Retroactive Anti-Jamming for MISO Broadcast Channels

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Abstract

Jamming attacks can significantly impact the performance of wireless communication systems. In addition to reducing the capacity, such attacks may lead to insurmountable overhead in terms of re-transmissions and increased power consumption. In this paper, we consider the multiple-input single-output (MISO) broadcast channel (BC) in the presence of a jamming attack in which a subset of the receivers can be jammed at any given time. Further, countermeasures for mitigating the effects of such jamming attacks are presented. The effectiveness of these anti-jamming countermeasures is quantified in terms of the degrees-of-freedom (DoF) of the MISO BC under various assumptions regarding the availability of the channel state information (CSIT) and the jammer state information at the transmitter (JSIT). The main contribution of this paper is the characterization of the DoF region of the two user MISO BC under various assumptions on the availability of CSIT and JSIT. Partial extensions to the multi-user broadcast channels are also presented.

1 Introduction

Wireless communication systems have now become ubiquitous and constitute a key component of the fabric of modern day life. However, the inherent openness of the wireless medium makes it susceptible to adversarial attacks. The vulnerabilities of the wireless system can be largely classified based on the capability of an adversary—

a) Eavesdropping attack, in which the eavesdropper (passive adversary) can listen to the wireless channel and try to infer information (which if leaked may severely compromise data integrity). The study of information theoretic security (or communication in presence of eavesdropping attacks) was initiated by Wyner [1], Csiszár and Körner [2]. Recently, there has been a resurgent interest in extending these results to multi-user scenarios. We refer the reader to a comprehensive tutorial [3] on this topic and the references therein.

b) Jamming attack, in which the jammer (active adversary) can transmit information in order to disrupt reliable data transmission or reception. While there has been some work in studying the impact of jamming on the capacity of point-to-point channels (such as [4–6]), the literature on information theoretic analysis of jamming attacks (and associated countermeasures) for multi-user channels is relatively sparse in comparison to the case of eavesdropping attacks.

In this paper, we focus on a class of time-varying jamming attacks over a fast fading multi-user multiple-input single-output (MISO) broadcast channel (BC), in which a transmitter equipped with $K$ transmit
antennas intends to send independent messages to $K$ single antenna receivers. While several jamming scenarios are plausible, we initiate the study of jamming attacks by focusing on a simple yet harmful jammer. In particular, we consider a jammer equipped with $K$ transmit antennas and at any given time instant, has the capability of jamming a subset of the receivers. We consider a scenario in which the jammers' strategy at any given time is random, i.e., the subset of receivers to be jammed is probabilistically selected. Furthermore, the jamming strategy varies in an independent and identically distributed (i.i.d.) manner across time\(^1\). Such random, time-varying jamming attacks may be inflicted either intentionally by an adversary or unintentionally, in different scenarios. We next highlight some plausible scenarios in which such random time varying jamming attacks could arise.

A resource constrained jammer that intentionally jams the receivers may conserve power by selectively jamming a subset (or none) of the receivers based on its available resources. Such a jammer can also choose to jam the receivers when it has information about channel sounding procedures (i.e., when this procedure occurs) and disrupts the communication only during those specific time instants. Interference from neighboring cells in a cellular system can act as a bottleneck to improve spectral efficiency and be particularly harmful for cell edge users. The interference seen from adjacent cells in such scenarios can be time varying depending on whether the neighboring cells are transmitting on the same frequency or not (which can change with time); and the spatial separation of the users from interfering cells. A frequency-selective jammer can disrupt communication on certain frequencies (carriers) in multi-carrier (for instance OFDM-based) systems. A jammer that has knowledge about the pilot signal-based synchronization procedures, can jam only those sub carriers that carry the pilot symbols in order to disrupt the synchronization procedure of the multi-carrier system [8].

Our analysis in this paper suggests that the transmitter and receivers based on the knowledge of the jammers' strategy, can reduce the effects of these jamming attacks by coding/transmitting across various jamming states (jamming state here can be interpreted as the subset of frequencies/sub-carriers that are jammed at a given time instant).

Interestingly, the MISO BC with a time-varying jamming attack can also be interpreted as a network with a time-varying topology. The concept of topological interference alignment has been recently introduced in [9] (also see [10], [11]) to understand the effects of time-varying topology on interference mitigation techniques such as interference alignment. In [10], the authors characterize the DoF by studying the interference management problem in such networks using a 1-bit delay-less feedback (obtained from the receivers) indicating the presence or absence of an interference link. The connection between jamming attacks considered in this paper and time-varying network topologies can be noted by observing the following: if at a given time, a receiver is jammed, then its received signal is completely drowned in the jamming signal (assuming jamming power as high as the desired signal) which is analogous to the channel (or link) to the jammed receiver being wiped out. For instance, in a 3-user MISO BC with a time-varying jamming attack, a total of $2^3 = 8$ topologies could arise (see Figure 1) over time: none of the receivers are jammed (one topology), all receivers are jammed (one topology), only one out of the three receivers is jammed (three topologies), or only two out of three receivers are jammed (i.e., three topologies). Interestingly, the retroactive anti-jamming techniques presented in this paper are philosophically related to topological interference alignment with alternating connectivity [10]. The common theme that emerges is that it is necessary to code across multiple jamming states (equivalently, topologies as in [10]) in order to achieve the optimal performance, which is measured in terms of degrees of freedom (capacity at high SNR).

The model considered in the paper also bears similarities with broadcast erasure channels studied in [12], [13] etc. The presence of a jamming signal ($J$) at a receiver implies that the information bearing signal ($X$)

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\(^1\)While we realize that perhaps more sophisticated jamming scenarios may arise in practice, as a first step, it is important to understand i.i.d jamming scenarios before studying the impact of more complicated attacks (such as time/signal correlated jammer, on-off jamming etc). Even in the i.i.d. jamming scenarios, interesting and non-trivial problems arise that we address in this paper in the context of broadcast channels.
is un-recoverable from the received signal ($Y = X + J + N$) in the context of degrees of freedom (since the pre-log of mutual information between $X$ and $Y$ would be zero as both signal and jamming powers become large). Hence, the presence of a jammer can be interpreted as an “erasure”. In the absence of a jammer (or no “erasure”), the signal $X$ can be recovered from $Y = X + N$ within noise distortion.

We study the impact of such random time-varying jamming attacks on the degrees-of-freedom (henceforth referred by DoF) region of the MISO BC. The DoF of a network can be regarded as an approximation of its capacity at high SNR and is also referred to as the pre-log of capacity. Even in the absence of a jammer, it is well known that the DoF is crucially dependent on the availability of channel state information at the transmitter (CSIT). The DoF region of the MISO BC has been studied under a variety of assumptions on the availability of CSIT including full (perfect and instantaneous) CSIT [14], no CSIT [15, 16], delayed CSIT [17, 18], compound CSIT [19], quantized CSIT [20], mixed (perfect delayed and partial instantaneous) CSIT [21] and asymmetric CSIT (perfect CSIT for one user, delayed CSIT for the other) [22]. To note the dependence of DoF on CSIT, we remark that a sum DoF of 2 is achieved in the 2-user MISO BC when perfect CSIT information is available [14], while it reduces to 1 (with statistically equivalent receivers) when no CSIT is available [16]. Interestingly it is shown in [17] that completely outdated CSIT in a fast fading channel is still useful and helps increase the DoF from 1 to $\frac{3}{2}$. Interesting extensions to the $K$-user case with delayed CSIT are also presented in [17]. In this paper, we denote the availability of CSIT (by CSI, we refer to the channel between the transmitter and the receiver, we do not assume the knowledge of the jammer’s channel at the transmitter or the receivers) through a variable $I_{\text{CSI}}$, which can take values either $P$, $D$ or $N$: where the state $I_{\text{CSI}} = P$ indicates that the transmitter has perfect and instantaneous channel state information at time $t$, the state $I_{\text{CSI}} = D$ indicates that the transmitter has perfect but delayed channel state information (i.e., it has knowledge of the channel realizations of time instants $\{1, 2, \ldots, t - 1\}$ at time $t$), and the state $I_{\text{CSI}} = N$ indicates that the transmitter has no channel state information.

As mentioned above, the impact of CSIT on the DoF of MISO broadcast channels has been explored for scenarios in which there is no adversarial time-varying interference. The novelty of this work is two fold: a) incorporating adversarial time-varying interference, and b) studying the joint impact of CSIT and the
knowledge about the absence/presence of interference at the transmitter (termed JSIT).

As we show in this paper, in the presence of a time-varying jammer, not only the CSIT availability but also the knowledge of jammer’s strategy significantly impacts the DoF. Indeed, if the transmitter is non-causally aware of the jamming strategy at time \( t \), i.e., if it knows which receiver (or receivers) is going to be disrupted at time \( t \), the transmitter can utilize this knowledge and adapt its transmission strategy by: either transmitting to a subset of receivers simultaneously (if only a subset of them are jammed/not-jammed) or conserving energy by not transmitting (if all the receivers are jammed).

However, such adaptation may not be feasible if there is delay in learning the jammer’s strategy. Feedback delays could arise in practice as the detection of a jamming signal would be done at the receiver (for instance, via a binary hypothesis test [23] in which the receiver could use energy detection to validate the presence/absence of a jammer in its vicinity). This binary decision could be subsequently fed back to the transmitter. In presence of feedback delays, the standard approach would be to exploit the time correlation in the jammer’s strategy to predict the current jammer’s strategy from the delayed measurements. The predicted jammer state could then be used in place of the true jammer state. However, if the jammer’s strategy is completely uncorrelated across time (which is the case if the jammers’ strategy is i.i.d), delayed feedback reveals no information about the current state, and a predict-then-adapt scheme offers no advantage. A third and perhaps worst case scenario could also arise in which the transmitter only has statistical knowledge of jammer’s strategy. This could be the case when the feedback links are unreliable or if the feedback links themselves are susceptible to jamming attacks, i.e., the outputs of feedback links are untrustworthy.

To take all such plausible scenarios into account, we formally model the jamming strategy via an independent and identically distributed (i.i.d.) random variable \( S(t) = (S_1(t), S_2(t), \ldots, S_K(t)) \); which we call the jammer state information (JSI) at time \( t \). Note here that in the context of the paper, the jammers’ state only indicates knowledge about the jammers’ strategy (i.e., which receivers are jammed) and not the channel between the jammer and receiver. At time \( t \), if the \( k \)th component of \( S(t) \), i.e., \( S_k(t) = 1 \), it indicates that receiver \( k \) is being jammed, and \( S_k(t) = 0 \) indicates that receiver \( k \) receives a jamming free signal. We denote the availability of jammer state information at the transmitter (JSIT) through a variable \( I_{JSIT} \), which (similar to \( I_{CSIT} \)) can take values either \( P \), \( D \) or \( N \); where the state \( I_{JSIT} = P \) indicates that the transmitter has perfect and instantaneous jammer state information \( (S_1(t), S_2(t), \ldots, S_K(t)) \) at time \( t \), the state \( I_{JSIT} = D \) indicates that the transmitter has delayed jammer state information (i.e., it has access to \( \{S_1(i), S_2(i), \ldots, S_K(i)\}_{i=1}^{t-1} \) at time \( t \)), and the state \( I_{JSIT} = N \) indicates that the transmitter does not have the exact realization of \( S(t) \) at its disposal. In all configurations above, it is assumed that the transmitter knows the statistics of \( S(t) \).

Summary of Main Results: Depending on the joint availability of channel state information (CSIT) and jammer state information (JSIT) at the transmitter, the variable \( I_{CSIT}I_{JSIT} \) can take 9 values and hence a total of 9 distinct scenarios can arise: \( PP, PD, PN, DP, DD, DN, NP, ND, \) and \( NN \). The main contributions of this paper are the following.

1. For the 2-user scenario, we characterize the exact DoF region for the \( PP, PD, PN, DP, DD, DN, NP, ND, \) and \( NN \) configurations.

2. For the DN and ND configurations in a 2-user MISO BC, we present novel inner bounds to the DoF regions.

3. The interplay between CSIT and JSIT and the associated impact on the DoF region in the various configurations is discussed. Specifically, the gain in DoF by transmitting across various jamming states and the loss in DoF due to the unavailability of CSI or JSI at the transmitter is quantified by the achievable sum DoF.

4. We extend the analysis in a 2-user MISO BC to a generic \( K \)-user MISO BC with such random time-
varying jamming attacks. The DoF region is completely characterized for the PP, PD, PN, NP and NN configurations. Further, novel inner bounds are presented for the sum DoF in DP and DD configurations. These bounds provide insights on the scaling of sum DoF with the number of receivers $K$.

The remaining parts of the paper are organized as follows. The system model is introduced in Section 2. The main contributions of the paper i.e., the Theorems describing the DoF regions in various (CSIT, JSIT) configurations for the 2-user and $K$-user MISO BC are illustrated in Sections 3 and 5 respectively and the corresponding converse proofs are presented in the Appendix. The coding (transmission) schemes achieving the optimal DoF regions are described in Sections 4, 5. Finally, conclusions are drawn in Section 6.

2 System Model

A $K$-user MISO broadcast channel with $K$ transmit antennas and $K$ single antenna receivers, is considered in the presence of a random, time-varying jammer. The system model for the $K = 2$ user case is shown in Fig. 2. The channel output at receiver $k$, for $k = 1, 2, \ldots, K$ at time $t$ is given as:

$$Y_k(t) = H_k(t)X(t) + S_k(t)G_k(t)J(t) + N_k(t), \quad (1)$$

where $X(t)$ is the $K \times 1$ channel input vector at time $t$ with

$$E \left( |X(t)|^2 \right) \leq P_T, \quad (2)$$

and $P_T$ is the power constraint on $X(t)$. In (1), $H_k(t) = [h_{1k}(t), h_{2k}(t), \ldots, h_{Kk}(t)]$ is the $1 \times K$ channel vector from the transmitter to the $k$th receiver at time $t$, $G_k(t)$ is the $1 \times K$ channel response from the jammer to receiver $k$ at time $t$ and $J(t)$ is the $K \times 1$ jammer’s channel input at time $t$ (a worst case scenario where the jammer has $K$ degrees-of-freedom to disrupt all $K$ parallel streams of data from the transmitter to the $K$ receivers). Without loss of generality, the channel vectors $H_k(t)$ and $G_k(t)$ are assumed to be sampled from any continuous distribution (for instance, Rayleigh) with an identity covariance matrix, and are i.i.d. across time. The additive noise $N_k(t)$ is distributed according to $\mathcal{CN}(0, 1)$ for $k = 1, \ldots, K$ and are assumed
to be independent of all other random variables. The random variable $S(t) = \{S_1(t), S_2(t), \ldots, S_K(t)\}$ that denotes the jammer state information $JSI$ at time $t$, is a $2^K$-valued i.i.d. random variable.

For example, in the 3-user MISO BC, the $JSI$ $S(t)$ is a 8-ary valued random variable taking values \{000, 001, 010, 011, 100, 101, 110, 111\} with probabilities $\{\lambda_{000}, \lambda_{001}, \lambda_{010}, \lambda_{011}, \lambda_{100}, \lambda_{101}, \lambda_{110}, \lambda_{111}\}$ respectively, for arbitrary $\{\lambda_{ijk} \geq 0\}_{i,j,k=0,0,1}$ such that $\sum_{i,j,k} \lambda_{ijk} = 1$. The jammer state $S(t)$ at time $t$ can be interpreted as follows:

- $S(t) = (0, 0, 0)$ : none of the receivers are jammed. This occurs with probability $\lambda_{000}$.
- $S(t) = \{(1, 0, 0) / (0, 1, 0) / (0, 0, 1)\}$ : only one receiver is jammed. This scenario occurs with probability $\lambda_{100}/\lambda_{010}/\lambda_{001}$ respectively. $S(t) = (1, 0, 0)$ indicates that the 1st receiver is jammed while the receivers 2 and 3 are not jammed.
- $S(t) = \{(1, 1, 0) / (1, 0, 1) / (0, 1, 0)\}$ : any two out of the three receivers are jammed. This happens with probability $\lambda_{110}/\lambda_{101}/\lambda_{011}$ respectively.
- $S(t) = (1, 1, 1)$ : all the receivers are jammed with probability $\lambda_{111}$.

Using the probability vector $\{\lambda_{000}, \lambda_{001}, \lambda_{010}, \lambda_{011}, \lambda_{100}, \lambda_{101}, \lambda_{110}, \lambda_{111}\}$, we define the marginal probabilities

$$
\begin{align*}
\lambda_1 &= \lambda_{000} + \lambda_{001} + \lambda_{010} + \lambda_{011}, \\
\lambda_2 &= \lambda_{000} + \lambda_{001} + \lambda_{100} + \lambda_{101}, \\
\lambda_3 &= \lambda_{000} + \lambda_{010} + \lambda_{100} + \lambda_{110},
\end{align*}
$$

where $\lambda_k \in [0, 1]$ denotes the total probability with which receiver $k$ is not jammed. For example, in the 3-user scenario, $\lambda_1$ indicates the total probability with which the 1st receiver is not jammed which happens when any one of the following events happen 1) none of the receivers are jammed with probability $\lambda_{000}$, 2) only the 2nd receiver is jammed with probability $\lambda_{010}$, 3) only 3rd receiver is jammed with probability $\lambda_{101}$ or 4) both the 2nd and 3rd receivers are jammed with probability $\lambda_{111}$. Similar definitions hold for the $K$-user MISO BC. In general, $S(t)$ is a $K \times 1$ vector where a 1(0) in the $k$th position indicates that the $k$th receiver is jammed (not-jammed).

