Two-step-only quadrature phase-shifting digital holography

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Received September 30, 2008; revised November 11, 2008; accepted December 4, 2008;
posted December 15, 2008 (Doc. ID 102164); published January 22, 2009

Conventional methods of quadrature phase-shifting holography require two holograms and either intensity distribution of the reference wave or that of the object wave to reconstruct an original object without the zero order and the twin-image noise in an on-axis holographic recording setup. We present a technique called two-step-only quadrature phase-shifting holography in which solely two quadrature-phase holograms are required. Neither reference-wave intensity nor an object-wave intensity measurement is needed in the technique.

OCIS codes: 090.0090, 090.1995.

Quadrature phase-shifting holography (QPSH), also called two-step phase-shifting holography, was first proposed by Gabor and Goss in the 1960s in their quest for 3D holographic microscopy [1]. A couple of years later, Burckhardt and Enloe proposed three-step phase-shifting holography when they investigated the television transmission of a hologram [2]. Since then, three-step and four-step phase-shifting holography have been used for modern applications [3–7]. Owing to the need for multiple phase-shift holograms to be recorded and the requirement for phase stability among the different phase shifts in order to obtain good quality image reconstruction in three- and four-step phase-shifting holography, quadrature phase-shifting techniques found advocates in recent years [8–12]. For quadrature phase-shifting holography, in order to reconstruct the original complex object wave without the zero-order light and the twin image, two quadrature-phase holograms and two intensity values (namely the object wave intensity and the reference wave intensity) are needed. Guo and Devaney [8] in the United States and Wang *et al.* [9] from China, about the same time, recently performed experiments to verify the idea of two-step phase shifting. Recently, Meng *et al.* [10] proposed an algorithm such that only two holograms and one intensity value, namely the reference intensity, are needed to extract the original complex object wave. Subsequently, the quadrature phase-shifting method has been carried out at one time using optical array devices [11]. Here, we report a technique where only two on-axis holograms are required to reconstruct the original complex object wave.

A typical quadrature phase-shifting holographic setup is shown in Fig. 1. When the slow axis of the quarter-wave plate (QWP) is parallel to the polarization state of the reference light, we record the first hologram. The second hologram is subsequently recorded in the condition that the fast axis of the QWP is parallel to the polarization state. Thus the QWP will introduce a phase shift of the reference wave in the two holograms by 90°.

Two quadrature-phase on-axis holograms are recorded sequentially and expressed as

$$I_{H1} = |R + \vartheta|^2 = I_O + 2\,\mathrm{Re}(\vartheta)R, \tag{1}$$

$$I_{H2} = |e^{j\pi/2}R + \vartheta|^2 = |jR + \vartheta|^2 = I_O + 2\,\mathrm{Im}(\vartheta)R, \tag{2}$$

where R is the amplitude of the reference light on the CCD camera, which is assumed to be a normal incident plane wave and taken to be a real constant for simplicity. ϑ denotes the complex amplitude of the object light on the CCD camera, and I_O is the zero-order light given by

$$I_O = R^2 + |\vartheta|^2. \tag{3}$$

Reconstruction of the hologram given by Eqs. (1) or (2) will give the twin-image noise and the spatially varying zero-order light on the reconstruction plane. However, if we construct a complex hologram, I_C, according to

$$\begin{aligned} I_C &= I_{H1} + jI_{H2} = I_O + jI_O + 2R[\mathrm{Re}(\vartheta) + j\,\mathrm{Im}(\vartheta)] \\ &= I_O + jI_O + 2R\vartheta, \end{aligned} \tag{4}$$

we obtain a twin-image-free hologram, as I_C is now proportional to ϑ in addition to the terms in the zero-order light. However, we can eliminate the contributions due to the zero-order light if we measure the intensity of R and ϑ before holographic recordings; i.e., I_O can be completely determined through measurements. Once I_O is found, we can finally obtain a twin-

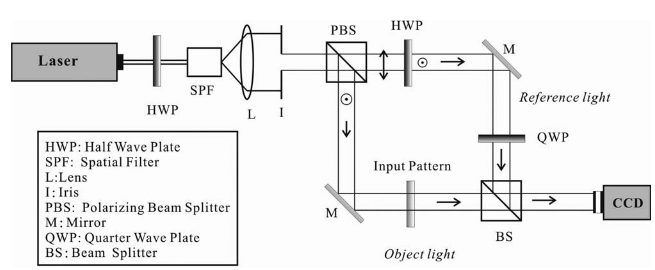

Fig. 1. Typical experimental setup of QPSH.

