^2)\right]\\ &= \frac{-ik_0}{2\pi z_0}\frac{ik_0}{2\pi z}\exp\left[\frac{ik_0}{2(z_0-z)}(x^2+y^2)\right].\end{aligned} \tag{12}$$

Equation (12) represents a converging spherical wave if $z < z_0$. If $z > z_0$, the wave is diverging. For $z=z_0$ the wave focuses to a real point source z_0 away from the hologram and is given by a delta function, $\delta(x,y)$. For the last term in Eq. (11) we have

$$\begin{aligned}\Psi_{\text{rec}}&\frac{ik_0}{2\pi z_0}\exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right] \otimes h(x,y;z)\\ &\propto \frac{-ik_0}{2\pi z_0}\frac{-ik_0}{2\pi z}\exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right] \otimes \exp\left[\frac{-ik_0}{2z}(x^2+y^2)\right]\\ &= \frac{-ik_0}{2\pi z_0}\frac{-ik_0}{2\pi z}\exp\left[\frac{-ik_0}{2(z_0+z)}(x^2+y^2)\right],\end{aligned} \tag{13}$$

which is a diverging wave with its virtual point source appearing to come from a distance z_0 behind the hologram. This reconstructed point source is at the location of the original point source object. The situation is illustrated in Fig. 4. The real point source is called the twin image of the virtual point source.

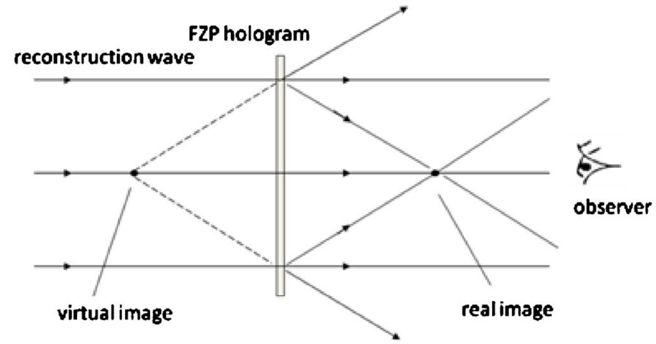

Fig. 4. Reconstruction of a Fresnel zone plate hologram with the existence of the twin-image (which is the real image reconstructed in the figure).

Let us see what happens if we have an object consisting of two point sources given by $b_0\delta(x,y)+b_1\delta(x-x_0,y-y_0)$, where b_0 and b_1 denote the amplitude of the two point sources. We assume that the two point sources are located z_0 away from the recording film. The object wave on the film then becomes

$$\Psi_0(x,y) = [b_0\delta(x,y) + b_1\delta(x-x_0,y-y_0)] \otimes h(x,y;z_0). \tag{14}$$

Using Eq. (14) the hologram becomes

$$\begin{aligned}H(x,y) = |\Psi_r + \Psi_0(x,y;z_0)|^2 = \Bigg|&a\exp(-ik_0z_0)\\ &+ b_0\exp(-ik_0z_0)\frac{ik_0}{2\pi z_0}\exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right]\\ &+ b_1\exp(-ik_0z_0)\frac{ik_0}{2\pi z_0}\exp\left[\frac{-ik_0}{2z_0}(x-x_0)^2\right.\\ &\left.\left.+ (y-y_0)^2\right]\right|^2.\end{aligned} \tag{15}$$

Equation (15) can be written in real form as

$$\begin{aligned}H(x,y) = C &+ \frac{ab_0k_0}{\pi z_0}\sin\left[\frac{k_0}{2z_0}(x^2+y^2)\right]\\ &+ \frac{ab_1k_0}{\pi z_0}\sin\left[\frac{k_0}{2z_0}[(x-x_0)^2+(y-y_0)^2]\right]\\ &+ 2b_0b_1\left(\frac{k_0}{2\pi z_0}\right)^2\cos\left\{\frac{k_0}{2z_0}[(x^2+y^2)-(x-x_0)^2\right.\\ &\left.-(y-y_0)^2]\right\},\end{aligned} \tag{16}$$

where C is a constant obtained as in Eq. (9). The second and third terms are the familiar Fresnel zone plate associated with each point source, and the last term is a cosinusoidal fringe grating whose origin is due to the nonlinear operation of photographic recording. Hence, there are a total of two Fresnel zone plates for two point sources. Only one term from each of the sinusoidal Fresnel zone plates contains the desired information because each contains the original light field for the two points. In general, the cosinusoidal fringe grating introduces noise on the reconstruction plane. Givens has given a general form of such a hologram due to n point sources.[12]