It is assumed that the jammer sends a signal with power equal to $P_T$ (the transmit signal power). This formulation attempts to capture the performance of the system in a time-varying interference (here jammer) limited scenario where the received interference power is as high as the transmit signal power $P_T$ (a worst case scenario where the receiver by no means can recover the symbol from the received signal). Furthermore, it is assumed that $\{J(t)\}_{t=1}^n$ is independent of $\{S(t)\}_{t=1}^n$. We denote the global channel state information (between transmitter and receivers) at time $t$ by $H(t) = \{H_1(t), H_2(t), \ldots, H_K(t)\}$. In all analysis that follows, we assume that both the receivers have complete knowledge of global channel vectors $\{H(t)\}_{t=1}^n$ and also of the jammer’s strategy $\{S(t)\}_{t=1}^n$, i.e., full CSIR and full JSIR (similar assumptions were made in earlier works, see [17], [24], [25] and references therein).

**Assumptions:** The following are the list of assumptions made in this paper.

- If CSIT exists (i.e., when $I_{CSIT} = P$ or $D$), the transmitter receives either instantaneous or delayed feedback from the receivers regarding the channel $H(t)$. In either scenario, neither the transmitter nor the receivers require knowledge of $G(t) = \{G_1(t), \ldots, G_K(t)\}$ i.e., the channel between the jammer and the receivers.
- If JSIT exists (i.e., when $I_{JSIT} = P$ or $D$), then the transmitter receives either instantaneous or delayed feedback about the jammers’ strategy i.e., $S(t)$.
• Irrespective of the availability/ un-availability of CSIT and JSIT, it is assumed that the transmitter has statistical knowledge of the jammer’s strategy (i.e., statistics of $S(t)$) which is assumed to be constant across time (these assumptions form the basis for future studies that deal with time varying statistics of a jammer).

• While the achievability schemes presented in Sections 4, 5 hold for arbitrary correlations between the random variables $S(t)$, $J(t)$, and $G(t)$, the converse proofs provided in the Appendix hold under the assumption that these random variables are mutually independent and when the elements of $J(t)$ are distributed i.i.d. as $\mathcal{CN}(0, P_r)$.

• The theorems, achievability schemes and the converse proofs presented in Sections 3–5 and the Appendix hold true for any continuous distributions that $H$ may assume. While these achievability schemes are valid for any distribution of the jammers’ signal $J(t)$, the converse proofs are presented for the case in which the jammers’ signal is Gaussian distributed.

For the $K$-user MISO BC, a rate tuple $(R_1, R_2, \ldots, R_K)$, with $R_k = \log(|W_k|)/n$, where $n$ is the number of channel uses, $W_k$ denotes the message for the $k$th receiver and $|W_k|$ represents the cardinality of $W_k$, is achievable if there exist a sequence of encoding functions $f(n)$ and decoding functions $g_k(n)$ $(Y^n_k, H^n, S^n)$ (one for each receiver) such that for all $k = 1, 2, \ldots, K$,

$$P (W_k \neq g_k^n (Y^n_k, H^n, S^n)) \leq n \epsilon_{kn},$$

where

$$\epsilon_{kn} \rightarrow 0 \text{ as } n \rightarrow \infty,$$

i.e., the probability of incorrectly decoding the message $W_k$ from the signal received at user $k$ converges to zero asymptotically. In (4), we have used the following shorthand notations $Y^n_k = (Y_k(1), \ldots, Y_k(n))$, $H^n = (H_1(1), \ldots, H_K(1), \ldots, H_1(n), \ldots, H_K(n))$ and $S^n = (S(1), S(2), \ldots, S(n))$. We are specifically interested in the degrees-of-freedom region $D$, defined as the set of all achievable pairs $(d_1, d_2, \ldots, d_K)$ with $d_k = \lim_{P_r \rightarrow \infty} \frac{R_k}{\log(P_r)}$. The encoding functions $f(n)$ that achieve the DoF described in Sections 3 and 5 depend on the availability of CSIT and JSIT i.e., on the variable $I_{CSIT}I_{JSIT}$. For example, in the DD (delayed CSIT, delayed JSIT) configuration, the encoding function takes the following form;

$$X(n) = f(n) (W_1, W_2, \ldots, W_K, H^{n-1}, S^{n-1}),$$

where the transmit signal $X(n)$ at time $n$, depends on the the past channel state $(H^{n-1})$ and jammer state $(S^{n-1})$ information available at the transmitter. However, in the NP configuration since the transmitter does not have knowledge about the channel (as no CSIT is available), it exploits the perfect and instantaneous knowledge about the jammers’ strategy $(S(t))$ by sending information exclusively to the unjammed receivers. As a result, the encoding function for the NP configuration can be represented as

$$X(n) = f(n) (W_1, W_2, \ldots, W_K, S^n).$$

The encoding functions across various channel and jammer states depend on the transmission strategies used and are discussed in more detail in Sections 4 and 5.

### 2.1 Review of Known Results

As mentioned earlier, the DoF region for the $K$-user MISO BC has been studied extensively in the absence of external interference. We briefly present some of those important results that are relevant to the work.
presented in this paper.

1. In the absence of jamming, the DoF region with perfect CSIT is given by,
\[ d_k \leq 1, \quad k = 1, 2, \ldots, K, \] 
and the achievable sum DoF is \( K \) [25].

2. With delayed CSIT, the DoF region in the absence of a jammer was characterized by Maddah-Ali and Tse in [17], and is given by
\[ \sum_{k=1}^{K} \frac{d_{\pi(k)}}{k} \leq 1, \] 
where \( \pi(K) \) is a permutation of the set of numbers \( \{1, 2, 3, \ldots, K\} \). In such a scenario, the sum DoF (henceforth referred to as DoF\(_{\text{MAT}}\)) is given by
\[ \text{DoF}_{\text{MAT}}(K) = \frac{K}{1 + \frac{1}{2} + \ldots + \frac{1}{K}}. \] 

3. The DoF region with no CSIT is given by
\[ \sum_{k=1}^{K} d_k \leq 1. \] 
and the sum DoF in this case reduces to 1 [25].

It is easy to see that the sum DoF achieved in a delayed CSIT scenario lies in between the sum DoF achieved in the perfect CSIT and no CSIT scenarios.

3 Main Results and Discussion

We first present DoF results for the 2-user MISO BC under various assumptions on the availability of CSIT and JSIT and discuss various insights arising from these results. In the 2-user case, the jammer state \( S(t) \) at time \( t \) can take one out of four values: 00, 01, 10, or 11, where
- \( S(t) = 00 \) indicates that none of the receivers are jammed, which happens with probability \( \lambda_{00} \),
- \( S(t) = 01 \) indicates that only receiver 1 is not jammed, which happens with probability \( \lambda_{01} \),
- \( S(t) = 10 \) indicates that only the 2nd receiver is un-jammed with probability \( \lambda_{01} \), and finally
- \( S(t) = 11 \) indicates that both the receivers are jammed with probability \( \lambda_{11} \).

In order to compactly present the results, we define the marginal probabilities
\[ \lambda_1 \triangleq \lambda_{00} + \lambda_{01}, \]
\[ \lambda_2 \triangleq \lambda_{00} + \lambda_{10}, \]
where \( \lambda_k \), for \( k = 1, 2 \) is the total probability with which receiver \( k \) is not jammed. In the sequel, Theorems 1-5 present the optimal DoF characterization for the (CSIT, JSIT) configurations PP, PD, PN, DP, DD, NP and
NN while Theorems 6 and 7 present non-trivial achievable schemes (novel inner bounds) for the DN and ND configurations.

**Theorem 1** The DoF region of the 2-user MISO BC for each of the CSIT-JSIT configurations PP, PD and PN is the same and is given by the set of non-negative pairs \((d_1, d_2)\) that satisfy

\[
\begin{align*}
  d_1 &\leq \lambda_1 \\
  d_2 &\leq \lambda_2.
\end{align*}
\]

**Theorem 2** The DoF region of the 2-user MISO BC for the CSIT-JSIT configuration DP, is given by the set of non-negative pairs \((d_1, d_2)\) that satisfy

\[
\begin{align*}
  d_1 &\leq \lambda_1 \\
  d_2 &\leq \lambda_2 \\
  2d_1 + d_2 &\leq 2\lambda_1 + \lambda_{10} \\
  d_1 + 2d_2 &\leq 2\lambda_2 + \lambda_{01}.
\end{align*}
\]

**Theorem 3** The DoF region of the 2-user MISO BC for the CSIT-JSIT configuration DD, is given by the set of non-negative pairs \((d_1, d_2)\) that satisfy

\[
\begin{align*}
  d_1 + \frac{d_2}{\lambda_1 + \lambda_2} &\leq 1 \\
  \frac{d_1}{\lambda_1 + \lambda_2} + \frac{d_2}{\lambda_2} &\leq 1.
\end{align*}
\]

**Theorem 4** The DoF region for the 2-user MISO BC for the CSIT-JSIT configuration NP, is given by the set of non-negative pairs \((d_1, d_2)\) that satisfy

\[
\begin{align*}
  d_1 &\leq \lambda_1 \\
  d_2 &\leq \lambda_2 \\
  d_1 + d_2 &\leq \lambda_{00} + \lambda_{01} + \lambda_{10}.
\end{align*}
\]

**Theorem 5** The DoF region of the 2-user MISO BC for the CSIT-JSIT configuration NN is given by the set of non-negative pairs \((d_1, d_2)\) that satisfy

\[
\begin{align*}
  d_1 + \frac{d_2}{\lambda_1 + \lambda_2} &\leq 1.
\end{align*}
\]

**Remark 1** [Redundancy of JSIT with Perfect CSIT] We note from Theorem 1 that when Perfect CSIT is available, the DoF region remains the same regardless of availability/un-availability of jammer state information at the transmitter. This implies that with perfect CSIT, only statistical knowledge about the jammer’s strategy suffices to achieve the optimal DoF region (note that it is assumed that the transmitter has statistical knowledge of the jammers’ strategy). The availability of perfect CSIT helps to avoid cross-interference in such a broadcast type communication system and thereby enables the receivers to decode their intended symbols whenever they are not jammed.
Remark 2 [Quantifying DoF Loss] When the transmitter has perfect knowledge about the jammers state i.e., perfect JSIT, it is seen that the sum DoF for the various configurations is

$$\text{Sum DoF (with Perfect JSIT)} = \begin{cases} 
\lambda_1 + \lambda_2, & \text{perfect CSIT,} \\
\lambda_1 + \lambda_2 - \frac{2}{3}\lambda_{00}, & \text{delayed CSIT,} \\
\lambda_1 + \lambda_2 - \lambda_{00}, & \text{no CSIT.}
\end{cases}$$

(24)

It is seen that the sum DoFs achieved in the DP and NP configurations are less than $\lambda_1 + \lambda_2$, the sum DoF achieved in the PP configuration. The loss in DoF due to delayed channel knowledge is $\frac{2}{3}\lambda_{00}$ and due to no channel knowledge is $\lambda_{00}$. As expected, the loss in the NP configuration is more than the corresponding DoF loss in the DP configuration due to the un-availability of CSIT. Interestingly, the loss in DoF due to delayed channel state information in the absence of a jammer is $2 - \frac{4}{3} = \frac{2}{3}$ (where $2 - \left(\frac{4}{3}\right)$ is the DoF achieved in a 2-user MISO BC with perfect (delayed) CSIT [17]), which, in the presence of a jammer, corresponds to the case when $\lambda_{00} = 1$ i.e., none of the receivers are jammed. Along similar lines, the DoF loss due to no CSIT is $2 - 1 = 1$ where 1 is the DoF achieved in the 2-user MISO BC when there is no CSIT [25] (in the absence of jamming). The loss in DoF converges to 0 as $\lambda_{00} \to 0$ i.e., the PP, DP and NP configurations are equivalent when the jammer disrupts either one or both the receivers at any given time.

Remark 3 [Separability with Perfect JSIT] When perfect JSIT is present, i.e., in the PP, DP and NP configurations, the transmitter does not need to code (transmit) across different jammer states; or in other words, the jammer’s states are separable. For instance, consider the case of delayed CSIT. In the absence of a jammer, the optimal DoF with delayed CSIT is 4/3 as shown in [17]. The optimal strategy in presence of a jammer and with perfect JSIT is the following: use the 00 state to achieve $\frac{4}{3}\lambda_{00}$ DoF by employing the MAT scheme [17] (transmission scheme to achieve the sum DoF given in (10), explained in Section 4), use 01 state to achieve $\lambda_{01}$ DoF by transmitting to receiver 1, use 10 state to achieve $\lambda_{10}$ DoF by transmitting to receiver 2. The state 11 yields 0 DoF since both the receivers are jammed. Thus, the net achievable DoF of this separation based strategy is given as: $\frac{4}{3}\lambda_{00} + \lambda_{01} + \lambda_{10} = \lambda_1 + \lambda_2 - \frac{2}{3}\lambda_{00}$. Similar interpretations hold with perfect CSIT and no CSIT. The transmission schemes that achieve these DoFs and make the jammers’ states separable are illustrated in more detail in Section 4.

Remark 4 [Marginal Equivalence] The DoF regions in Theorems 1, 3 and 5 only depend on the marginal probabilities ($\lambda_1, \lambda_2$) with which each receiver is not jammed. This implies that two different jamming strategies with statistics, $\{\lambda_{00}, \lambda_{01}, \lambda_{10}, \lambda_{11}\}$ and $\{\lambda'_{00}, \lambda'_{01}, \lambda'_{10}, \lambda'_{11}\}$ result in the same DoF regions for PP, PD, PN, DD and NN configurations as long as $\lambda_{00} + \lambda_{01} = \lambda'_{00} + \lambda'_{01} = \lambda_1$ and $\lambda_{00} + \lambda_{10} = \lambda'_{00} + \lambda'_{10} = \lambda_2$.

In the next two Theorems, we present achievable DoF regions for the remaining configurations DN and ND respectively. It should be noticed that ignoring the availability of delayed CSIT in the DN configuration and the availability of delayed JSIT in the ND configuration, the DoF region described by Theorem 5 can always be achieved. However, the novel inner bounds presented in Theorems 6,7 show that the achievable DoF can be improved by synergistically using the delayed feedback regarding CSIT and JSIT.

Theorem 6 An achievable DoF region for the 2-user MISO BC for the CSIT-JSIT configuration DN, is given as follows.

For $\frac{|\lambda_1 - \lambda_2|}{\lambda_1 \lambda_2} \leq 1$, following region is achievable

$$d_1 + \frac{2 \max(1, \lambda_1/\lambda_2) - 1}{1 + \lambda_2} d_2 \leq \lambda_1$$

(25)

$$\frac{2 \max(1, \lambda_2/\lambda_1) - 1}{1 + \lambda_1} d_1 + d_2 \leq \lambda_2.$$
For $\frac{\lambda_1 - \lambda_2}{\lambda_1 \lambda_2} > 1$, following region is achievable

$$\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} \leq 1. \quad (27)$$

Though the optimal DoF region for the DN configuration remains unknown, we propose a novel inner bound (achievable scheme) to the DoF region as specified in Theorem 6. This scheme is based on a coding scheme (alternative to the original transmission scheme proposed in [17]) to achieve DoF of $\frac{4}{3}$ for the 2-user MISO BC in the absence of jamming attacks. This alternative scheme is discussed in Section 4.

**Theorem 7** An achievable DoF region for the 2-user MISO BC in the CSIT-JSIT configuration ND, is given by the set of non-negative pairs $(d_1, d_2)$ that satisfy

$$\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_0 + \lambda_{01} + \lambda_{10}} \leq 1 \quad (28)$$

$$\frac{d_1}{\lambda_{00} + \lambda_{01} + \lambda_{10}} + \frac{d_2}{\lambda_2} \leq 1. \quad (29)$$

By noticing that $\lambda_{00} + \lambda_{01} + \lambda_{10} \geq \max(\lambda_1, \lambda_2)$, it can be seen that the DoF region described by Theorem 7 is better than the region described by Theorem 5 i.e., the region achieved in the NN configuration can be improved by utilizing the delayed JSIT information. Also, the DoF achievable in the ND configuration is a subset of the DoF achieved in the DD configuration. This is because $\lambda_1 + \lambda_2 \geq \lambda_{00} + \lambda_{01} + \lambda_{10}$. However, in scenarios where $\lambda_{00} = 0$, the DoF region achieved by these two configurations is the same. Thus the converse proof in the Appendix that shows the optimality of the DoF region achieved in the DD configuration also holds true for the ND scenario when $\lambda_{00} = 0$. This equivalence will be explained further in Section 4.

Table 1 summarizes the mapping between the (CSIT, JSIT) configurations and the theorems that specify their DoF. The coding schemes that achieve the corresponding degrees of freedom regions are detailed in Section 4 and the corresponding converse proofs are presented in the Appendix.