0146-9592/09/030250-3/$15.00

image- and zero-order-free hologram by subtracting I_O+jI_O from I_C; i.e., we have

$$H_{\mathrm{PHS}}=(I_{H1}-I_O)+j(I_{H2}-I_O). \quad (5)$$

In essence, that is what Guo and Devaney and Wang *et al.* have done experimentally. In this approach, two quadrature-phase holograms and two intensity values have to be measured—a total of four recordings. We shall refer such method as the "standard" quadrature phase-shifting digital holography. Could we achieve the same goal; i.e., to obtain a twin-image-free reconstruction in addition to the elimination of the zero-order light, by making fewer than four recordings? We show such goal can be achieved with the recording of only two holograms.

From the two captured quadrature-phase holograms, we shall now derive an expression for I_O, which is a function of R^2. This is advantageous because we can simply measure R^2 in addition to the two holograms to obtain the twin-image- and zero-order-free hologram. By taking square of the absolute value of both side of Eq. (4), we obtain a quadratic equation in I_O:

$$2I_O^2-(4R^2+2I_{H1}+2I_{H2})I_O+(I_{H1}^2+I_{H2}^2+4R^4)=0. \quad (6)$$

The solutions to the above quadratic equation can be easily found to be

$$I_O=\frac{2R^2+I_{H1}+I_{H2}}{2}\pm\frac{\sqrt{(2R^2+I_{H1}+I_{H2})^2-2(I_{H1}^2+I_{H2}^2+4R^4)}}{2}. \quad (7)$$

Thus the zero-order light can be found without additional measurements, provided the intensity of the reference light is known and the +/− sign can be decided. Note that with the exception of +/− sign replaced by the minus sign in the equation this result is identical to the one given by Meng *et al.* [10] when the phase shift is exactly equal to $\pi/2$. Since only one of the signs is the correct solution, we shall determine it now. From Eqs. (1) and (2), we calculate

$$I_{H1}+I_{H2}+2R^2=2I_O+2[R+\mathrm{Re}(\vartheta)+\mathrm{Im}(\vartheta)]R, \quad (8)$$

and from which, we can write

$$I_O=\frac{2R^2+I_{H1}+I_{H2}}{2}-[R+\mathrm{Re}(\vartheta)+\mathrm{Im}(\vartheta)]R. \quad (9)$$

Comparing Eq. (7) with Eq. (9), we find that if $R+\mathrm{Re}(\vartheta)+\mathrm{Im}(\vartheta)=F>0$, the minus sign should be chosen. The quantity, F, will be positive everywhere if the intensity of reference light is large enough, which is generally true in practice. If R is not large enough to ensure the requirement $F>0$ anywhere, the calculated I_O at some pixels will be incorrect, thus inducing noise in the reconstructed image. To determine how large the value of R should be with respect to the object wave, we define an amplitude ratio, A, as

$$A\equiv\frac{R}{\frac{1}{2}[\max(|\vartheta|)+\min(|\vartheta|)]}, \quad (10)$$

and a parameter called correlation factor, CF, as

$$\mathrm{CF}\equiv\frac{[|E_R|^2\otimes I_{\mathrm{orig}}]_{\mathrm{peak}}}{\sum|E_R|^2}, \quad (11)$$

where $\otimes$ denotes the operation of cross correlation; I_{orig} is the intensity of the original object, Σ is a summation of all data pixels, and E_R denotes the calculated complex amplitude of the object light in the reconstruction plane at different intensity values of the reference light R^2. Hence, E_R is a function of R^2, and the calculation of E_R is based on Eq. (5) with Eq. (7) for different value of R^2. If the zero-order light calculated from Eq. (7) is correct, the reconstructed image should be similar to the original object. We shall now show a series of results based on the following conditions. The wavelength of the laser is 0.6328 μm, and the object is 5 cm from the CCD. The number of the pixels of the object is 750 (H) by 710 (V) with pixel size of 3.3 μm, which translates to the size of the object of 2.48 mm × 2.34 mm.

In Fig. 2, we plot CF versus A^2. The curve of QPSH is for reconstruction by the standard quadrature phase-shifting digital holography, while the curves for QPSH− and QPSH+ are reconstructions by using Eqs. (5) and (7) when the reference wave is measured in addition to the two holograms. Note that the signs − and + after QPSH denote the sign used in Eq. (7). The straight line, obtained by the standard quadrature phase-shifting digital holographic technique is our reference line, as this technique represents an ideal reconstruction. From the figure, obviously QPSH− is the correct choice, but if the intensity of the reference light is too small (less than $A^2=1$), the quality of the reconstruction begins to decline as the CF factor begins to dip.