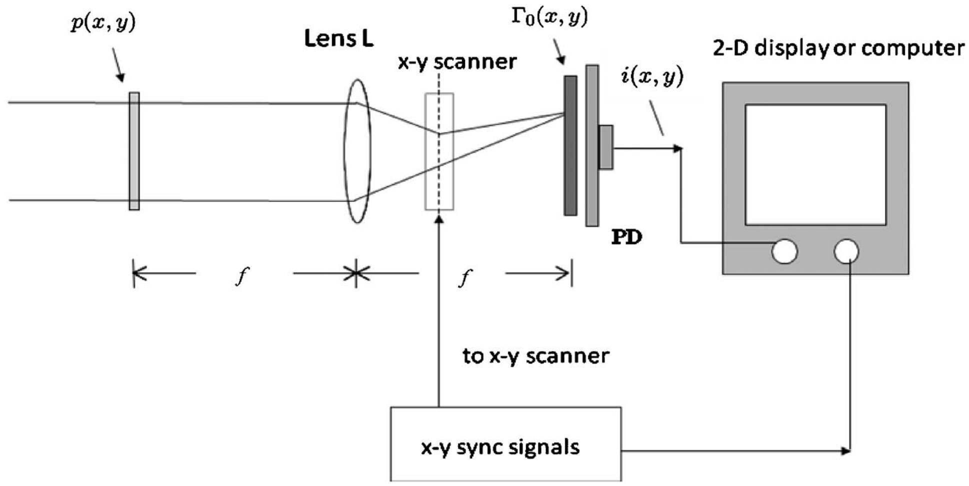

Fig. 5. Standard optical scanning system for imaging. The object transparency $\Gamma_0(x,y)$ is scanned two-dimensionally by an optical beam. The photodetector PD converts the light into electrical information, which is displayed or stored on a computer as a 2D record. Note that the pupil function $p(x,y)$ located in the front focal plane of lens L with focal length f can modify the shape of the scanning beam.

IV. OPTICAL SCANNING HOLOGRAPHY

Optical scanning holography is a form of electronic (or digital) holography.[13,14] The latter is a general name that refers to the fact that holographic recording is done electronically, thereby avoiding the nonreal-time darkroom processing of the film. Digital holography traditionally employs a CCD camera for recording. Optical scanning holography is a real-time technique in which holographic information of a 3D object can be acquired by using a single 2D optical scan where scattered light from the object is detected by a photodetector. Hence, optical scanning holography is a form of digital holography. Optical scanning holography was first proposed by Poon and Korpel,[15] and the original idea was later formulated in Ref. 16. The technique was eventually called optical scanning holography to emphasize that holographic recording can be achieved by active optical scanning.[17] Applications of optical scanning holography include scanning holographic microscopy, 3D image recognition, 3D optical remote sensing, 3D TV and display, and 3D cryptography.[18] Optical scanning holography involves the principle of optical heterodyne scanning. We shall therefore discuss optical scanning first.

In Fig. 5 we show a typical optical scanning imaging system such as a laser scanning microscope. A collimated laser beam is projected through the x-y optical scanner to scan out the input object specified by transparency $\Gamma_0(x,y)$. The photodetector converts the light to an electrical signal that contains the processed information for the scanned object. If the scanned electrical signal is digitally stored (in a computer) in synchronization with the 2D scan signals of the scanning mechanism (such as the x-y scanning mirrors), the stored record is a processed 2D image of the scanned object.