### 4 Achievability Proofs

Here, we present the transmission schemes achieving the bounds mentioned in Theorems 1-7.

<table>
<thead>
<tr>
<th>CSIT</th>
<th>JSIT</th>
<th>Configuration ($I_{CSIT} I_{JSIT}$)</th>
<th>Theorem</th>
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<tbody>
<tr>
<td>Perfect</td>
<td>Delayed</td>
<td>PP, PD, PN</td>
<td>Theorem 1</td>
</tr>
<tr>
<td>Perfect</td>
<td>None</td>
<td>DP, DD, DN</td>
<td>Theorem 2, Theorem 3</td>
</tr>
<tr>
<td>Delayed</td>
<td>Perfect</td>
<td>DP, DD, DN</td>
<td>Theorem 2, Theorem 3</td>
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<tr>
<td>None</td>
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<td>NP, ND, NN</td>
<td>Theorem 4, Theorem 7, Theorem 5</td>
</tr>
<tr>
<td>None</td>
<td>None</td>
<td>NP, ND, NN</td>
<td>Theorem 4, Theorem 7, Theorem 5</td>
</tr>
</tbody>
</table>

Table 1: CSIT, JSIT configurations and corresponding theorems.
4.1 Perfect CSIT

In this sub-section schemes achieving the DoF for PP, PD and PN configurations are discussed. It is clear that the following ordering holds:

\[ \text{DoF}_{\text{PP}} \subseteq \text{DoF}_{\text{PD}} \subseteq \text{DoF}_{\text{PN}}, \]  

i.e, the DoF is never reduced when JSI (i.e., S(t)) is available at the transmitter.

4.1.1 Perfect CSIT, Perfect JSIT (PP):

In this configuration the transmitter has perfect and instantaneous knowledge of CSIT and JSIT. Further, since the jammers’ states (4 in this case) are i.i.d across time, the transmitter’s strategy in this configuration is also independent across time. This is further explained below.

- When \( S(t) = 11 \), i.e., when both the receivers are jammed, the transmitter does not send any information symbols to the receivers as they are completely disrupted by the jamming signals.

- When \( S(t) = 01 \), i.e., the case when only the 2nd receiver is jammed and the 1st receiver is un-jammed, the transmitter sends

\[ X(t) = \begin{bmatrix} a \\ 0 \end{bmatrix}, \]  

where \( a \) is an information symbol intended for the 1st receiver. In this case, the receiver 1 gets

\[ Y_1(t) = H_1(t)X(t) + N_1(t) = h_{11}(t)a + N_1(t), \]  

and the 2nd receiver gets

\[ Y_2(t) = H_2(t)X(t) + G_2(t)J(t) + N_2(t). \]  

The 2nd receiver cannot recover its symbols because it is disrupted by the jamming signals. However, since the 1st receiver is un-jammed, it can recover the intended symbols within noise distortion\(^2\).

- \( S(t) = 10 \), i.e., the case when only the 1st receiver is jammed and the 2nd receiver is un-jammed. This is the converse case of the jammers’ state \( S(t) = 01 \). In this scenario, the transmitter sends

\[ X(t) = \begin{bmatrix} 0 \\ b \end{bmatrix}, \]  

where \( b \) is an information symbol intended for the 2nd receiver. The 2nd receiver can recover the symbol \( b \) within noise distortion.

- Finally, for the jammer state \( S(t) = 00 \), i.e., none of the receivers are jammed, the transmitter can increase the DoF by sending symbols to both the receivers. This is achieved by using the knowledge of the perfect and instantaneous channel state information. In such a scenario, the transmitter employs a pre-coding based zero-forcing transmission strategy as illustrated below. The transmitter sends

\[ X(t) = B_1(t)a + B_2(t)b, \]  

\(^2\)Throughout the paper, it is assumed that the receivers are capable of recovering their symbols within noise distortion whenever they are not jammed (a valid assumption given that the DoF characterization is done for \( P_T \to \infty \)).
where \( \mathbf{B}_1(t) \) and \( \mathbf{B}_2(t) \) are \( 2 \times 1 \) auxiliary pre-coding vectors such that \( \mathbf{H}_1(t)\mathbf{B}_2(t) = 0 \) and \( \mathbf{H}_2(t)\mathbf{B}_1(t) = 0 \) (i.e., there is no interference caused at a user due to the un-intended information symbols). Thus, the received signals at the users are given by

\[
Y_1(t) = \mathbf{H}_1(t)\mathbf{B}_1(t)a + N_1(t) \quad (36)
\]

\[
Y_2(t) = \mathbf{H}_2(t)\mathbf{B}_2(t)b + N_2(t) \quad (37)
\]

which are decoded at the receivers using available CSIR (jamming signal \( J(t) \) is not present in the received signal since \( S_1(t) = S_2(t) = 0 \)).

Based on the above transmission scheme, it is seen that each receiver can decode the intended information symbols whenever they are not jammed. Since, the 1st receiver is not jammed in the states \( S(t) = 00 \) and \( S(t) = 01 \), which happen with probabilities \( \lambda_{00}, \lambda_{01} \) respectively (i.e., it can recover symbols for \( \lambda_{00} + \lambda_{01} \) fraction of the total transmission time), the DoF achieved is \( \lambda_1 = \lambda_{00} + \lambda_{01} \). Similarly, the DoF achieved by the 2nd receiver is \( \lambda_2 = \lambda_{00} + \lambda_{10} \). Thus the DoF pair \( (\lambda_1, \lambda_2) \) described by Theorem 1 is achieved using this transmission scheme.

### 4.1.2 Perfect CSIT, Delayed JSIT (PD):

Unlike in the PP configuration, the transmitters’ strategy in the PD configuration is not independent (or not separable) across various time instants due to the unavailability of instantaneous JSIT. However, we show that using the knowledge of perfect and instantaneous CSIT and the delayed knowledge of JSIT, the DoF pair \( (d_1, d_2) = (\lambda_1, \lambda_2) \) can still be achieved. Since the transmitter has delayed knowledge about the jammers strategy, it adapts its transmission scheme at time \( t \) based on the feedback it receives about the jammers’ strategy at time \( t - 1 \) i.e., \( S(t - 1) \). This transmission scheme is briefly explained here.

Let \( \{a_1, a_2\} \) denote the symbols to be sent to the 1st receiver and \( \{b_1, b_2\} \) to the 2nd receiver. Since the transmitter has perfect knowledge about the channel or CSIT, it creates pre-coding vectors \( \mathbf{B}_1(t) \) and \( \mathbf{B}_2(t) \) such that \( \mathbf{H}_1(t)\mathbf{B}_2(t) = 0 \) and \( \mathbf{H}_2(t)\mathbf{B}_1(t) = 0 \) (similar to the PP configuration). For example, at \( t = 1 \), it sends

\[
\mathbf{X}(1) = \mathbf{B}_1(1)a_1 + \mathbf{B}_2(1)b_1. \quad (39)
\]

- If the d-JSIT about the jammer’s state at \( t = 1 \) indicates that none of the receivers were jammed i.e., \( S(1) = 00 \), then the transmitter sends new symbols \( a_2 \) and \( b_2 \) as

\[
\mathbf{X}(2) = \mathbf{B}_1(2)a_2 + \mathbf{B}_2(2)b_2, \quad (40)
\]

at time \( t = 2 \) because both the receivers can decode their intended symbols \( a_1 \) and \( b_1 \) within noise distortion in the absence of jamming signals.

- If the jammer’s state at \( t = 1 \) suggests that only the 1st receiver was jammed i.e., \( S(t) = 10 \), then the transmitter sends

\[
\mathbf{X}(2) = \mathbf{B}_1(2)a_1 + \mathbf{B}_2(2)b_2, \quad (41)
\]

in order to deliver the undelivered symbol to the 1st receiver and a new symbol for the 2nd receiver (since it was not jammed at \( t = 1 \)).
When the feedback about the jammers’ state at \( t = 1 \) indicates that \( S(1) = 01 \), the coding scheme used when \( S(t) = 10 \) is reversed (roles of the receivers are flipped) and the transmitter sends a new symbol to the 1st receiver and the undelivered symbol to the 2nd receiver as

\[
X(2) = B_1(2)a_2 + B_2(2)b_1.
\]  

(42)

If both the receivers were jammed i.e., \( S(1) = 11 \), then the transmitter re-transmits the symbols for the both the receivers as

\[
X(2) = B_1(2)a_1 + B_2(2)b_1.
\]

(43)

By extending this transmission scheme to multiple time instants, the DoF described by Theorem 1 is also achieved in the PD configuration (since the receivers 1 and 2 get jamming free symbols whenever they are not jammed which happen with probabilities \( \lambda_1 \) and \( \lambda_2 \) respectively).

### 4.1.3 Perfect CSIT, No JSIT (PN):

In this section, we sketch the achievability of the pair \((d_1, d_2) = (\lambda_1, \lambda_2)\) for the PN configuration. We first note that for a scheme of block length \( n \), for sufficiently large \( n \), only \( \lambda_2 n \) symbols will be received cleanly (i.e., not-jammed) at receiver \( k \), since at each time instant the \( k \)th receiver gets a jamming free signal with probability \( \lambda_k \). As the transmitter is statistically aware of jammers’ strategy, it only sends \( \lambda_k n \) symbols for receiver \( k \) over the entire transmission period. It overcomes the problem of no feedback by sending pre-coded random linear combinations (LC) of these \( \{\lambda_k n\}_{k=1,2} \) symbols at each time instant. Notice here the difference between the schemes suggested for the PD and PN configurations. Due to the availability of JSIT, albeit in a delayed manner in the PD configuration, the transmitter can deliver information symbols to the receivers in a timely fashion without combining the symbols. This is not the case in the PN configuration.

The proposed scheme for PN configuration is illustrated below.

Let \( \{a_j\}_{j=1}^{\lambda_1 n} \) and \( \{b_j\}_{j=1}^{\lambda_2 n} \) denote the information symbols intended to be sent to receiver 1 and 2 respectively. Having the knowledge of \( \{H_1(t), H_2(t)\} \), the transmitter sends the following input at time \( t \):

\[
X(t) = B_1(t)f_t(a_1, \ldots, a_{\lambda_1 n}) + B_2(t)g_t(b_1, \ldots, b_{\lambda_2 n}),
\]

(44)

where \( f_t(\cdot), g_t(\cdot) \) are random linear combinations\(^3\) of the respective \( \lambda_1 n \) and \( \lambda_2 n \) symbols; and the \( B_1(t), B_2(t) \) are \( 2 \times 1 \) precoding vectors (similar to the ones used in PP and PD configurations). Thus, the received signals at time \( t \) are given as

\[
Y_1(t) = H_1(t)B_1(t)f_t(a_1, \ldots, a_{\lambda_1 n}) + S_1(t)G_1(t)J(t) + N_1(t)
\]

\[
Y_2(t) = H_2(t)B_2(t)g_t(b_1, \ldots, b_{\lambda_2 n}) + S_2(t)G_2(t)J(t) + N_2(t).
\]

Each receiver can decode all these symbols upon successfully receiving \( \lambda_k n \) linearly independent combinations\(^4\) transmitted using the zero-forcing strategy. Using this scheme, each receiver can decode \( \lambda_k n \) symbols over \( n \) time instants using the received \( \lambda_k n \) LCs. Hence \((d_1, d_2) = (\lambda_1, \lambda_2)\) is achievable. The proposed scheme is in similar spirit to the random network coding used in broadcast packet erasure channels where the receivers collect sufficient number of packets before being able to decode their intended information

---

\(^3\)The random coefficients are assumed to be known at the receivers. The characterization of the overhead involved in this process is beyond the scope of this paper.

\(^4\)Note here that in order to be able to decode all \( \lambda_1 n \) symbols, we need \( \lambda_1 n \) linearly independent combinations of \( \lambda_1 n \) symbols. For example to be able to decode \( a_1, a_2, a_3 \), 3 LCs say \( f_1(a_1, a_2, a_3), f_2(a_1, a_2, a_3), f_3(a_1, a_2, a_3) \) are sufficient.
Remark 5 For all possible (CSIT, JSIT) configurations, the DoF pairs: $(d_1, d_2) = (\lambda_1, 0)$ and $(d_1, d_2) = (0, \lambda_2)$ are achievable. This is possible via a simple scheme in which the transmitter sends random LC's of $\lambda_k n$ symbols to only the $k$th receiver throughout the transmission interval. The $k$th receiver can decode $\lambda_k n$ symbols in $n$ time slots given the fact that it receives jamming free LCs with probability $\lambda_k$. As Theorem 5 suggests, for the case in which the transmitter has neither CSI nor JSI (i.e., in the NN configuration), the optimal strategy is to alternate between transmitting symbols exclusively to only one receiver.

Remark 6 Although the PP, PD and PN configurations are equivalent in terms of the achievable DoF region, they may not be equivalent in terms of the achievable capacity region. For instance, it can be seen in the PN configuration that the intended symbols can be decoded only after sufficient linear combinations of the intended symbols are received. However, this is not the case in the other configurations. In PP and PD configurations, the receivers can decode their intended symbols instantaneously whenever they are not jammed. Thus with respect to the receivers, the decoding delay is maximum in the case of PN configuration while it is the least in the PP and PD configurations. In addition, with respect to the transmitter, re-transmissions are not required in the PP configuration while they are necessary in the case of the PD and PN configurations to ensure that the receivers get their intended symbols. Thus it must not be confused that the PP, PD and PN configurations are equivalent.

4.2 Delayed CSIT

The DoF region of a 2-user MISO BC using delayed-CSIT has been studied in the absence of a jammer [17]. A 3-stage scheme was proposed by the authors in [17] to increase the optimal DoF from 1 (no CSIT) to $\frac{4}{3}$. We briefly explain this scheme here.

4.2.1 Scheme achieving $\text{DoF} = \frac{4}{3}$ in the absence of jamming

At $t = 1$, the transmitter sends

$$X(1) = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix},$$

(45)

where $a_1, a_2$ are symbols intended for the 1st receiver. The outputs at the receivers (within noise distortion) at $t = 1$ are given as

$$Y_1(1) = H_1(1) \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = h_{11}(1)a_1 + h_{21}(1)a_2 \triangleq F_1(a_1, a_2)$$

(46)

$$Y_2(1) = H_2(1) \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = h_{12}(1)a_1 + h_{22}(1)a_2 \triangleq F_2(a_1, a_2),$$

(47)

where $H_k(t) = [h_{1k}(t) \ h_{2k}(t)]$ for $k = 1, 2$ and $h_{1k}(t), h_{2k}(t)$ represent the channel between the 2 transmit antennas and the $k$th receive antenna. The LC at 2nd receiver is not discarded, instead it is used as side information in Stage 3. In Stage 2 the transmitter creates a symmetric situation at the 2nd receiver by transmitting $b_1, b_2$, the symbols intended for the 2nd receiver.

$$X(2) = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}.$$
The outputs at the receivers at $t = 2$ are given as

$$Y_1(2) = \mathbf{H}_1(2) \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = h_{11}(2)b_1 + h_{21}(2)b_2 \triangleq G_1(b_1, b_2)$$

$$Y_2(2) = \mathbf{H}_2(2) \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = h_{12}(2)b_1 + h_{22}(2)b_2 \triangleq G_2(b_1, b_2).$$

(49)  
(50)

Similar to stage 1, the undesired LC at receiver 1 is not discarded. The transmitter is aware of the LCs $F_1, F_2, G_1, G_2$ via delayed CSIT. At this point, each receiver has one LC that is not intended for them, but is useful if it is delivered at the other receiver. Having access to $F_2$ along with $F_1$ will enable the 1st receiver to decode its intended symbols. Similarly, the 2nd receiver can decode its $b$-symbols using $G_1$ and $G_2$. To achieve this, the transmitter multicasts

$$X(3) = \begin{bmatrix} F_2(a_1, a_2) + G_1(b_1, b_2) \\ 0 \end{bmatrix}$$

(51)

at $t = 3$ to the receivers. Upon successfully receiving this symbol within noise distortion, the receivers can recover $F_2(a_1, a_2)$ and $G_1(b_1, b_2)$ using the available side information (the side information can be cancelled from the new LC). Thus each receiver has 2 LCs of 2 intended symbols. Using this transmission scheme the receivers can decode 2 symbols each in 3 time slots. Thus the optimal DoF ($\frac{4}{3}, \frac{4}{3}$) is achieved using this transmit strategy. Hereafter, this scheme is referred to as the “MAT scheme”.

Below we present transmission schemes to achieve optimal DoF in the presence of jamming signals, specifically in scenarios where the jamming state information (JSIT) is either available instantaneously or with a delay or is not available i.e., for the DP, DD and DN configurations. The following relationship holds true,

$$\text{DoF}_{\text{DN}} \subseteq \text{DoF}_{\text{DD}} \subseteq \text{DoF}_{\text{DP}}.$$  

(52)

4.2.2 Delayed CSIT, Perfect JSIT (DP):

As seen in Fig. 3, the following DoF pairs $(d_1, d_2) = (\lambda_1, 0)$, $(\lambda_1, \lambda_{10})$, $(\frac{2}{3}\lambda_{00} + \lambda_{01}, \frac{2}{3}\lambda_{00} + \lambda_{10})$ and $(\lambda_{01}, \lambda_2)$, $(0, \lambda_2)$ are achievable in the DP configuration. The DoF pairs $(\lambda_1, 0)$ and $(0, \lambda_2)$ are readily achievable by transmitting to only receiver 1 (resp. receiver 2). Here, we present transmission schemes to achieve the DoF pairs $(\frac{2}{3}\lambda_{00} + \lambda_{01}, \frac{2}{3}\lambda_{00} + \lambda_{10})$, $(\lambda_1, \lambda_{10})$ and $(\lambda_{01}, \lambda_2)$.