Based on Eqs. (5) and (7) and the guidance from Fig. 2, we conclude that we can use two quadrature-phase holograms and a premeasured reference wave intensity to obtain a twin-image- and zero-order-free hologram if $A^2\geq 1$; i.e., the reconstruction is as good as that given by standard QPSH. We shall now pro-

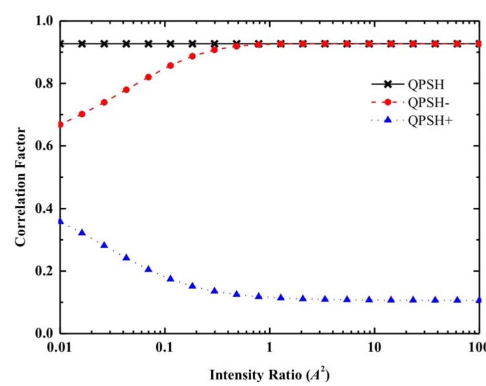

Fig. 2. (Color online) Correlation factors of various phase-shift holograms versus intensity ratio.

pose an algorithm to determine the reference wave intensity directly from the two holograms without the actual need to record the reference wave intensity at all. If successful, this will increase the applicability of the quadrature-phase method, as the technique no longer depends on the premeasure of the reference intensity before holographic recordings. We shall call such overall two-step phase-shifting method two-step-only. We find the reference light intensity (without actual measurement) by searching all the most possible values. From common sense, R^2 should be within 0 and the maximum of I_{H1}. We then test the resultant hologram by using different R^2. We define the image reconstructed by I_{H1} as the target image, E_T, as it contains the correct object information although the zero-order light still exists. Then we calculate a parameter called normalized correlation peak, NCP, which is defined as

$$\mathrm{NCP} \equiv \frac{\mathrm{real}\{[E_T \otimes E_R]_{\max}\}}{[E_R \otimes E_R]_{\max}}, \quad (12)$$

where real{.} denotes the real part of {.}. We then plot $R^2/\max(I_{H1})$ or the relative intensity of the reference light versus NCP and set the criterion such that the peak of the curve locates the actual value of R^2. We call the actual value the setup value in the legend of subsequent plots. With this found value of R^2 we can reconstruct the hologram based on Eqs. (5) and (7).

Based on the idea of NCP discussed, Fig. 3 shows the NCP curve for different A values, and Fig. 4 shows the reconstruction by using the peak value found in the NCP method with the A values corresponding to those of Fig. 3. Note that in Fig. 3 the peak value of the curve correctly locates the setup value, which is the unknown value of R^2 used in the simulations. The technique we propose here to obtain the reconstructed images, shown in Fig. 4, is called two-step-only quadrature phase-shifting holography (TSO-QPSH) to emphasize the fact that only two holograms are needed to perform zero-order- and twin-image-free reconstruction under on-axis holographic recording. For $A \geqslant 1$, the NCP curve predicting the reconstruction [see Figs. 4(a) and 4(b)] based on TSO-QPSH is the same as that calculated by standard QPSH. For the values when the reference intensity value is not larger than that of the object wave intensity, there are errors in the reconstruction as seen from Fig. 4(c).

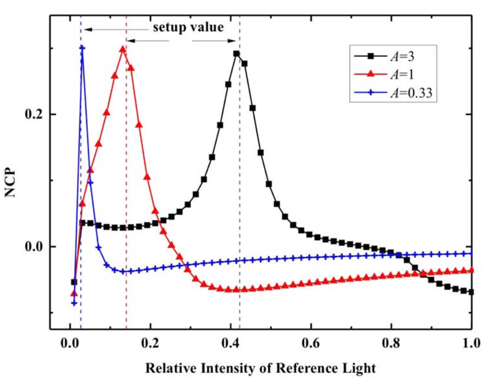

Fig. 3. (Color online) NCP curves for different A based on TSO-QPSH.

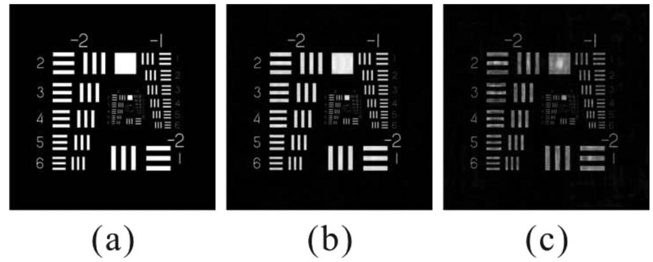

Fig. 4. Image reconstruction for different A based on TSO-QPSH: (a) $A=3$, (b) $A=1$, (c) $A=0.33$.

We have proposed two-step-only quadrature phase-shifting holography, in which only two quadrature-phase holograms are needed to reconstruct a zero-order- and twin-image-free hologram. Such technique precludes the need from recording either the reference wave or the object wave intensity before recording of the quadrature-phase holograms. Consequently, TSO-QPSH simplifies the setup of QPSH. The conditions and abilities of TSO-QPSH have also been discussed and verified. Results showed that TSO-QPSH works as good as the standard QPSH under typical holographic recording conditions ($A \geqslant 1$).

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