Assume that the scanning optical beam is specified by a complex field $\boldsymbol{\Psi}_b(x,y)$ on the object transparency, and the complex field reaching the photodetector is $\Gamma_0(x',y')\boldsymbol{\Psi}_b(x'-x,y'-y)$. The coordinate shifts in the argument of $\boldsymbol{\Psi}_b(x'-x,y'-y)$ signify the motion relative to the transparency. The photodetector then delivers a current $i(x,y)$ as the output by spatially integrating the intensity, $|\boldsymbol{\Psi}_b(x'-x,y'-y)\Gamma_0(x',y')|^2$, over the active area S of the detector. The current is displayed on the monitor as a 2D record:

$$i(x,y) \propto \int\int_S |\Gamma_0(x',y')\boldsymbol{\Psi}_b(x'-x,y'-y)|^2 dx'dy', \quad (17)$$

where $x(t)$ and $y(t)$ represent the instantaneous position of the scanning beam. For uniform scanning speed, v, of the beam, we have $x(t)=vt$ and $y(t)=vt$. In terms of correlations in two dimensions for real signals f and g, we have[18]

$$f(x,y) \oplus g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x',y')g(x+x',y+y')dx'dy', \quad (18)$$

and Eq. (17) can be written as

$$i(x,y) = |\boldsymbol{\Psi}_b(x,y)|^2 \oplus |\Gamma_0(x,y)|^2. \quad (19)$$

Note that the beam field distribution $\boldsymbol{\Psi}_b(x,y)$ and the pupil function $p(x,y)$ located in the front focal plane of Lens L (see Fig. 5) are related by a Fourier transform. For instance, assume that the light field that illuminates the pupil function is uniform and $p(x,y)=1$. Then $\boldsymbol{\Psi}_b(x,y)$ becomes a delta function. Equation (17) consequently gives an exact replica of $|\Gamma_0(x,y)|^2$.

In general, $|\Gamma_0(x,y)|^2$ is processed by $|\boldsymbol{\Psi}_b(x,y)|^2$, even though the object $\Gamma_0(x,y)$ originally may be complex. The fact that the object's intensity, $|\Gamma_0(x,y)|^2$, is manipulated by a real and non-negative quantity, $|\boldsymbol{\Psi}_b(x,y)|^2$, is known as incoherent optical image processing.[18] Such an optical scanning system cannot manipulate any phase information. Because holography requires the preservation of the phase, we need to find a way to preserve the phase information during photodetection if we expect to use optical scanning to record holographic information. The solution to this problem is optical scanning heterodyning. In the latter the object is scanned by a time-dependent Fresnel zone plate, which is the

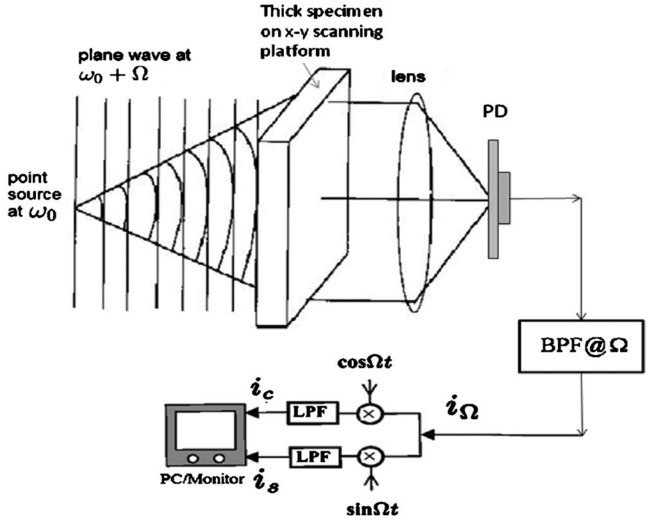

Fig. 6. Optical scanning holography setup. The thick specimen is scanned two-dimensionally by the combination of a spherical wave and a plane wave of different frequencies. The photodetector PD collects all the light and delivers a heterodyne signal at frequency Ω. The heterodyne signal is then electronically processed to give two processed outputs. BPF is a bandpass filter and LPF is a lowpass filter.

superposition of a spherical wave and a plane wave of different temporal frequencies. The top part of Fig. 6 shows the configuration. The lens is used as a light collector that collects all of the transmitted light to the photodetector. In the actual experimental setup, the plane wave and the spherical wave are combined, though a beam splitter and the spherical wave can be derived from a focusing laser beam. The temporal frequency difference Ω between the two waves can be provided by using an acousto-optic modulator[8,19] or an electro-optic modulator[8] in the path of the plane wave.