Due to the availability of perfect JSIT, the transmitters strategy is independent across time i.e., the transmitter uses a different strategy based on the jammers’ state. Thus the transmission scheme can be divided into 4 different strategies based on the jammers’ state $S(t)$ which is detailed below.

- When the jammers’ state $S(t) = 00$, the transmitter uses the MAT scheme which was described earlier. Since this state is seen with probability $\lambda_{00}$ and the DoF achieved by the MAT scheme in the presence of delayed CSIT is $(\frac{2}{3}, \frac{2}{3})$, the overall DoF achieved whenever this jammer state is seen is given by $(\frac{2}{3}\lambda_{00}, \frac{2}{3}\lambda_{00})$.

Instead of using the MAT scheme, if the transmitter chooses to send symbols exclusively to only one receiver, then the DoF pair $(\lambda_{00}, 0)$ or $(0, \lambda_{00})$ is achieved depending on whether it chooses the 1st or the 2nd receiver (notice the DoF loss by using this strategy).

- When $S(t) = 01$, the jammer transmits symbols only to the 1st receiver (since the 2nd receiver cannot recover its symbols due to jamming) which can recover the intended symbol within noise distortion. Since this state is seen with probability $\lambda_{01}$, the DoF achievable in this state is given by $(\lambda_{01}, 0)$.
• The state $S(t) = 10$ is the converse of the previous state $S(t) = 01$ with the roles of the two receivers flipped. Thus the DoF achieved in this state is $(0, \lambda_{10})$.

• When the jammers’ state is $S(t) = 11$, none of the receivers can recover the symbols as their received signals are completely disrupted by the jamming signals. Thus the transmitter does not send symbols whenever this jamming state occurs.

Since the jammers’ states are disjoint, the overall DoF achieved in the DP configuration is given by the pair $(d_1, d_2) = (\frac{2}{3} \lambda_{00} + \lambda_{01}, \frac{2}{3} \lambda_{00} + \lambda_{10})$ if it chooses to use the MAT scheme. Else the DoF pairs, $(d_1, d_2) = (\lambda_1, \lambda_{10})$ or $(d_1, d_2) = (\lambda_{01}, \lambda_2)$ are achievable. This completes the achievability scheme for the DP configuration. Hence, the DoF region mentioned by Theorem 2 is achieved.

As mentioned earlier, if perfect JSIT is available, the transmitter does not have to transmit/ code across different jammers’ states in order to achieve DoF gains. In other words, the jammers’ states are separable due to availability of perfect JSIT. As will be seen next, this separability no longer holds true in the case of DD and DN configurations and hence necessitate transmitting across various jamming states. These transmission schemes thereby introduce decoding delays at the intended receivers.

4.2.3 Delayed CSIT, Delayed JSIT (DD):

In this subsection, we propose a transmission scheme that achieves the following $(d_1, d_2)$ pair (which corresponds to intersection of (18) and (19), see Fig. 4):

$$ (d_1, d_2) = \left( \frac{\lambda_1}{\lambda_1 + \lambda_2}, \frac{\lambda_2}{\lambda_1 + \lambda_2} \right) $$

In this scheme, the decoding process follows once the transmission of the symbols has finished and the receivers have all required linear combinations of the symbols which are used to decode the symbols. The decoding process using the linear combinations is explicitly mentioned in the transmission schemes below. This algorithm operates in three stages. In stage 1, the transmitter sends symbols intended only for receiver

![Diagram of DoF region with delayed CSIT and perfect JSIT](image-url)
Stage 1—In this stage, the transmitter intends to deliver \( n_1 \) a-symbols, in a manner such that each a-symbol is received at at least one of the receivers (either 1st or 2nd receiver). At every time instant the transmitter sends two symbols on two transmit antennas. A pair of symbols (say \( a_1 \) and \( a_2 \)) are re-transmitted until they are received at at least one receiver (this knowledge is available via d-JSIT). Any one of the following four scenarios can arise:

1. **Event 00**: none of the receivers are jammed (which happens with probability \( \lambda_{00} \)). As an example, suppose that at time \( t \), if the transmitter sends \((a_1, a_2)\): then receiver 1 gets \( F_1(a_1, a_2) \) and receiver 2 gets \( F_2(a_1, a_2) \). The fact that the event 00 occurred at time \( t \) is known at time \( t + 1 \) via d-JSIT; and the LCs \((F_1(a_1, a_2), F_2(a_1, a_2))\) can be obtained at the transmitter at time \( t + 1 \) via d-CSIT. The goal of stage 3 would be to deliver \( F_2(a_1, a_2) \) to receiver 1 by exploiting the fact that it is already received at receiver 2. Thus, at time \( t + 1 \), the transmitter sends two new symbols \((a_3, a_4)\).

2. **Event 01**: receiver 1 is not jammed, while receiver 2 is jammed (which happens with probability \( \lambda_{01} \)). As an example, suppose that at time \( t \), if the transmitter sends \((a_1, a_2)\): then receiver 1 gets \( F_1(a_1, a_2) \) and receiver 2’s signal is drowned in the jamming signal. The fact that the event 01 occurred at time \( t \) is known at time \( t + 1 \) via d-JSIT; and the LC \( F_1(a_1, a_2) \) can be obtained at the transmitter at time \( t + 1 \) via d-CSIT. Thus, at time \( t + 1 \), the transmitter sends a fresh symbol \( a_3 \) on one antenna; and a LC of \((a_1, a_2)\); say \( \tilde{F}_1(a_1, a_2) \); such that \( F_1(a_1, a_2) \) and \( \tilde{F}_1(a_1, a_2) \) constitute two linearly independent combinations of \((a_1, a_2)\). In summary, at time \( t + 1 \), the transmitter sends \((a_3, \tilde{F}_1(a_1, a_2))\).
3. Event 10: receiver 2 is not jammed, while receiver 1 is jammed (which happens with probability $\lambda_{10}$). As an example, suppose that at time $t$, if the transmitter sends $(a_1, a_2)$: then receiver 1’s signal is drowned in the jamming signal, whereas receiver 2 gets $F_2(a_1, a_2)$. The fact that the event 10 occurred at time $t$ is known at time $t+1$ via d-CSIT; and the LC $F_2(a_1, a_2)$ can be obtained at the transmitter at time $t+1$ via d-CSIT. The goal of stage 3 would be to deliver $F_2(a_1, a_2)$ to receiver 1 by exploiting the fact that it is already received at receiver 2. Thus, at time $t+1$, the transmitter sends a fresh symbol $a_3$ on one antenna; and a LC of $(a_1, a_2)$: say $\tilde{F}_2(a_1, a_2)$; such that $F_2(a_1, a_2)$ and $\tilde{F}_2(a_1, a_2)$ constitute two linearly independent combinations of $(a_1, a_2)$. In summary, at time $t+1$, the transmitter sends $(a_3, \tilde{F}_2(a_1, a_2))$.

4. Event 11: both receivers are jammed (which happens with probability $\lambda_{11}$). Using d-CSIT, transmitter knows at time $t+1$ that the event 11 occurred and hence at time $t+1$, it re-transmits $(a_1, a_2)$ on the two transmit antennas.

The above events are disjoint, so in one time slot, the average number of useful LCs delivered to at least one receiver is given by

$$E[\# \text{ of LC's delivered}] = 2\lambda_{00} + \lambda_{01} + \lambda_{10} \triangleq \phi.$$ 

Hence, the expected time to deliver one LC is

$$\frac{1}{\phi} = \frac{1}{2\lambda_{00} + \lambda_{01} + \lambda_{10}} \triangleq \frac{1}{\lambda_1 + \lambda_2}. \tag{54}$$

The time spent in this stage to deliver $n_1$ LCs is

$$N_1 = \frac{n_1}{\lambda_1 + \lambda_2}. \tag{55}$$

Since receiver 1 is not jammed in events 00 and 01, i.e., for $\lambda_1$ fraction of the time, it receives only $\lambda_1 N_1$ LCs. The number of undelivered LCs is $n_1 - \lambda_1 N_1 = \frac{\lambda_2 n_1}{\lambda_1 + \lambda_2}$. These LCs are available at receiver 2 (corresponding to events 00 and 10) and are known to the transmitter via d-CSIT. This side information created at receiver 2 is not discarded, instead it is used in Stage 3 of the transmission scheme.

Stage 2 - In this stage, the transmitter intends to deliver $n_2$ b-symbols, in a manner such that each symbol is received at at least one of the receivers. Stage 1 is repeated here with the roles of the receivers 1 and 2 interchanged. On similar lines to Stage 1, the time spent in this stage is

$$N_2 = \frac{n_2}{\lambda_1 + \lambda_2}. \tag{56}$$

The number of LCs received at receiver 2 is $\lambda_2 N_2$ and the number of LCs not delivered to receiver 2 but are available as side information at receiver 1 is $n_2 - \lambda_2 N_2 = \frac{\lambda_1 n_2}{\lambda_1 + \lambda_2}$.

Remark 7 At the end of these 2 stages, following typical situation arises: $F(a_1, a_2)$ (resp. $G(b_1, b_2)$) is a LC intended for receiver 1 (resp. 2) but is available as side information at receiver 2 (resp. 1). Notice that these LCs must be transmitted to the complementary receivers so that the desired symbols can be decoded. In Stage 3, the transmitter sends a random LC of these symbols, say $L = l_1 F(a_1, a_2) + l_2 G(b_1, b_2)$ where $l_1, l_2$ that form the new LC are known to the transmitter and receivers a priori. Now, assuming that only receiver 2 (resp. 1) is jammed, $L$ is received at receiver 1 (resp. 2) within noise distortion. Using this LC, it can recover $F(a_1, a_2)$ (resp. $G(b_1, b_2)$) from $L$ since it already has $G(b_1, b_2)$ (resp. $F(a_1, a_2)$) as side information. When no receiver is jammed, both the receivers are capable of recovering $F(a_1, a_2), G(b_1, b_2)$ simultaneously.

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*Such situations correspond to events 00 and 01 in Stage 1; and events 00, 10 in Stage 2.*
Stage 3—In this stage, the undelivered LCs to each receiver are transmitted using the technique mentioned above. Let us assume that $\mathcal{F}_1(a_1, a_2)$ and $\mathcal{G}_1(b_1, b_2)$ are LCs available as side information at receivers 2 and 1 respectively. The transmitter sends $\mathcal{L}(\mathcal{F}_1, \mathcal{G}_1)$, a LC of these symbols on one transmit antenna, with the eventual goal of multicasting this LC (i.e., send it to both receivers). The following events, as specified earlier in Stages 1 and 2, are also possible while in this stage.

Event 00: Suppose at time $t$, if the transmitter sends $\mathcal{L}(\mathcal{F}_1, \mathcal{G}_1)$, then both the receivers get this LC within noise distortion. With the capability to recover $\mathcal{L}(\mathcal{F}_1, \mathcal{G}_1)$ within a scaling factor, the receivers 1 and 2 decode their intended LCs $\mathcal{F}_1$ and $\mathcal{G}_1$ respectively using the side informations $\mathcal{G}_1$ and $\mathcal{F}_1$ that are available with them. Since the intended LCs are delivered at the intended receivers, the transmitter, at time $t + 1$, sends a new LC $\tilde{\mathcal{L}}(\tilde{\mathcal{F}}_1, \tilde{\mathcal{G}}_1)$.

Event 01: Since receiver 2 is jammed, its signal is drowned in the jamming signal while receiver 1 gets $\mathcal{L}(\mathcal{F}_1, \mathcal{G}_1)$ and is capable of recovering $\mathcal{F}_1$ using $\mathcal{G}_1$ available as side information. The fact that event 01 occurred is known to the transmitter at time $t + 1$ via d-JSIT. Thus, at time $t + 1$, the transmitter sends a new LC $\tilde{\mathcal{L}}(\tilde{\mathcal{F}}_1, \tilde{\mathcal{G}}_1)$ since $\mathcal{G}_1$ has not yet been delivered to receiver 2.

Event 10: This event is similar to event 01, with the roles of the receivers 1 and 2 interchanged. Hence, receiver 2 is capable of recovering $\mathcal{G}_1$ from $\mathcal{L}(\mathcal{F}_1, \mathcal{G}_1)$ while receiver 1’s signal is drowned in the jamming signal. Thus at time $t + 1$, the transmitter sends a new LC $\tilde{\mathcal{L}}(\tilde{\mathcal{F}}_1, \tilde{\mathcal{G}}_1)$ since $\mathcal{F}_1$ has not yet been delivered to receiver 1.

Event 11: Using d-JSIT, transmitter knows at time $t + 1$ that the event 11 occurred and hence at time $t + 1$, it re-transmits $\mathcal{L}(\mathcal{F}_1, \mathcal{G}_1)$ on one of its transmit antennas.

Since, all the events are disjoint, in one time slot, the average number of LCs delivered to receiver 1 is given by

$$E[\# \text{ of LC's delivered to user 1}] = \lambda_{00} + \lambda_{01} \triangleq \lambda_1.$$
Hence, the expected time to deliver one LC to receiver 1 in this stage is $\frac{1}{\lambda_1}$. Given that $\frac{\lambda_2 n_1}{\lambda_1 + \lambda_2}$ LCs are to be delivered to receiver 1 in this stage, the time taken to achieve this is $\frac{\lambda_2 n_1}{\lambda_1 \lambda_2}$. Interchanging the roles of the users, the time taken to deliver $\frac{\lambda_1 n_2}{\lambda_1 + \lambda_2}$ LCs to receiver 2 is $\frac{\lambda_1 n_2}{\lambda_2 \lambda_1}$. Thus the total time required to satisfy the requirements of both the receivers in Stage 3 is given by

$$N_3 = \max\left(\frac{\lambda_2 n_1}{\lambda_1 (\lambda_1 + \lambda_2)}, \frac{\lambda_1 n_2}{\lambda_2 (\lambda_1 + \lambda_2)}\right). \quad (57)$$

The optimal DoF achieved in the DD configuration is readily evaluated as

$$d_1 = \frac{n_1}{N_1 + N_2 + N_3}, \quad d_2 = \frac{n_2}{N_1 + N_2 + N_3}. \quad (58)$$

Substituting for $\{N_i\}_{i=1,2,3}$ from (55)-(57), we have,

$$d_k = \frac{n_k}{\frac{n_1}{\lambda_1 + \lambda_2} + \frac{n_2}{\lambda_1 + \lambda_2} + \max\left(\frac{\lambda_2 n_1}{\lambda_1 \lambda_1 + \lambda_2}, \frac{\lambda_1 n_2}{\lambda_2 \lambda_1 + \lambda_2}\right)}, \quad k = 1, 2. \quad (59)$$

Using $\eta = \frac{n_1}{n_1 + n_2}$, we have

$$d_1 = \frac{1}{\lambda_1 + \lambda_2} + \max\left(\frac{\lambda_2 \eta}{\lambda_1 \lambda_1 + \lambda_2}, \frac{\lambda_1 (1-\eta)}{\lambda_2 \lambda_1 + \lambda_2}\right)$$

$$d_2 = \frac{1}{\lambda_1 + \lambda_2} + \max\left(\frac{\lambda_2 \eta}{\lambda_1 \lambda_1 + \lambda_2}, \frac{\lambda_1 (1-\eta)}{\lambda_2 \lambda_1 + \lambda_2}\right). \quad (60)$$

Eliminating $\eta$ from the above two equations, yields the $(d_1, d_2)$ pair given in (53).

**Remark 8** It is seen that only JSI at time $t$ is necessary for the transmitter to make a decision on the LCs to be transmitted at time $t+1$ in Stage 3. Also, it is worth noting that the outer most points on the DoF region described by Theorem 3 (for a given $\lambda_1$, $\lambda_2$) are obtained for different values of $\eta \in [0, 1]$. Another interesting point to note here is that if $\lambda_1 = \lambda_2 = 1$, (which is possible only if $\lambda_{00} = 1$) i.e none of the receivers are jammed, the DoF achieved is $\frac{4}{3}$ which is the optimum DoF achieved in a d-CSIT scenario for the 2-user MISO broadcast channel as shown by Maddah-Ali and Tse in [17].

**4.2.4 Delayed CSIT, No JSIT (DN):**

One of the novel contributions of this paper is developing a new coding/transmission scheme for the DN configuration. Before we explain the proposed scheme, we first present a modified MAT scheme (original MAT scheme proposed in [17]) that achieves a DoF of $\frac{4}{3}$ in a 2-user MISO BC (in the absence of jamming).

**Modified MAT Scheme:** Consider a 2-user MISO BC where the transmitter intends to deliver $a$-symbols ($a_1, a_2$) to the 1st receiver and $b$-symbols ($b_1, b_2$) to the 2nd receiver respectively. The MAT scheme proposed in [17] was illustrated earlier in Section 4.2. Here we first revise the modified MAT scheme to achieve the same results.

At $t = 1$, the transmitter sends

$$X(1) = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}, \quad (61)$$

on its two transmit antennas. The outputs (within noise distortion) at the 2 receivers are given as (ignoring
unavailability of JSIT configuration, the transmitter sends random LCs of the intended symbols to both the users to overcome the MAT achieved by the transmitter at time $t$. CSIR received symbols can be grouped in this manner as the receivers have $\lambda$ received symbols.

At this point, receiver 1 has one LC of intended symbols $F_1$ that it received at time $t = 1$. Thus at the end of 3 time instants, the receivers 1 and 2 have $F_1$, $F_2$ and $G_1$, $G_2$ respectively, that help them decode their intended symbols. Thus using this transmission scheme, 4 symbols are decoded at the receivers in 3 time slots that leads to a sum DoF of $\frac{3}{2}$ which is also the DoF achieved by the MAT scheme in the 2-user MISO BC with delayed CSIT.

**Proposed Scheme for DN:** It is clearly seen that the modified MAT scheme presented above cannot be directly extended to the case where the jammer disrupts the receivers. Below, we present a novel 3-stage transmission strategy to achieve the DoF described by Theorem 6. The transmitter uses the statistical knowledge of the jammers strategy to deliver symbols to both the receivers in this configuration (as feedback information about the undelivered symbols is not available at the transmitter). Similar to the PN configuration, the transmitter sends random LCs of the intended symbols to both the users to overcome the unavailability of JSIT.

Let $(1 + \lambda_1)n$ and $(1 + \lambda_2)n$ (the reason for choosing $(1 + \lambda_k)n$, $k = 1, 2$, as the length of symbol sequence will be clear as we proceed through the algorithm) denote the total number of symbols the transmitter intends to deliver to receivers 1 and 2 respectively, where $\lambda_1, \lambda_2$ indicate the probability with which the
receivers are not disrupted by the jammer. In this scheme, we assume that the decoding process follows once the transmission of the symbols has finished and the receivers have all required linear combinations of the symbols which are used to decode the symbols. So each receiver needs \((1 + \lambda_1)n\), \((1 + \lambda_2)n\) LCs respectively to completely decode their symbols.

- **Stage 1**: The transmitter forms random LCs of the \((1 + \lambda_1)n\) a-symbols and \((1 + \lambda_2)n\) b-symbols symbols intended for both the receivers. Let us denote these LCs by \((a_1, a_2, \ldots, a_{(1+\lambda_1)n})\) and \((b_1, b_2, \ldots, b_{(1+\lambda_2)n})\) respectively (these are the actual transmitted symbols and are similar to the a-symbols and b-symbols mentioned earlier in the modified MAT scheme). In Stage 1, the transmitter combines these a-symbols and b-symbols and sends them over \(n\) time instants (please refer to the modified MAT scheme to see how combination of a-symbols and b-symbols are sent). Since the receivers 1, 2 are not jammed with a probability \(\lambda_1, \lambda_2\) respectively, they receive \(\lambda_1n\) and \(\lambda_2n\) combinations of a-symbols and b-symbols over \(\tau_1 = n\) time instants.

As mentioned, the transmitter does not have knowledge about the LCs undelivered to the receivers. However, using d-CSIT, it can reconstruct the LCs that would have been received at each receiver irrespective of whether they are jammed or not. For example, let us denote these LCs by \(F_1, F_2, G_1, G_2\) that correspond to combinations of \(a_1, a_2, b_1\) and \(b_2\) (refer to modified MAT scheme). Irrespective of whether \(F_1 + G_1\) is received at receiver 1 or not, the LC \(F_2\) is useful for it as it will act as an additional LC that helps decode its intended symbols. Similar reasoning holds for receiver 2 with respect to the symbol \(G_1\). But because these LCs have been received at the un-intended receiver, these act as side information which are used in the stages 2 and 3 of the algorithm.

- **Stage 2**: In this stage, the transmitter multicasts \(F\)-type LCs that would have been received at receiver 2 (irrespective of whether it is jammed or not, the transmitter can reconstruct them using d-CSIT). This is now available at the 1st receiver with a probability \(\lambda_1\) and with probability \(\lambda_2\) at the 2nd receiver. This is useful for both the receivers as it is a useful LC of intended symbols for the 1st receiver while it can be used to remove the side information at receiver 2 to recover its intended LC if at all it was received in the past (note that this is not useful for the 2nd receiver, if a LC consisting of
(a_1, \ldots, a_{(1+\lambda_1)n}) \quad (b_1, \ldots, b_{(1+\lambda_2)n}) \quad \rightarrow \quad \text{Un - Jammed receiver}

\rightarrow \quad \text{Jammed receiver}

Stage 1: Transmit $a + b$ $\bar{a} + \bar{b}$

Stage 2: Multicast $f_i$

Stage 3: Multicast $g_j$

$\tau_2 = \max \left( \frac{1}{\lambda_1}, \frac{1}{\lambda_2} \right) n. \quad \text{(67)}$

- Stage 3: This stage is the complement of the Stage 2, where the transmitter sends the $G$-type LCs that would have been received at the 1st receiver, but are useful to both of them. Thus the total time spent in Stage 3 is given by

$\tau_3 = \max \left( \frac{1}{\lambda_1}, \frac{1}{\lambda_2} \right) n. \quad \text{(68)}$

DoF analysis: At the end of the proposed 3-stage algorithm, notice that both the receivers have $(1 + \lambda_1)n$ and $(1 + \lambda_2)n$ intended LCs. Since $(1 + \lambda_1)n$ random LCs are sufficient to decode $(1 + \lambda_1)n$ symbols, at the end of this stage 3, both the receivers have successfully decoded all intended symbols. Thus

Figure 7: Coding with delayed CSIT and no JSIT.
the DoF is given by

\[
d_1 = \frac{(1 + \lambda_1)n}{\tau_1 + \tau_2 + \tau_3}
\]

(69)

\[
d_1 = \frac{(1 + \lambda_1)n}{n + \max\left(\frac{n}{\lambda_1}, \frac{n}{\lambda_2}\right) + \max\left(\frac{n}{\lambda_1}, \frac{n}{\lambda_2}\right)}
\]

(70)

\[
d_1 = \frac{(1 + \lambda_1)}{1 + 2\max\left(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}\right)}
\]

(71)

On similar lines, we have

\[
d_2 = \frac{(1 + \lambda_2)}{1 + 2\max\left(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}\right)},
\]

(72)

which is the DoF region given by Theorem 6.

Theorems 5 and 6, suggest that the DoF in the DN configuration can be increased only when the region described in Theorem 5 is a subset of the region described in Theorem 6. This is possible only when

\[
\frac{(1 + \lambda_2)\lambda_1}{2\max(1, \lambda_1/\lambda_2 - 1)} \geq \lambda_2
\]

(73)

\[
\frac{(1 + \lambda_1)\lambda_2}{2\max(1, \lambda_2/\lambda_1 - 1)} \geq \lambda_1.
\]

In other words, the proposed scheme for the DN configuration can achieve DoF gains over the naive TDMA-
based scheme if and only if $\lambda_1, \lambda_2$ satisfy (obtained by solving the above two equations)

$$\frac{|\lambda_1 - \lambda_2|}{\lambda_1 \lambda_2} \leq 1. \quad (74)$$

Fig. 8 shows the Sum DoF achieved using the naive TDMA scheme and the proposed scheme for the DN configuration. Since the transmitter has statistical knowledge about the jammers strategy, it can choose to use the naive scheme or the novel scheme based on the values of $\lambda_1, \lambda_2$.

4.3 No CSIT

The following relationship holds true,

$$\text{DoF}_{\text{NN}} \subseteq \text{DoF}_{\text{ND}} \subseteq \text{DoF}_{\text{NP}}. \quad (75)$$

i.e, the DoF is never reduced when JSI is available at the transmitter.

4.3.1 No CSIT, Perfect JSIT (NP):

As seen in Fig. 9, the following DoF pairs $(d_1, d_2) = (\lambda_1, 0), (\lambda_1, \lambda_{10}), (\lambda_{01}, \lambda_2)$, and $(0, \lambda_2)$ are achievable in the NP configuration. The DoF pairs $(\lambda_1, 0)$ and $(0, \lambda_2)$ are readily achievable using the naive scheme mentioned before where the transmitter sends symbols exclusively to the receiver that is not jammed (when the transmitter sends $n$ symbols to the $k$th receiver using the knowledge of perfect JSIT, it receives $\lambda_k n$ symbols since it is not jammed with probability $\lambda_k$). The remaining DoF pairs, $(\lambda_1, \lambda_{10})$ and $(\lambda_{01}, \lambda_2)$ are achieved via the transmission schemes suggested in the DP configuration for the corresponding DoF pairs.
4.3.2 No CSIT, Delayed JSIT (ND):

Here, we present a 3-stage scheme that achieves the DoF region given by Theorem 7. This scheme is similar to the algorithm proposed for the DD configuration. In stage 1, the transmitter sends symbols intended for receiver 1 alone and keeps re-transmitting them until it is received (jamming free signal) at at least one receiver. On similar lines, the transmitter sends symbols intended only for receiver 2 in Stage 2. Stage 3 consists of transmitting the undelivered symbols to the intended receiver. However, since there is no CSI available at the transmitter, the algorithm proposed for the DD configuration cannot be applied here. The modified 3-stage algorithm is presented henceforth.

Stage 1—In this stage, the transmitter intends to deliver $n_1$ $a$-symbols, in a manner such that each symbol is received at at least one of the receivers. At every time instant the transmitter sends one symbol on one of its transmit antennas. This message is re-transmitted until it is received at least one receiver. Any one of the following four scenarios can arise:

Event 00: none of the receivers are jammed (which happens with probability $\lambda_{00}$). As an example, suppose that at time $t$, if the transmitter sends $a_1$: then receiver 1 gets $F_1(a_1)$ and receiver 2 gets $F_2(a_1)$ (note that these are scaled versions of the transmit signal corrupted by white Gaussian noise and are recovered by the receivers within noise distortion). The fact that the event 00 occurred at time $t$ is known at time $t+1$ via d-JSIT. The transmitter ignores the side information created at receiver 2, since the intended symbol is delivered to receiver 1. The transmitter sends a new symbol $a_2$ at time $t+1$.

Event 01: receiver 1 is not jammed, while receiver 2 is jammed (which happens with probability $\lambda_{01}$). As an example, suppose that at time $t$, if the transmitter sends $a_1$: then receiver 1 gets $F_1(a_1)$ and receiver 2’s signal is drowned in the jamming signal. The fact that the event 01 occurred at time $t$ is known at time $t+1$ via d-JSIT. Since the intended $a$-symbol is delivered to receiver 1, at time $t+1$, the transmitter sends a new symbol $a_2$ from the message queue of symbols intended for receiver 1.

Event 10: receiver 2 is not jammed, while receiver 1 is jammed (which happens with probability $\lambda_{10}$). As an example, suppose that at time $t$, if the transmitter sends $a_1$: then receiver 1’s signal is drowned in the
jamming signal, whereas receiver 2 gets $F_2(a_1)$. The fact that the event 10 occurred at time $t$ is known at time $t + 1$ via d-JSIT. Since the receivers have CSI and JSI, receiver 1 is aware of the message received at receiver 2 within noise distortion. This message is not discarded, but instead used as side information and is delivered to the receiver 1 in Stage 3.

**Event 11:** both receivers are jammed (which happens with probability $\lambda_{11}$). Using d-JSIT, transmitter knows at time $t + 1$ that the event 11 occurred and hence at time $t + 1$, it re-transmits $a_1$ on one of its transmit antennas.

The above events are disjoint, so in one time slot, the average number of useful messages delivered to at least one receiver is given by

$$E[\# \text{ of symbols delivered}] = \lambda_{00} + \lambda_{01} + \lambda_{10} \triangleq \phi.$$  

Hence, the expected time to deliver one LC is

$$\frac{1}{\phi} = \frac{1}{\lambda_{00} + \lambda_{01} + \lambda_{10}}.$$  

(76)

**Summary of Stage 1:**

- The time spent in this stage to deliver $n_1$ LCs is
  $$N_1 = \frac{n_1}{\phi}.$$  

(77)

- Since receiver 1 is not jammed in events 00 and 01, i.e., with probability $\lambda_1$, it receives only $\lambda_1 N_1$ symbols.

- The number of undelivered symbols is $n_1 - \lambda_1 N_1 = \frac{\lambda_0 + n_1}{\phi}$. These symbols are available at receiver 2 (corresponding to the event 10) and are known to the transmitter via d-JSIT. This side information created at receiver 2 is not discarded, instead it is used in Stage 3 of the transmission scheme.

- The loss in DoF in this configuration due to the unavailability of CSIT is observed by noticing the expected number of symbols delivered in the ND configuration which is given by $\lambda_{00} + \lambda_{01} + \lambda_{10}$ while it is $2\lambda_{00} + \lambda_{01} + \lambda_{10}$ in the DD configuration as seen in (54).

**Stage 2**—In this stage, the transmitter intends to deliver $n_2$ b-symbols, in a manner such that each symbol is received at *at least* one of the receivers. Stage 1 is repeated here with the roles of the receivers 1 and 2 interchanged. On similar lines to Stage 1, the time spent in this stage is

$$N_2 = \frac{n_2}{\phi}.$$  

(78)

The number of symbols received at receiver 2 is $\lambda_2 N_2$ and the number of symbols not delivered to receiver 2 but are available as side information at receiver 1 is $n_2 - \lambda_2 N_2 = \frac{\lambda_0 + n_2}{\phi}$.

**Remark 9** At the end of these 2 stages, following typical situation arises: $F(a_1)$ (resp. $G(b_1)$) is a symbol intended for receiver 1 (resp. 2) but is available as side information at receiver 2 (resp. 1)\(^6\). Notice that these symbols must be transmitted to the complementary receivers so that the desired symbols can be decoded. The transmitter, via delayed-JSIT, is aware of the symbols that are not delivered to the receivers (however the transmitter is not required to be aware of $F$ (resp. $G$) since the receivers have this knowledge and that $F$ (resp. $G$) is the noise corrupted version of one symbol $a_1$ (resp. $b_1$)). In Stage 3, the transmitter

\(^6\)Such situations correspond to the event 10 in Stage 1; and the event 01 in Stage 2.
sends a random LC of these symbols, say \( \mathcal{L} = h_1 \mathcal{F}(a_1) + h_2 \mathcal{G}(b_1) \) where \( h_1, h_2 \) that form the new LC are known to the transmitter and receivers \textit{a priori}. Now, assuming that only receiver 2 (resp. 1) is jammed, \( \mathcal{L} \) is received at receiver 1 (resp. 2) within noise distortion. Using this LC, it can recover \( \mathcal{F}(a_1) \) (resp. \( \mathcal{G}(b_1) \)) from \( \mathcal{L} \) since it already has \( \mathcal{G}(b_1) \) (resp. \( \mathcal{F}(a_1) \)) as side information. When no receiver is jammed, both the receivers are capable of recovering \( \mathcal{F}(a_1), \mathcal{G}(b_1) \) simultaneously.

\textbf{Stage 3}—In this stage, the undelivered symbols to each receiver are transmitted using the technique mentioned above. Let us assume that \( \mathcal{F}_1(a_1) \) and \( \mathcal{G}_1(b_1) \) are symbols available as side information at receivers 2 and 1 respectively. The transmitter sends \( \mathcal{L}(\mathcal{F}_1, \mathcal{G}_1) \), a LC of these symbols on one transmit antenna, with the eventual goal of multicasting this LC (i.e., send it to \textit{both} receivers). The following events, as specified earlier in Stages 1 and 2, are also possible while in this stage.

\textbf{Event 00}: Suppose at time \( t \), if the transmitter sends \( \mathcal{L}(\mathcal{F}_1, \mathcal{G}_1) \), then both the receivers get this LC within noise distortion. With the capability to recover \( \mathcal{L}(\mathcal{F}_1, \mathcal{G}_1) \) within a scaling factor, the receivers 1 and 2 decode their intended messages \( \mathcal{F}_1 \) and \( \mathcal{G}_1 \) respectively using the side informations \( \mathcal{G}_1 \) and \( \mathcal{F}_1 \) that are available with them. Since the intended messages are delivered at the intended receivers, the transmitter, at time \( t + 1 \), sends a new LC of two new symbols \( \hat{\mathcal{L}}(\hat{\mathcal{F}}_1, \hat{\mathcal{G}}_1) \).

\textbf{Event 01}: Since receiver 2 is jammed, its signal is drowned in the jamming signal while receiver 1 gets \( \mathcal{L}(\mathcal{F}_1, \mathcal{G}_1) \) and is capable of recovering \( \mathcal{F}_1 \) using \( \mathcal{G}_1 \) available as side information. The fact that event 01 occurred is known to the transmitter at time \( t + 1 \) via d-JSIT. Thus, at time \( t + 1 \), the transmitter sends a new LC \( \hat{\mathcal{L}}(\hat{\mathcal{F}}_1, \hat{\mathcal{G}}_1) \) since \( \mathcal{G}_1 \) has not yet been delivered to receiver 2.