Assume that the spot of the focusing laser beam that generates the spherical wave is a distance z from the object transparency $\Gamma_0(x,y;z)$. We can express the scanning beam pattern at the transparency as

$$\boldsymbol{\Psi}_b(x,y;z) = a\exp[i(\boldsymbol{\omega}_0+\Omega)t] + \frac{ik_0b}{2\boldsymbol{\pi}z}\exp\left[\frac{-ik_0}{2z}(x^2+y^2)\right]\exp(i\boldsymbol{\omega}_0t), \quad (20)$$

where $\boldsymbol{\omega}_0$ and $\boldsymbol{\omega}_0+\Omega$ are the frequencies of the spherical wave and the plane wave, respectively. As in Eq. (17), the photodetector generates a current given by

$$i(x,y) = \int\int_S |\Gamma_0(x',y';z)\boldsymbol{\Psi}_b(x'-x,y'-y;z)|^2 dx'dy'. \quad (21)$$

If we substitute Eq. (20) into Eq. (21) and keep only the heterodyne current (by using a bandpass filter tuned at the heterodyne frequency Ω), we have

$$i_\Omega(x,y) = ab\frac{k_0}{\boldsymbol{\pi}z}\sin\left[\Omega t + \frac{k_0}{2z}(x^2+y^2)\right] \oplus |\Gamma_0(x,y;z)|^2. \quad (22)$$

This heterodyne current shows that the information of the object is carried by the heterodyne frequency. To extract the information associated with the object, we can electronically mix it with a sine and a cosine function at the heterodyne frequency to obtain the in-phase and the quadrature components of the heterodyne current, respectively. We have

$$i_\Omega(x,y)\times\sin\Omega t = \frac{abk_0}{2\boldsymbol{\pi}z}\left\{\cos\left[\frac{k_0}{2z}(x^2+y^2)\right] \oplus |\Gamma_0(x,y;z)|^2 - \cos\left[2\Omega t + \frac{k_0}{2z}(x^2+y^2)\right] \oplus |\Gamma_0(x,y;z)|^2\right\}, \quad (23)$$

where we have used the identity $\sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$. By using an electronic lowpass filter, we can extract the first term of Eq. (23) and finally obtain the cosine-Fresnel zone plate hologram:

$$i_s(x,y) \propto H_{\cos}(x,y) = \frac{k_0}{2\boldsymbol{\pi}z}\cos\left[\frac{k_0}{2z}(x^2+y^2)\right] \oplus |\Gamma_0(x,y;z)|^2. \quad (24)$$

Similarly, if a cosine function in Eq. (23) is used, that is, $\cos\Omega t$, we obtain the product of sine and cosine functions. After lowpass filtering, we have the sine-Fresnel zone plate hologram:

$$i_c(x,y) \propto H_{\sin}(x,y) = \frac{k_0}{2\boldsymbol{\pi}z}\sin\left[\frac{k_0}{2z}(x^2+y^2)\right] \oplus |\Gamma_0(x,y;z)|^2. \quad (25)$$

Electronic processing is shown in the lower part of Fig. 6.

Equations (24) and (25) are for a planar object $|\Gamma_0(x,y;z)|^2$ located a distance z from the scanning point source. If we consider a 3D object as a collection of planes along the z-direction, the holograms of the 3D object become

$$H_{\cos}(x,y) = \int \frac{k_0}{2\boldsymbol{\pi}z}\cos\left[\frac{k_0}{2z}(x^2+y^2)\right] \oplus |\Gamma_0(x,y;z)|^2 dz; \quad (26a)$$

$$H_{\sin}(x,y) = \int \frac{k_0}{2\boldsymbol{\pi}z}\sin\left[\frac{k_0}{2z}(x^2+y^2)\right] \oplus |\Gamma_0(x,y;z)|^2 dz. \quad (26b)$$

That is, we integrate the results of Eqs. (24) and (25) along z assuming that the 3D object is weakly scattering. Equation (26) is the major result of optical scanning holography. For each 2D scanning of the 3D object, we have two holograms.