\textbf{Event 10}: This event is similar to event 01, with the roles of the receivers 1 and 2 interchanged. Hence, receiver 2 is capable of recovering \( \mathcal{G}_1 \) from \( \mathcal{L}(\mathcal{F}_1, \mathcal{G}_1) \) while receiver 1’s signal is drowned in the jamming signal. Thus at time \( t + 1 \), the transmitter sends a new LC \( \hat{\mathcal{L}}(\hat{\mathcal{F}}_1, \hat{\mathcal{G}}_1) \) since \( \mathcal{F}_1 \) has not yet been delivered to receiver 1.

\textbf{Event 11}: Using d-JSIT, transmitter knows at time \( t + 1 \) that the event 11 occurred and hence at time \( t + 1 \), it re-transmits \( \mathcal{L}(\mathcal{F}_1, \mathcal{G}_1) \) on one of its transmit antennas.

Since, all the events are disjoint, in one time slot, the average number of LCs delivered to receiver 1 is given by

\[
E[\# \text{ of symbols delivered to user 1}] = \lambda_{00} + \lambda_{01} = \lambda_1.
\]

Hence, the expected time to deliver one symbol to receiver 1 in this stage is \( \frac{1}{\lambda_1} \). Given that \( \frac{\lambda_0 n_1}{\phi} \) symbols are to be delivered to receiver 1 in this stage, the time taken to achieve this is \( \frac{\lambda_0 n_1}{\lambda_1 \phi} \). Interchanging the roles of the users, the time taken to deliver \( \frac{\lambda_0 n_2}{\phi} \) symbols to receiver 2 is \( \frac{\lambda_0 n_2}{\lambda_2 \phi} \). Thus the total time required to satisfy the requirements of both the receivers in Stage 3 is given by

\[
N_3 = \max \left( \frac{\lambda_0 n_1}{\lambda_1 \phi}, \frac{\lambda_0 n_2}{\lambda_2 \phi} \right).
\]

The optimal DoF achieved in the DD configuration is readily evaluated as

\[
d_1 = \frac{n_1}{N_1 + N_2 + N_3}, \quad d_2 = \frac{n_2}{N_1 + N_2 + N_3}.
\]

Substituting \( \{N_i\}_{i=1,2,3} \) from (77)–(79), we have,

\[
d_1 = \frac{1}{\phi} + \max \left( \frac{\lambda_0 n_1}{\lambda_1 \phi}, \frac{\lambda_0 (1-\eta)}{\lambda_2 \phi} \right) \eta,
\]

\[
d_2 = \frac{1}{\phi} + \max \left( \frac{\lambda_0 n_2}{\lambda_1 \phi}, \frac{\lambda_0 (1-\eta)}{\lambda_2 \phi} \right) (1-\eta).
\]
where $\eta = \frac{n_1 + n_2}{n_1 + n_2}$. Eliminating $\eta$ from the above two equations, yields the DoF region given by Theorem 7. The DoF pairs $(\lambda_1, 0)$ and $(0, \lambda_2)$ are achieved by using the transmission strategy proposed for the NN configuration below.

4.3.3 No CSIT, No JSIT (NN):

The DoF for the NN configuration is given by Theorem 5 and the simple time sharing scheme achieves $\text{DoF}_{\text{NN}}$. For completeness, we briefly explain the transmission scheme used in this configuration. We first explain the achievability of the DoF pair: $(d_1, d_2) = (\lambda_1, 0)$. To this end, note that receiver 1 is jammed in an i.i.d. manner with probability $(1 - \lambda_1)$. This implies that for a scheme of sufficiently large duration $n$, it will receive $\lambda_1 n$ jamming free information symbols (corresponding to those instants in which $S_1(t) = 0$). However, in the NN configuration (no CSIT and no JSIT), the transmitter is not aware of the symbols which are received without being jammed. In order to compensate for the lack of this knowledge, it sends random linear combinations (LCs) (the random coefficients are assumed to be known at the receivers [13]) of $\lambda_1 n$ symbols over $n$ time slots. For sufficiently large $n$, receiver 1 obtains $\lambda_1 n$ jamming free LCs and hence it can decode these symbols. Thus the DoF pair $(\lambda_1, 0)$ is achievable. Similarly, by switching the role of the receivers, the pair $(0, \lambda_2)$ is also achievable. Finally, the entire region in Theorem 5 is achievable by time sharing between these two strategies.

5 Extensions to Multi-receiver MISO Broadcast Channel

We present extensions of the 2-user case to that of a multi-user broadcast channel. In particular, for the $K$-user scenario, the total number of possible jammer states is $2^K$, which can be interpreted as:

$$2^K = \binom{K}{0} + \binom{K}{1} + \ldots + \binom{K}{K}.$$  \hfill (82)

In such a scenario, the jammer state $S(t)$ at time $t$ is a length $K$ vector with each element taking values 0 or 1. We present the optimal DoF regions for the PP, PD and PN configurations in Theorem 8 and for the NN configuration in Theorem 9. For the DP and DD configurations, we present lower bounds on the sum DoF under a class of symmetric jamming strategies. Furthermore, we illustrate the impact of jamming and the availability of JSIT (either instantaneous or delayed) by comparing the DoF achievable in these configurations with the DoF achieved in the absence of jamming (with delayed CSIT) i.e., $\text{DoF}_{\text{MAT}}(K)$ (defined in Section 2) [17]. For most of the configurations, the achievability schemes are straightforward extensions of the coding schemes presented in the 2-user case. Hence, in the interest of space, we do not outline these schemes again.

**Theorem 8** The DoF region of the $K$-user MISO BC for each of the (CSIT, JSIT) configurations PP, PD and PN is the same and is given by the set of non-negative pairs $(d_1, \ldots, d_K)$ that satisfy

$$d_k \leq \lambda_k, \quad k = 1, \ldots, K,$$  \hfill (83)

where $\lambda_k$ is the probability with which the $k$th receiver is not jammed.

The achievability of this DoF region is a straightforward extension of the scheme proposed in Section 4 for the 2-user MISO BC for the corresponding $I_{\text{CSIT}}I_{\text{JSIT}}$ configurations.
Theorem 9  The DoF region of the $K$-user MISO BC for the (CSIT, JSIT) configuration NN is given as

$$\sum_{k=1}^{K} \frac{d_k}{\lambda_k} \leq 1. \quad (84)$$

The achievability of this DoF region is also an extension of the transmission scheme proposed for the NN configuration in Section 4 for the 2-user MISO BC. This is a simple time sharing scheme (TDMA) where the transmitter sends information to only one receiver among the $K$ receivers at any given time instant.

For the DP and DD configurations, we consider a symmetric scenario in which any subset of receivers are jammed symmetrically i.e.,

$$\lambda_s = \lambda_{\pi(s)}, \quad (85)$$

where $\lambda_s$ is the probability that $S(t) = s$ at any given time $t$ and $\pi(s)$ denotes any permutation of the $K$ length jamming state vector $S(t) = s$. In particular, for $K = 3$, this assumption corresponds to

$$\lambda_{001} = \lambda_{010} = \lambda_{100}, \quad \text{and} \quad \lambda_{011} = \lambda_{101} = \lambda_{110}. \quad (86)$$

From (3) and (86), it is seen that

$$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_{000} + \lambda_{001} + \lambda_{010} + \lambda_{011}, \quad (87)$$

i.e., the marginal probabilities of the receivers being jammed (un-jammed) are the same. For the $K$-user case, we have

$$\lambda_1 = \lambda_2 \ldots = \lambda_K. \quad (88)$$

Let $||s||_1$ denote the 1-norm of the $K$-length vector $s$. In other words, $||s||_1$ indicates the total number of 1's seen in the vector $s$ and hence $0 \leq ||s||_1 \leq K$. We denote $\eta_j$ as the total probability with which any $j$ receivers are jammed i.e.,

$$\eta_j = \Pr(||s||_1 = j), \quad j = 0, 1, 2, \ldots, K, \quad (89)$$

where $\Pr(\mathcal{E})$ indicates the probability of occurrence of event $\mathcal{E}$. By definition, we have $\sum_{j=0}^{K} \eta_j = 1$ and we collectively define these probabilities as the $(K+1) \times 1$ vector $\eta = [\eta_0, \eta_1, \ldots, \eta_K]^T$. For instance, $\eta_0 = 1$ corresponds to the no jamming scenario i.e., none of the receivers are jammed. For $K = 3$, we have

$$\eta_0 = \lambda_{000}, \quad \eta_1 = \lambda_{001} + \lambda_{010} + \lambda_{100}$$

$$\eta_2 = \lambda_{011} + \lambda_{101} + \lambda_{110}, \quad \eta_3 = \lambda_{111}. \quad (90)$$

It is easily verified that $\eta_0 + \eta_1 + \eta_2 + \eta_3 = 1$. From (3), (86)-(90), it is seen that $\lambda_i = \eta_0 + \frac{2}{3} \eta_1 + \frac{1}{3} \eta_2$, for $i = 1, 2, 3$. In general, it can be shown that

$$\lambda_1 = \lambda_2 = \cdots = \lambda_K = \left(\sum_{j=0}^{K} \left(\frac{K-j}{K}\right) \eta_j\right) \triangleq \lambda_\eta. \quad (91)$$

Theorem 10  An achievable sum DoF of the $K$-user MISO BC for the (CSIT, JSIT) configuration DP is
given as\footnote{DoF} \[ \text{DoF}^\text{Ach}_\text{DP}(\eta, K) = \sum_{j=0}^{K} \eta_j \text{DoF}_\text{MAT}(K - j). \] (92)

We note from Theorem 10 that when perfect JSIT is available, the sum DoF in (92) is achieved by transmitting only to the unjammed receivers. The transmission scheme that achieves this sum DoF is the $K$-user extension of the scheme presented for the DP configuration in Section 4.

**Theorem 11** An achievable sum DoF of the $K$-user MISO BC for the (CSIT, JSIT) configuration DD is given as

\[ \text{DoF}^\text{Ach}_\text{DD}(\eta, K) = \left( \sum_{j=0}^{K} \left( \frac{K - j}{K} \right) \eta_j \right) \text{DoF}_\text{MAT}(K) \triangleq \lambda_\eta \text{DoF}_\text{MAT}(K). \] (93)

**Remark 10** The DoF result in (93) has the following interesting interpretation: consider a simpler problem in which only two jamming states are present: $S(t) = 00 \cdots 0$ (none of the receivers are jammed) with probability $\lambda_\eta$ and $S(t) = 11 \cdots 1$ (all receivers are jammed) with probability $1 - \lambda_\eta$. In addition, assume that the transmitter has perfect JSIT. In such a scenario, the transmitter can use the MAT scheme (for the $K$-user case) for $\lambda_\eta$ fraction of time to achieve $\lambda_\eta \text{DoF}_\text{MAT}$ degrees-of-freedom (this scenario is equivalent to jamming state $S(t) = 00$ in the DP configuration for a 2-user scenario which is discussed in Section 4) which is precisely as shown in (93). Even though equivalence of these distinct problems is not evident \textit{a priori}, the DoF result indicates the benefits of using JSIT, although it is completely delayed.

It is reasonable to expect that the DoF achievable in the DP configuration will be higher than the DoF that can be achieved in the DD configuration. This can be readily shown as

\[ \text{DoF}^\text{Ach}_\text{DP}(\eta, K) = \sum_{i=0}^{K} \eta_i \text{DoF}_\text{MAT}(K - i) \]
\[ = \sum_{i=0}^{K} \left[ \left( \frac{K - i}{K} \right) \eta_i \left( \frac{K}{K - i} \right) \text{DoF}_\text{MAT}(K - i) \right] \]
\[ = \sum_{i=0}^{K} \left[ \left( \frac{K - i}{K} \right) \eta_i \left( \frac{K}{K - i} \right) \frac{K - i}{1 + \frac{1}{2} + \cdots + \frac{1}{K - i}} \right] \]
\[ = \sum_{i=0}^{K} \left[ \left( \frac{K - i}{K} \right) \eta_i \frac{K}{1 + \frac{1}{2} + \cdots + \frac{1}{K - i}} \right] \]
\[ \geq \sum_{i=0}^{K} \left[ \left( \frac{K - i}{K} \right) \eta_i \frac{K}{1 + \frac{1}{2} + \cdots + \frac{1}{K}} \right] \]
\[ = \sum_{i=0}^{K} \left[ \left( \frac{K - i}{K} \right) \eta_i \text{DoF}_\text{MAT}(K) \right] \]
\[ = \text{DoF}^\text{Ach}_\text{DD}(\eta, K). \] (94)
Fig. 11 shows the DoF comparison between DP and the DD configurations for a special case in which any subset of receivers is jammed with probability $\lambda_s = \frac{1}{2K}, \forall s$ i.e.,

$$\eta_j = \frac{k}{2K}. \quad (95)$$

It is seen that the sum DoF achieved in these configurations increases with the number of users, $K$. The additional DoF achievable in the DP configuration compared to the DD configuration increases with $K$ and is lower bounded by$^8$

$$\text{DoF}_{DP}^\text{Ach}(\eta, K) - \text{DoF}_{DD}^\text{Ach}(\eta, K) \geq \frac{K - 1}{4 \left(1 + \frac{1}{2} + \ldots + \frac{1}{K}\right)^2} \overset{K \to \infty}{\longrightarrow} \infty. \quad (96)$$

Also, it can be shown that the DoF gap between $\text{DoF}_{MAT}(K)$ and $\text{DoF}_{DP}^\text{Ach}(\eta, K)$ is lower bounded by$^9$

$$\text{DoF}_{MAT}(K) - \text{DoF}_{DP}^\text{Ach}(\eta, K) \geq \frac{K}{2 \left(1 + \frac{1}{2} + \ldots + \frac{1}{K}\right)} - \frac{K \left(2^K - 1\right)}{2^K \left(1 + \frac{1}{2} + \ldots + \frac{1}{K}\right)^2} \overset{K \to \infty}{\longrightarrow} \infty. \quad (97)$$

These bounds illustrate the dependence of the sum DoF on the availability of perfect JSIT in a multi-user MISO BC in the presence of jamming attacks. For example, since the transmitter has instantaneous knowledge of the users that are jammed (at any given instant) in the DP configuration, it can conserve

$^8$For large values of $K$, the expression $1 + \frac{1}{2} + \ldots + \frac{1}{K} \to \log(K)$. Hence the right side expression of (96) behaves as $\frac{K}{(\log(K))^2} \overset{K \to \infty}{\longrightarrow} \infty$.

$^9$For large $K$, the expression on the right side of (97) behaves as $\frac{K}{\log(K)} - \frac{K \log(K)}{(\log(K))^2}$ which tends to $\infty$ as $K \to \infty$. 

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energy by only transmitting to the un-jammed receivers. However since no such information is available in the DD configuration, the transmitter has to transmit across different jamming scenarios (different subsets of receivers jammed) in such a configuration to realize DoF gains over naive transmission schemes. The sum DoF achieved in these configurations is much larger than the DoF achieved using a naive transmission scheme (DoF = λη) where the transmitter sends information to only one user at any given time instant without using CSIT or JSIT. The coding schemes that achieve the sum DoF in (92) and (93) are detailed in Section 4.

5.1 Achievability Scheme for DD configuration in K-user scenario

Before we explain the DoF achievability scheme for the K-user DD configuration, we briefly explain the DD configuration for the 2-user MISO BC for a special case in which the users are un-jammed with equal probability i.e,

\[ \lambda = \lambda_1 = \lambda_2 \triangleq \lambda_{01} = \lambda_{10}. \]  

(98)

In such a scenario, a simple 2-phase scheme can be developed to achieve the optimal sum DoF of \( \frac{8\lambda^3}{3} \) (this is seen by substituting \( \lambda_1 = \lambda_2 = \lambda \) and \( n_1 = n_2 \) in (53)). We define order 1 symbols as the set of symbols intended to only 1 receiver while order 2 symbols as the ones that are intended at both the receivers. Phase 1 of the algorithm only uses order 1 symbols while the order 2 symbols are used in the 2nd phase. We define \( \text{DoF}_1(2, \lambda) \) as the DoF of the 2-user MISO BC to deliver order 1 symbols in the case where the receivers are un-jammed with probability \( \lambda \). On similar lines, \( \text{DoF}_2(2, \lambda) \) is the DoF of the system in delivering the order 2 symbols to both the receivers.