V. EXAMPLE OF THE OPTICAL SCANNING HOLOGRAPHIC RECORDING OF POINT SOURCES

As an example, we let the object be a pinhole a distance z_0 from the scanning point source, that is, $|\Gamma_0(x,y;z)|^2 = \boldsymbol{\delta}(x,y)\boldsymbol{\delta}(z-z_0)$. From Eq. (26b) we have

$$H_{\sin}(x,y) = \int \frac{k_0}{2\pi z} \sin\left[\frac{k_0}{2z}(x^2+y^2)\right] \oplus \boldsymbol{\delta}(x,y)\boldsymbol{\delta}(z-z_0)dz$$
$$= \frac{k_0}{2\pi z_0} \sin\left[\frac{k_0}{2z_0}(x^2+y^2)\right] \oplus \boldsymbol{\delta}(x,y)$$
$$= \frac{k_0}{2\pi z_0} \sin\left[\frac{k_0}{2z_0}(x^2+y^2)\right], \quad (27)$$

where we have used the sampling property of the delta function to evaluate the integration along z. The expression in Eq. (27) is almost Eq. (9) with the exception of the DC term, and hence is the holographic recording of a single point without any DC.

What happens when we have a two-point object given by $|\Gamma_0(x,y;z)|^2 = \boldsymbol{\delta}(x,y)\boldsymbol{\delta}(z-z_0) + \boldsymbol{\delta}(x-x_0,y-y_0)\boldsymbol{\delta}(z-z_0)$. We use this form in Eq. (26b) and obtain

$$H_{\sin}(x,y) = \frac{k_0}{2\pi z_0} \sin\left[\frac{k_0}{2z_0}(x^2+y^2)\right] + \frac{k_0}{2\pi z_0} \sin\left[\frac{k_0}{2z_0}[(x-x_0)^2+(y-y_0)^2]\right]. \quad (28)$$

Equation (28) represents two Fresnel zone plates without the undesired intermodulation term shown in Eq. (16). This removal of the intermodulaton term is possible because holographic information is generated by heterodyne scanning. In addition, because of heterodyning, we have two holograms [see Eq. (26)] for each 2D scan. The cosine-Fresnel zone plate hologram is not redundant and is useful for the elimination of the twin-image.

We can form the complex hologram given by

$$H_{\pm}^{c}(x,y) = \beta[H_{\cos}(x,y) \pm iH_{\sin}(x,y)], \quad (29)$$

where β is a constant. Reconstruction of such a complex hologram will not give rise to a twin-image. As an example, the complex hologram for a single point object is given by substituting Eqs. (24) and (25) into Eq. (29) with $\beta=1$,

$$H_{-}^{c}(x,y) = \frac{k_0}{2\pi z_0} \exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right], \quad (30)$$

where we have taken the negative sign in Eq. (29).

Equation (30) is for the complex field [see Eq. (6)], aside from a constant, due to a single point source. This hologram contains only the desired information of the point source because there is no term leading to the twin-image and the annoying background noise due to intermodulation, which unavoidably comes from conventional holographic recording [see Eq. (8)]. To see how only the desirable point source is restored, we reconstruct the complex hologram by the illumination of a plane wave. Using the notation of Eq. (11), we have

$$\boldsymbol{\Psi}_{\mathrm{rec}} H_{-}^{c}(x,y) \otimes h(x,y;z) \propto \frac{k_0}{2\pi z_0} \exp\left[\frac{-ik_0}{2z_0}(x^2+y^2)\right] \otimes \frac{ik_0}{2\pi z} \exp\left[\frac{-ik_0}{2z}(x^2+y^2)\right] \propto \boldsymbol{\delta}(x,y) \quad (31)$$

for $z=-z_0$. We see that a virtual point object has been reconstructed at a distance z_0 behind the hologram, and there is no real point object (the twin image) formed in front of the hologram. In addition, there is no zeroth-order beam. If we use the + sign from Eq. (29), we obtain $H_{+}^{c}(x,y)$ and upon the illumination of a plane wave of such a hologram, we will reconstruct only a real point object in front of the hologram.