- **Phase 1:** Phase 1 consists of 2-stages one each for both the users. In each of these stages, symbols intended for a particular user are transmitted such that they are received at either receiver. Since each receiver is un-jammed with a probability \( \lambda \), it receives \( \lambda d \) symbols intended for itself and \( \lambda d \) symbols of the other user which is used as side information in the 2nd phase of this algorithm. Here \( d \) is the time duration of each stage of this phase. Since a total of \( n \) symbols are transmitted in each stage, we have

\[ 2\lambda d = n \implies d = \frac{n}{2\lambda}. \]  

(99)

The total time spent in this phase is \( 2d = \frac{n}{\lambda} \). At the end of this phase, each user has \( \lambda d = \frac{n}{2} \) intended symbols and \( \frac{n}{2} \) symbols intended for the other user. Using these \( \frac{n}{2} \) side information symbols available at both the users, the transmitter can form \( \frac{n}{2} \) LCs of these symbols which are transmitted in the 2nd phase of the algorithm. These LCs are required by both the users that help them decode their intended symbols. Thus we have

\[ \text{DoF}_1(2, \lambda) = \frac{2n}{\lambda + \frac{n}{\text{DoF}_2(2, \lambda)}}. \]  

(100)

- **Phase 2:** The \( \frac{n}{2} \) LCs of the side information symbols created at the transmitter are multicasted in this phase until both the receivers receive all the LCs. These LCs help the receivers decode their intended symbols using the available CSIR and the side information created in the 1st phase of the algorithm. Since each receiver is jammed with probability \( (1 - \lambda) \), the expected time taken to deliver a order 2
symbol to any receiver is $\frac{1}{\lambda}$. Hence the total time spent in this stage is

$$\frac{n}{2} \max \left( \frac{1}{\lambda}, \frac{1}{\lambda} \right) = \frac{n}{2\lambda}. \tag{101}$$

Using the above result we can calculate $\text{DoF}_2(2, \lambda)$ as

$$\text{DoF}_2(2, \lambda) = \frac{n}{\frac{n}{2\lambda}} = \lambda. \tag{102}$$

Hence the sum $\text{DoF}$ of the 2-user MISO BC is given by

$$\text{DoF}_1(2, \lambda) = \frac{2n}{\frac{n}{\lambda} + \frac{2}{\lambda}} \tag{103}$$

$$= \frac{4\lambda}{3}, \tag{104}$$

which is also the sum $\text{DoF}$ obtained from (53) for the specified scenario. This algorithm also builds up the platform for developing the transmission scheme for the $K$-receiver MISO BC whose $\text{DoF}$ is given by (93).

An interesting observation can be made from this result. If the jammer attacks either both or none of the receivers at any given time (i.e., $\lambda_{01} = \lambda_{10} = 0$) such that the total probability with which the receivers are jammed together is $(1 - \lambda)$ (and hence the probability with which they are not jammed is $\lambda$), the $\text{DoF}$ achievable is $\frac{4}{3} \lambda$ ( $\frac{4}{3}$ is the optimal $\text{DoF}$ achieved in a 2-receiver MISO BC with d-CSIT [17]). This is shown in Fig. 12. Though such an equivalence is not seen apriori, the sum $\text{DoF}$ achieved by this transmission scheme shows that a synergistic benefit is achievable over a long duration of time if all the possible jammer states are used jointly.

Figure 12: State Equivalence when $\lambda_{01} = \lambda_{10}$.
5.1.1 $K$-User:

In this subsection, we present a $K$-phase transmission scheme that achieves the DoF described in Theorem 11. The achievability of Theorem 11 is based on the synergistic usage of delayed CSIT and delayed JSIT by exploiting side-information created at the un-jammed receivers in the past and transmitting linear combinations of such side-information symbols in the future. Before we explain the scheme for this configuration, we first give a brief description of the transmission scheme that achieves $\text{DoF}_{\text{MAT}}(K)$ for the $K$-user MISO BC with delayed CSIT and in the absence of any jamming attacks [17]. Hereafter this scheme is referred to as the MAT scheme.

A $K$-phase transmission scheme is presented in [17] to achieve $\text{DoF}_{\text{MAT}}(K)$. The transmitter has information about the symbols (or linear combinations of the transmitted symbols) available at the receivers via delayed-CSIT. The first phase of the algorithm sends symbols intended for each receiver. The side-information (symbols that are desired at a user but are available at other users) created at the receivers are used in the subsequent phases of the algorithm to create higher order symbols (symbols required by $>1$ receivers) [17], thereby increasing the DoF.

Specifically, $(K - j + 1)j$ order $j$ symbols (symbols intended for $j \leq K$ receivers) are chosen in the $j$th phase to create $j_{j+1}^{(K)}$ order $(j+1)$ symbols that are necessary for $(j+1) \leq K$ receivers and are used in the $(j+1)$th phase of the algorithm. Using this, a recursive relationship between $\text{DoF}$ of the $j$th and $(j+1)$th phases is obtained as [17, eq. (28)],

$$\text{DoF}_j(K) = \frac{(K - j + 1)j^{(K)}}{\binom{K}{j} + j_{j+1}^{(K)}},$$  \hspace{1cm} (105)

where $\text{DoF}_j(K)$ is the DoF of the $K$-user MISO BC to deliver order $j$ symbols. This recursive relationship then leads to the DoF for a $K$-user MISO BC given by $\text{DoF}_{\text{MAT}}(K)$. See [17] for a complete description of the coding scheme.

It is assumed that the decoding process takes place when the receivers have received sufficient linear combinations (LCs) of the intended symbols required to decode their symbols. For example, $n$ jamming free LCs are sufficient to decode $n$ symbols at a receiver. The synergistic benefits of transmitting over different jamming states in these configurations is achievable in the long run by exploiting the knowledge about the present and past jamming states.

Before we present the proposed scheme, notations necessary for the proposed multi-phase transmission scheme are presented. Let $\text{DoF}_j(\eta, K)$ denote the $\text{DoF}$ of the $K$-user MISO BC to deliver order $j$ symbols to the users in a scenario where the receivers are jamming free with equal probability $\lambda_\eta$ given by (91) which is a function of $\eta = [\eta_0, \eta_1, \ldots, \eta_K]$.

We show that in the presence of a jammer, the following relationship (analogous to (105)) holds:

$$\text{DoF}_j(\eta, K) = \frac{(K - j + 1)j^{(K)}}{\binom{K}{j} + \frac{j_{j+1}^{(K)}}{\text{DoF}_{j+1}(\eta, K)}},$$  \hspace{1cm} (106)

Using (106), it can be shown that the $\text{DoF}$ of a $K$-user MISO BC in the presence of such a jamming attack is given by

$$\text{DoF}_{\text{DD}}(\eta, K) \triangleq \text{DoF}_1(\eta, K) = \lambda_\eta \text{DoF}_{\text{MAT}}(K),$$  \hspace{1cm} (107)

where $\text{DoF}_{\text{MAT}}(K)$ is given by (10). We initially present the transmission scheme for the 1st phase and later generalize it for the $j$th ($j \leq K$) phase.
Phase 1: Phase 1 of the coding scheme consists of $K$-stages, one for each receiver. In these stages, symbols intended for each user are transmitted in their respective stages. For instance, let $(a_1, a_2, \ldots, a_K)$ represent the symbols to be delivered to the 1st receiver. The transmitter sends these symbols on its $K$ transmit antennas during the 1st stage. The receivers get jamming free LCs of these symbols when they are not jammed. Each of these $K$-stages end when the LCs intended for a particular receiver are received jamming free by at least one of the $K$ receivers. This information (i.e., which LC was received and whether it was received unjammed or not) is available at the transmitter using d-CSIT and d-JSIT.

Let $d$ denote the duration of one such stage. A particular receiver is not jammed with probability $\lambda_d$, and hence $\lambda_d d$ jamming free LCs are available at each of the $K$ receivers. Since $K$ jamming free LCs suffice to decode $K$ symbols, we enforce $K \times \lambda_d d = K \Rightarrow d = \frac{1}{\lambda_d}$. Since there are $K$ such stages in the 1st phase, the total time duration of this phase is $\tau_1 = \frac{K}{\lambda_d}$. At the end of this phase, each receiver requires $(K - 1)\lambda_d d = (K - 1)$ additional jamming free LCs that are available at the other receivers to decode its symbols. Each receiver has order 1 LCs (side information) that are required by the other receivers. These order 1 LCs are used to create order 2 LCs which are subsequently used in the 2nd phase of the algorithm. Notice that the total number of $(K - 1)K$ order 1 LCs available at the end of this phase can be used to create $(K - 1)\frac{K}{2}$ order 2 LCs that are used in the 2nd phase of the algorithm. Thus the DoF can be represented as

$$\text{DoF}_1(\eta, K) = \frac{K^2}{\tau_1 + \tau_2},$$

where $\tau_2$ is the total time taken to deliver $(K - 1)\frac{K}{2}$ order 2 LCs to the receivers and is given by

$$\tau_2 = \frac{(K-1)K}{2 \text{DoF}_2(\eta, K)}.$$

Thus the DoF$_1(\eta, K)$ is given by

$$\text{DoF}_1(\eta, K) = \frac{K^2}{\frac{K}{\lambda_d} + \frac{(K-1)K}{2 \text{DoF}_2(\eta, K)}} = \frac{K}{\frac{1}{\lambda_d} + \frac{(K-1)K}{2 \text{DoF}_2(\eta, K)}}.$$

Notice here that this conforms with the recursion given in (106).

Phase $j$: In the $j$th phase the transmitter sends $(K - j + 1)$ order $j$-symbols on its $(K - j + 1)$ transmit antennas. The $j$th phase has $\binom{K}{j}$ such stages one each for the $\binom{K}{j}$ different subsets of $j \leq K$ receivers. It can be shown that $(j + 1)$ order $j$ jamming free symbols (LCs) can be used to create $j$ symbols (LCs) of order $(j + 1)$. Equivalently, $1$ order $j$ symbol helps to create $\frac{1}{j+1}$ order $(j + 1)$ symbols. Hence, $(K - j + 1)$ order $j$ jamming free symbols transmitted in the $j$th phase, help to create $(K - j)\frac{1}{j+1}$ order $(j + 1)$ symbols which are subsequently transmitted in the $(j + 1)$th phase of the algorithm.

Since each receiver is not jammed with probability $\lambda_n$, the average time required to deliver an order $j$ symbol (LC) is $\frac{1}{\lambda_n}$. The total time duration of this phase is $\frac{\binom{K}{j}}{\lambda_n}$ since we have $\binom{K}{j}$ stages. Thus the $j$th phase transmits $(K - j + 1)\binom{K}{j}$ jamming free symbols of order $j$ in $\frac{\binom{K}{j}}{\lambda_n}$ time slots and generates $j\binom{K}{j}$ order $(j + 1)$ symbols which are delivered to the receivers in the subsequent phases. The $K$th phase transmits symbols of order $K$ and does not create any new symbols (LCs). Thus we have

$$\text{DoF}_j(\eta, K) = \frac{(K - j + 1)\binom{K}{j}}{\frac{\binom{K}{j}}{\lambda_n} + \text{DoF}_{j+1}(\eta, K)}.$$

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Using this recurrence relation we can show that

\[ \text{DoF}_1(\eta, K) = \lambda \eta \frac{K}{1 + \frac{1}{2} \cdots + \frac{1}{K}}. \]  

(112)

6 Conclusions

In this paper, the MISO broadcast channel has been studied in the presence of a time-varying jammer. We introduced a new variable JSIT to indicate the presence or absence of information regarding the jammer. From our results, the interplay between CSIT and JSIT and associated impact on the DoF regions are illuminated. For the case in which there is perfect CSIT, by employing a randomized zero-forcing precoding scheme, the DoF region remains the same irrespective of the availability/un-availability of JSIT. On the other hand, for the case of delayed CSIT and JSIT, our results show that both the jammer and channel state information must be synergistically used in order to provide DoF gains. Whenever there is perfect JSIT, it is seen that the jammers’ states are separable and the optimal strategy is to send information symbols independently across different jamming states. The result for the NN configuration quantifies the DoF loss in case of unavailability of JSIT and CSIT. The results for the K-user MISO BC indicate the scaling of the sum DoF with the number of users in the presence of jamming attacks. Finally, several interesting open questions and directions emerge out of this work. We outline some of these below.

1. It remains unclear if the inner bounds to the DoF region for the DN and ND configurations are optimal. The exact DoF region for these configurations remains an interesting open problem. The DoF region achieved by the DD configuration in the 2-user MISO BC serves as an outer bound for both DN and ND configurations. Improving both these inner and outer bounds for the DN and ND configurations is a challenging problem.

2. For the DD configuration, a 3-stage scheme is proposed to achieve the optimal DoF region. In the 3rd stage of this coding scheme, the transmitter did not require any CSIT or JSIT. This raises an interesting question: what is the minimum fraction of time over which CSIT and JSIT must be acquired in order to achieve the optimal DoF. A similar problem has been considered in the absence of a jammer [25], in which the minimum amount of CSIT required to achieve a particular DoF value is characterized.

3. Finally, the results presented in this paper can possibly be extended to scenarios where the jammers’ statistics are not stationary. While the analysis presented in this paper assumes that the jammers’ states are i.i.d and that its statistics are constant across time, it would be interesting to understand the behavior of DoF regions in a scenario where the jammers’ states are correlated across time and also possibly correlated with the transmit signals.

7 Appendix

7.1 Converse Proof for Theorem 1

We first present the proof for the bounds \( d_1 \leq \lambda_1 \) and \( d_2 \leq \lambda_2 \) for the (CSIT, JSIT) configuration PP. Clearly, these bounds would also continue to serve as valid outer bounds for the worse configurations PD and PN. Since these bounds are symmetric, it suffices to prove that \( d_1 \leq \lambda_1 \). We have the following sequence of
bounds for receiver 1:

\[ nR_1 = H(W_1) = H(W_1|H^n, S^n_1, S^n_2) \]
\[ = I(W_1; Y^n_1|H^n, S^n_1, S^n_2) + H(W_1|Y^n_1, H^n, S^n_1, S^n_2) \]
\[ \leq I(W_1; Y^n_1|H^n, S^n_1, S^n_2) + n\epsilon_n \]
\[ = h(Y^n_1|H^n, S^n_1, S^n_2) - h(Y^n_1|W_1, H^n, S^n_1, S^n_2) + n\epsilon_n \]
\[ \leq n \log(P_T) - h(Y^n_1|W_1, H^n, S^n_1, S^n_2) + n\epsilon_n \]
\[ \leq n \log(P_T) - h(Y^n_1|X^n, W_1, H^n, S^n_1, S^n_2) + n\epsilon_n \]
\[ = n \log(P_T) - h(S^n_1 G^n_1 J^n_1 + N^n_1|X^n, W_1, H^n, S^n_1, S^n_2) + n\epsilon_n \]
\[ \leq n \log(P_T) - n(\lambda_{10} + \lambda_{11}) \log(P_T) + n\epsilon_n \]
\[ = n(1 - \lambda_{10} - \lambda_{11}) \log(P_T) + n\epsilon_n \]
\[ = n(\lambda_{00} + \lambda_{01}) \log(P_T) + n\epsilon_n \]
\[ = n\lambda_1 \log(P_T) + n\epsilon_n, \]  

where (115) follows from Fano's inequality, (121) is obtained from the fact that \( \Pr(S_1(t) = 1) = (\lambda_{11} + \lambda_{10}) \) and the assumption that the jammer’s signal is AWGN with power \( P_T \). Normalizing (124) by \( n \log(P_T) \), and taking the limit \( n \to \infty \) and then \( P_T \to \infty \), we obtain

\[ d_1 \leq \lambda_1. \]  

On similar lines since user 2 is jammed with probability \( (\lambda_{11} + \lambda_{01}) \), it can be readily proved that

\[ d_2 \leq (\lambda_{00} + \lambda_{10}) = \lambda_2. \]  

### 7.2 Converse Proof for Theorem 3

We next provide the proof for the (CSIT, JSIT) configuration DD, in which the transmitter has delayed CSIT and delayed JSIT. In this case, we prove the bound:

\[ \frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_1 + \lambda_2} \leq 1. \]  

Let \( \Omega = (H^n, S^n_1, S^n_2) \) denote the global CSIT and JSIT for the entire block length \( n \). We next enhance the original MISO broadcast channel and make it physically degraded by letting a genie provide the output of receiver 1 to receiver 2. Formally, in the new MISO BC, receiver 1 has \( (Y^n_1, \Omega) \) and receiver 2 has \( (Y^n_1, Y^n_2, \Omega) \). We next note that for a physically degraded BC, it is known from [26] that feedback from the receivers does not increase the capacity region. We can therefore remove delayed CSIT and delayed JSIT from the transmitter without decreasing the capacity region of the enhanced MISO BC. The capacity region for this model serves as an outer bound to the capacity region of the original MISO BC.

Henceforth, we will focus on the model in which receiver 1 has \( (Y^n_1, \Omega) \), receiver 2 has \( (Y^n_1, Y^n_2, \Omega) \) and most importantly, the transmitter has no CSIT and no JSIT.

For such a model, we next state the following key property, which we call as the statistical equivalence property (denoted in short by SEP):

\[ h(H_1(t)X(t) + N_1(t)) = h(H_2(t)X(t) + N_2(t)). \]
This property follows from the following facts:

1. $\mathbf{H}_1(t)$ and $\mathbf{H}_2(t)$ are drawn from the same distribution.
2. $N_1(t)$ and $N_2(t)$ are statistically equivalent, i.e., drawn from the same distribution.
3. $\mathbf{X}(t)$ is independent of $(\mathbf{H}_1^n, \mathbf{H}_2^n, N_1^n, N_2^n)$.