VI. SUMMARY AND REMARKS

The hologram of a point source object has been discussed, and we showed that a Fresnel zone plate is the hologram of a point object. We then demonstrated the reconstruction of the Fresnel zone plate and showed that in addition to the reconstruction of the original point object, a twin-image has been formed. For a two-point object, the recorded hologram is a collection of two zone plates and a grating, with the two zone plates corresponding to each of the two point sources and the grating resulting from the interference or intermodulation of the two point sources. The two zone plates corresponding to the two point sources generate the original two point sources, but also create twin-images, and the grating arising from the intermodulation in general introduces noise on the reconstruction plane.

We then introduced optical scanning holography, which is unconventional in the sense that its holographic recording of a point source object gives a sine-Fresnel zone plate hologram and a cosine-Fresnel zone plate hologram. From the two Fresnel zone plate holograms, we can construct a complex hologram that does not create a twin-image upon reconstruction. For multiple point sources, no intermodulations are recorded. This explanation of holographic recording is called the zone plate approach and was pointed out by Givens as an alternative approach to the understanding of holography.[12] At that time, it was pointed out the approach was "slightly less than perfect" because the zone plate approach could not reproduce the hologram generated by Gabor's approach.[10] We found, however, that the zone plate approach can be implemented and is a better way to generate holograms because no intermodulation is recorded and we can eliminate the twin-image when a complex hologram is used.

For a complete discussion of optical scanning holography and its current applications, readers are referred to Ref. 18. In the Appendix, we show some examples of optical scanning holography-based holograms and their numerical reconstruction.

APPENDIX: EXAMPLES OF OPTICAL SCANNING HOLOGRAPHY-BASED HOLOGRAMS AND THEIR NUMERICAL RECONSTRUCTION

In this Appendix we first formulate optical scanning holography in the frequency domain. We then show some numerical results. An example of the MATLAB code can be found in Ref. 18. For some basic use of MATLAB in optical imaging, readers are encouraged to consult Ref. 20.

For a planar intensity object located a distance z_0 from the point source of the scanning beam, the holograms obtained according to Eqs. (24) and (25) are

$$H_{\cos}(x,y) = \frac{k_0}{2\pi z_0} \cos\left[\frac{k_0}{2z_0}(x^2+y^2)\right] \otimes |\Gamma_0(x,y;z_0)|^2, \quad (A1)$$

and

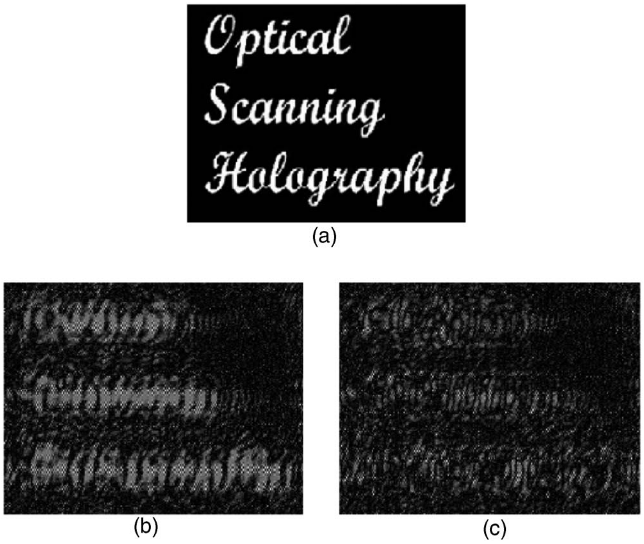

Fig. 7. (a) Original planar intensity object, (b) sine-hologram of (a), and (c) cosine-hologram of (a). The holograms are based on Eq. (A7) with $z_0/2k_0=2.0$.

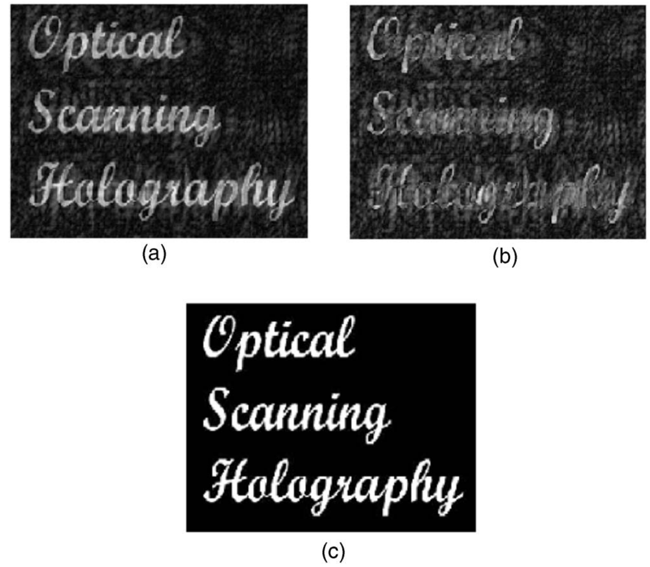

Fig. 8. (a) Reconstruction of Fig. 7(b), (b) reconstruction of Fig. 7(c), and (c) reconstruction of the complex hologram constructed according to Eq. (A3). These reconstructions are based on Eq. (A11) with $z_0/2k_0=2.0$. Note that twin-image noise exists in (a) and (b).

$$H_{\sin}(x,y)=\frac{k_0}{2\pi z_0}\sin\left[\frac{k_0}{2z_0}(x^2+y^2)\right]\otimes|\Gamma_0(x,y;z_0)|^2. \tag{A2}$$

We have replaced the correlation operation by the convolution operation, which is possible because the cosine- and sine-Fresnel zone plates are symmetrical. We then construct a complex hologram according to the general guideline given by Eq. (29). We construct

$$\begin{aligned}H_+^c(x,y)&=-i[H_{\cos}(x,y)+iH_{\sin}(x,y)]\\&=\frac{-ik_0}{2\pi z_0}\exp\left[\frac{ik_0}{2z_0}(x^2+y^2)\right]\otimes|\Gamma_0(x,y;z_0)|^2,\end{aligned} \tag{A3}$$

where we have chosen $\beta=-i$ for convenience.

We define the 2D Fourier transform as

$$\begin{aligned}F\{g(x,y)\}&=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}g(x,y)\exp(ik_xx+ik_yy)dxdy\\&=G(k_x,k_y)\ \text{(Ref. 8)},\end{aligned} \tag{A4}$$

where k_x and k_y are spatial frequencies[9,18] corresponding to the spatial coordinates x and y, respectively. (Spatial frequencies are commonly referred to as wave numbers in physics.) By taking the 2D transform of Eq. (A3), we have

$$\begin{aligned}F\{H_+^c(x,y)\}&=F\left\{\frac{-ik_0}{2\pi z_0}\exp\left[\frac{ik_0}{2z_0}(x^2+y^2)\right]\right\}F\{|\Gamma_0(x,y;z_0)|^2\}\\&=OTF_{\text{osh}}(k_x,k_y;z_0)F\{|\Gamma_0(x,y;z_0)|^2\},\end{aligned} \tag{A5}$$

where we have used the identity $F\{g_1(x,y)\otimes g_2(x,y)\}=F\{g_1(x,y)\}\times F\{g_2(x,y)\}$ to obtain the second equality in Eq. (A5). The first term of the right side of Eq. (A5) can be shown to give

$$OTF_{\text{osh}}(k_x,k_y;z_0)=\exp\left[\frac{-iz_0}{2k_0}(k_x^2+k_y^2)\right], \tag{A6}$$

which is the optical transfer function of optical scanning holography.[8] For a given planar intensity object, $|\Gamma_0(x,y;z_0)|^2$ we calculate the complex hologram according to Eq. (A5), and aside from a constant, the sine- and cosine-holograms can be obtained by taking the real and imaginary part of the spatial domain of Eq. (A5); that is,

$$H_{\sin}(x,y)=\text{Re}[F^{-1}\{OTF_{\text{osh}}(k_x,k_y;z_0)F\{|\Gamma_0(x,y;z_0)|^2\}\}] \tag{A7a}$$

and

$$H_{\cos}(x,y)=\text{Im}[F^{-1}\{OTF_{\text{osh}}(k_x,k_y;z_0)F\{|\Gamma_0(x,y;z_0)|^2\}\}], \tag{A7b}$$

where F^{-1} denotes the inverse Fourier transform operation. A complex hologram can then be constructed according to Eq. (29) to obtain $H_\pm^c(x,y)$. A reconstruction of the hologram at a plane located z_0 from the hologram is calculated according to

$$H_{\text{any}}(x,y)\otimes h(x,y;z_0), \tag{A8}$$

where $H_{\text{any}}(x,y)$ represents the sine-hologram, the cosine-hologram, or the complex hologram. We implement the reconstruction in the frequency domain and write Eq. (A8) as

$$F^{-1}\{F\{H_{\text{any}}(x,y)\}F\{h(x,y;z_0)\}\}. \tag{A9}$$

It can be shown that

$$\begin{aligned}F\{h(x,y;z_0)\}&=\exp(-ik_0z_0)\exp\left[\frac{iz_0}{2k_0}(k_x^2+k_y^2)\right]\\&=[OTF_{\text{osh}}(k_x,k_y;z_0)]^*,\end{aligned} \tag{A10}$$

where we have neglected the constant phase term $\exp(-ik_0z_0)$ to obtain the last step. Hence, the reconstruction formula given by Eq. (A9) can be written as

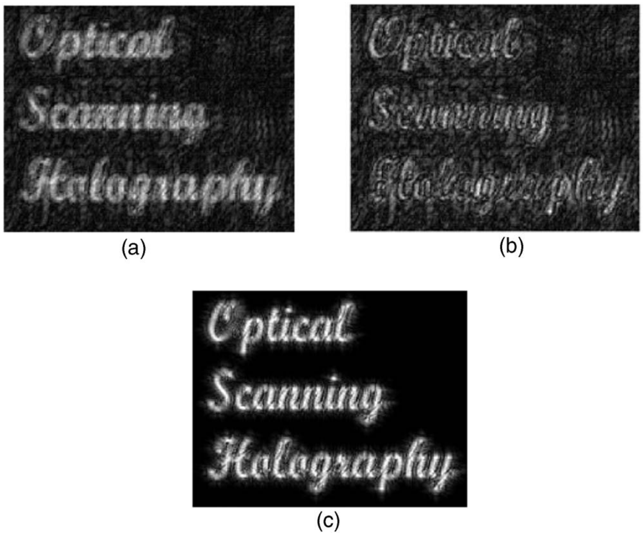

Fig. 9. (a) Reconstruction of Fig. 7(b), (b) reconstruction of Fig. 7(c), and (c) reconstruction of the complex hologram constructed according to Eq. (A3). These reconstructions are based on Eq. (A11) with $z_0/2k_0=1.95$. Note that twin-image noise exists in (a) and (b). These reconstructions are generally out of focus.

$$\text{Reconstruction} = F^{-1}\{F\{H_{\text{any}}(x,y)\}[OTF_{\text{osh}}(k_x,k_y;z_0)]^*\}. \tag{A11}$$

In summary, we have formulated optical scanning holography in the frequency domain. To construct holograms, we incorporate Eq. (A6) into Eqs. (A7) and (29). For reconstruction we use Eq. (A11).

In Fig. 7 we show the original planar intensity object and its sine- and cosine-holograms. Both of the holograms based on Eq. (A7) are generated using $z_0/2k_0=2$, which is proportional to the recording distance z_0. In Fig. 8 we show the reconstruction of the sine-hologram, the cosine-hologram, and the complex hologram constructed based on Eq. (A3). These reconstructions are based on Eq. (A11) with $z_0/2k_0=2$. In Fig. 9, we show reconstruction in different transverse planes using $z_0/2k_0=1.95$. Note that these reconstructions illustrate defocusing of the original object.

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