With these in place, we have the following sequence of bounds for receiver 1:

\[
R_1 = H(W_1) = H(W_1|\Omega) \\
\leq I(W_1; Y_1^n|\Omega) + n\epsilon_n \\
= h(Y_1^n|\Omega) - h(Y_1^n|W_1, \Omega) + n\epsilon_n \\
\leq n \log(P_T) - h(Y_1^n|W_1, \Omega) + n\epsilon_n. \\
\]

We now focus on the second term appearing in (132):

\[
h(Y_1^n|W_1, \Omega) = \sum_{t=1}^{n} h(Y_{1t}|W_1, \Omega, Y_{1t}^{t-1}) \\
\geq \sum_{t=1}^{n} h(Y_{1t}|W_1, \Omega, Y_{1t}^{t-1}, Y_{2t}^{t-1}) \\
= \sum_{t=1}^{n} h(Y_{1t}|S_1(t), S_2(t), W_1, \Omega \setminus \{S_1(t), S_2(t)\}, Y_{1t}^{t-1}, Y_{2t}^{t-1}) \\
= \sum_{t=1}^{n} \left[ \lambda_{00} h(Y_{1t}|S_1(t) = 0, S_2(t) = 0, U_t) \\
+ \lambda_{01} h(Y_{1t}|S_1(t) = 0, S_2(t) = 1, U_t) \\
+ \lambda_{10} h(Y_{1t}|S_1(t) = 1, S_2(t) = 0, U_t) \\
+ \lambda_{11} h(Y_{1t}|S_1(t) = 1, S_2(t) = 1, U_t) \right] \\
= \sum_{t=1}^{n} \left[ \lambda_{00} h(\mathbf{H}_1(t)\mathbf{X}(t) + N_1(t)|U_t) \\
+ \lambda_{01} h(\mathbf{H}_1(t)\mathbf{X}(t) + N_1(t)|U_t) \\
+ \lambda_{10} h(\mathbf{H}_1(t)\mathbf{X}(t) + \mathbf{G}_1(t)\mathbf{J}(t) + N_1(t)|U_t) \\
+ \lambda_{11} h(\mathbf{H}_1(t)\mathbf{X}(t) + \mathbf{G}_1(t)\mathbf{J}(t) + N_1(t)|U_t) \right] \\
\geq \sum_{t=1}^{n} \left[ \lambda_{00} + \lambda_{01} h(\mathbf{H}_1(t)\mathbf{X}(t) + N_1(t)|U_t) \\
+ \lambda_{10} + \lambda_{11} h(\mathbf{H}_1(t)\mathbf{X}(t) + \mathbf{G}_1(t)\mathbf{J}(t) + N_1(t)|U_t) \right]. \\
\]
\[
= \sum_{t=1}^{n} \left[ (\lambda_{00} + \lambda_{01}) h(H_1(t)X(t) + N_1(t)|U_i) + (\lambda_{10} + \lambda_{11}) h(G_1(t)J(t) + N_1(t)) \right]
\]

\[
\geq \sum_{t=1}^{n} \left[ (\lambda_{00} + \lambda_{01}) h(H_1(t)X(t) + N_1(t)|U_i) + (\lambda_{10} + \lambda_{11}) \log(P_T) \right]
\]

\[
= (\lambda_{00} + \lambda_{01}) \sum_{t=1}^{n} \eta_t + n(\lambda_{10} + \lambda_{11}) \log(P_T)
\]
We next bound each one of the four terms in (155) as follows:

\[
\begin{align*}
    h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 0, U_i) \\
    &= \sum_{t=1}^{n} \left[ \lambda_{00} h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 0, U_i) \\
    &\quad + \lambda_{01} h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 1, U_i) \\
    &\quad + \lambda_{10} h(Y_{1t}, Y_{2t} | S_1(t) = 1, S_2(t) = 0, U_i) \\
    &\quad + \lambda_{11} h(Y_{1t}, Y_{2t} | S_1(t) = 1, S_2(t) = 1, U_i) \right].
\end{align*}
\] (155)

In summary, we have

\[
\begin{align*}
    h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 0, U_i) \\
    &= h(H_1(t) X(t) + N_1(t), H_2(t) X(t) + N_2(t) | S_1(t) = 0, S_2(t) = 0, U_i) \\
    &\leq h(H_1(t) X(t) + N_1(t) | S_1(t) = 0, S_2(t) = 0, U_i) \\
    &\quad + h(H_2(t) X(t) + N_2(t) | S_1(t) = 0, S_2(t) = 0, U_i) \\
    &= h(H_1(t) X(t) + N_1(t) | U_i) + h(H_2(t) X(t) + N_2(t) | U_i) \\
    &= 2\eta_t,
\end{align*}
\] (156)

where in (159), we have made use of the (conditional version of) statistical equivalence property (SEP) for the two receivers as stated in (128).

\[
\begin{align*}
    h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 1, U_i) \\
    &= h(H_1(t) X(t) + N_1(t), H_2(t) X(t) + G_2(t) J(t) + N_2(t) | S_1(t) = 0, S_2(t) = 1, U_i) \\
    &\leq h(H_1(t) X(t) + N_1(t) | S_1(t) = 0, S_2(t) = 1, U_i) \\
    &\quad + h(H_2(t) X(t) + G_2(t) J(t) + N_2(t) | S_1(t) = 0, S_2(t) = 1, U_i) \\
    &\leq h(H_1(t) X(t) + N_1(t) | U_i) + \log(P_T) \\
    &= \eta_t + \log(P_T).
\end{align*}
\] (157)

In summary, we have

\[
\begin{align*}
    h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 0, U_i) &\leq 2\eta_t \quad (158) \\
    h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 1, U_i) &\leq \eta_t + \log(P_T) \quad (159) \\
    h(Y_{1t}, Y_{2t} | S_1(t) = 1, S_2(t) = 0, U_i) &\leq \eta_t + \log(P_T) \quad (160) \\
    h(Y_{1t}, Y_{2t} | S_1(t) = 1, S_2(t) = 1, U_i) &\leq 2\log(P_T). \quad (161)
\end{align*}
\]

Substituting these back in (155), we obtain

\[
\begin{align*}
    h(Y_{1t}^n, Y_{2t}^n | W_1, \Omega) &\leq n(\lambda_{01} + \lambda_{10} + 2\lambda_{11}) \log(P_T) + (\lambda_{01} + \lambda_{10} + 2\lambda_{00}) \sum_{t=1}^{n} \eta_t \\
    &\leq n\sum_{t=1}^{n} \eta_t + n\epsilon_n. \quad (162)
\end{align*}
\] (162)

Upon substituting (162) back in (151), we have the following bound on $R_2$:

\[
\begin{align*}
    nR_2 &\leq (\lambda_{01} + \lambda_{10} + 2\lambda_{00}) \sum_{t=1}^{n} \eta_t + n\epsilon_n \\
    &= (\lambda_1 + \lambda_2) \sum_{t=1}^{n} \eta_t + n\epsilon_n \quad (163)
\end{align*}
\] (163)

where $\lambda_1 = \lambda_{01} + \lambda_{10} + 2\lambda_{00}$ and $\lambda_2 = \lambda_{11}$.
In summary, from (146) and (170), we can write

\[ nR_1 \leq n\lambda_1 \log(P_T) - \lambda_1 \sum_{t=1}^{n} \eta_t + n\epsilon_n \]  

\[ nR_2 \leq (\lambda_1 + \lambda_2) \sum_{t=1}^{n} \eta_t + n\epsilon_n \]  

(171)  

(172)

Eliminating the term \( \sum_{t=1}^{n} \eta_t \), we obtain

\[ \frac{nR_1}{\lambda_1} + \frac{nR_2}{(\lambda_1 + \lambda_2)} \leq n \log(P_T) + n\epsilon_n' \]  

(173)

Normalizing by \( n \log(P_T) \), and taking the limits \( n \to \infty \) and then \( P_T \to \infty \), we obtain the bound:

\[ \frac{d_1}{\lambda_1} + \frac{d_2}{(\lambda_1 + \lambda_2)} \leq 1. \]  

(174)

Reversing the role of receivers 1 and 2, i.e., making receiver 2 degraded with respect to receiver 1, we can similarly obtain the other bound

\[ \frac{d_1}{(\lambda_1 + \lambda_2)} + \frac{d_2}{\lambda_2} \leq 1. \]  

(175)

This completes the proof of the converse for Theorem 3.

### 7.3 Converse Proof for Theorem 2

We next provide the proof for the (CSIT, JSIT) configuration DP, in which the transmitter has delayed CSIT and perfect (instantaneous) JSIT. In this case, we prove the bound:

\[ 2d_1 + d_2 \leq 2\lambda_{00} + 2\lambda_{01} + \lambda_{10} \]  

(176)

Let \( \Omega = (H^n, S^n_1, S^n_2) \) denote the global CSIT and JSIT for the entire block length \( n \). As in the proof for Theorem 3, we enhance the original MISO broadcast channel and make it physically degraded by letting a genie provide the output of receiver 1 to receiver 2. Formally, in the new MISO BC, receiver 1 has \( (Y^n_1, \Omega) \) and receiver 2 has \( (Y^n_1, Y^n_2, \Omega) \). We next note that for a physically degraded BC, it is known from [26] that feedback from the receivers does not increase the capacity region. We can therefore remove delayed CSIT from the transmitter without decreasing the capacity region of the enhanced MISO BC. The capacity region for this model serves as an outer bound to the capacity region of the original MISO BC.

Henceforth, we will focus on the model in which receiver 1 has \( (Y^n_1, \Omega) \), receiver 2 has \( (Y^n_1, Y^n_2, \Omega) \) and most importantly, the transmitter has no CSIT. Note that unlike in proof for Theorem 3, in this case we cannot remove the assumption of perfect JSIT. Recall that in the proof of Theorem 3, in this case we made use of the following relationships (which we called as the statistical equivalence property):

\[ h(H_1(t)X(t) + N_1(t)|S_1(t) = i, S_2(t) = j, U_t) = h(H_2(t)X(t) + N_2(t)|S_1(t) = i', S_2(t) = j', U_t). \]  

(177)
for \(i, i', j, j' \in \{0, 1\}\). In this case, we can only use a stricter version of the statistical equivalence property:

\[
h(\mathbf{H}_1(t)\mathbf{X}(t) + N_1(t)|S_1(t) = 0, S_2(t) = 0, U_t) = h(\mathbf{H}_2(t)\mathbf{X}(t) + N_2(t)|S_1(t) = 0, S_2(t) = 0, U_t). \tag{178}
\]

The reason is that for the DP configuration, due to the fact that the transmitter has perfect JSIT, the marginal probabilities \(p(X(t)|S_1(t) = i, S_2(t) = j, U_t)\) can depend explicitly on \((i, j)\), the realization of jammer’s strategies at time \(t\), which was not the case in Theorem 3.

With these in place, we have the following sequence of bounds for receiver 1:

\[
nR_1 \leq n \log(P_T) - h(Y^n_1|W_1, \Omega) + n\epsilon_n. \tag{179}
\]

We next focus on the second term in (179):

\[
h(Y^n_1|W_1, \Omega) = \sum_{i=1}^{n} h(Y_{1i}|W_1, \Omega, Y_1^{t-1}) \geq \sum_{i=1}^{n} h(Y_{1i}|W_1, \Omega, Y_1^{t-1}, Y_2^{t-1}) \tag{180}
\]

\[
= \sum_{i=1}^{n} h(Y_{1i}|S_1(t), S_2(t), W_1, \Omega \setminus \{S_1(t), S_2(t)\}, Y_1^{t-1}, Y_2^{t-1}) \tag{181}
\]

\[
= \sum_{i=1}^{n} h(Y_{1i}|S_1(t), S_2(t), U_t)
\]

\[
= \sum_{i=1}^{n} \left[ \lambda_{00} h(Y_{1i}|S_1(t) = 0, S_2(t) = 0, U_t) + \lambda_{01} h(Y_{1i}|S_1(t) = 0, S_2(t) = 1, U_t) + \lambda_{10} h(Y_{1i}|S_1(t) = 1, S_2(t) = 0, U_t) + \lambda_{11} h(Y_{1i}|S_1(t) = 1, S_2(t) = 1, U_t) \right]
\]

\[
\geq \sum_{i=1}^{n} \left[ \lambda_{00} \underbrace{h(\mathbf{H}_1(t)\mathbf{X}(t) + N_1(t)|S_1(t) = 0, S_2(t) = 0, U_t)}_{\geq 0} + \lambda_{01} \underbrace{h(\mathbf{H}_1(t)\mathbf{X}(t) + G_1(t)\mathbf{J}(t) + N_1(t)|S_1(t) = 1, S_2(t) = 0, U_t)}_{\geq \log(P_T)} + \lambda_{10} \underbrace{h(\mathbf{H}_1(t)\mathbf{X}(t) + G_1(t)\mathbf{J}(t) + N_1(t)|S_1(t) = 1, S_2(t) = 1, U_t)}_{\geq \log(P_T)} + \lambda_{11} \underbrace{h(\mathbf{H}_1(t)\mathbf{X}(t) + G_1(t)\mathbf{J}(t) + N_1(t)|S_1(t) = 1, S_2(t) = 1, U_t)}_{\geq \log(P_T)} \right] \tag{182}
\]
\[ = \lambda_0 \sum_{t=1}^{n} \eta_t^{(00)} + \lambda_1 \sum_{t=1}^{n} \eta_t^{(01)} + n(\lambda_{10} + \lambda_{11}) \log(P_T), \]  

where in (182), we used the fact that elements of \( J_T \) are i.i.d. with variance \( P_T \), and in (183), we have defined

\[ \eta_t^{(00)} \triangleq h(H_1(t)X(t) + N_1(t) | S_1(t) = 0, S_2(t) = 0, U_t) \]  

(184)

\[ \eta_t^{(01)} \triangleq h(H_1(t)X(t) + N_1(t) | S_1(t) = 0, S_2(t) = 1, U_t). \]  

(185)

Substituting (183) in (179), we obtain

\[ nR_1 \leq n(\lambda_{00} + \lambda_{01}) \log(P_T) - \lambda_{00} \sum_{t=1}^{n} \eta_t^{(00)} - \lambda_{01} \sum_{t=1}^{n} \eta_t^{(01)} + n\epsilon_n \]  

(186)

We next focus on the receiver 2 which has access to both \( Y_1^n \) and \( Y_2^n \). We can obtain the following bound similar to the one obtained in the proof for Theorem 3:

\[ nR_2 \leq h(Y_1^n, Y_2^n | W_1, \Omega) - n(\lambda_{01} + \lambda_{10} + 2\lambda_{11}) \log(P_T) + n\epsilon_n, \]  

(187)

We next expand the first term in (187) as follows:

\[
 \begin{aligned}
 h(Y_1^n, Y_2^n | W_1, \Omega) &= \sum_{t=1}^{n} h(Y_{1t}, Y_{2t} | W_1, \Omega, Y_1^{t-1}, Y_2^{t-1}) \\
 &= \sum_{t=1}^{n} h(Y_{1t}, Y_{2t} | S_1(t), S_2(t), W_1, \Omega \setminus \{S_1(t), S_2(t)\}, Y_1^{t-1}, Y_2^{t-1})_{(189)} \\
 &= \sum_{t=1}^{n} h(Y_{1t}, Y_{2t} | S_1(t), S_2(t), U_t)_{(190)} \\
 &= \sum_{t=1}^{n} \left[ \lambda_{00} h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 0, U_t) \\
 &+ \lambda_{01} h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 1, U_t)_{(189)} \\
 &\leq h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 1, U_t) + \log(P_T) \\
 &+ \lambda_{10} h(Y_{1t}, Y_{2t} | S_1(t) = 1, S_2(t) = 0, U_t)_{(189)} \\
 &\leq 2\log(P_T) \\
 &+ \lambda_{11} h(Y_{1t}, Y_{2t} | S_1(t) = 1, S_2(t) = 1, U_t)_{(189)} \\
 &\leq 2\log(P_T) \\
 \right] \\
 &\leq \sum_{t=1}^{n} \left[ \lambda_{00} h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 0, U_t) \\
 &+ \lambda_{01} h(Y_{1t} | S_1(t) = 0, S_2(t) = 1, U_t) \\
 &+ \lambda_{10} h(Y_{1t} | S_1(t) = 1, S_2(t) = 0, U_t) \\
 &+ \lambda_{11} h(Y_{1t} | S_1(t) = 1, S_2(t) = 1, U_t)_{(189)} \\
 &\leq \lambda_{00} h(Y_{1t}, Y_{2t} | S_1(t) = 0, S_2(t) = 0, U_t) \\
 &+ \lambda_{01} h(Y_{1t} | S_1(t) = 0, S_2(t) = 1, U_t) \\
 &+ \lambda_{10} h(Y_{1t} | S_1(t) = 1, S_2(t) = 0, U_t) \\
 &+ (\lambda_{01} + 2\lambda_{10} + 2\lambda_{11}) \log(P_T) \\
 \right] 
\end{aligned}
\]
\[
\begin{align*}
\leq \sum_{t=1}^{n} \left[ \lambda_{00} h(Y_1 \mid S_1(t) = 0, S_2(t) = 0, U_i) \\
+ \lambda_{00} h(Y_2 \mid S_1(t) = 0, S_2(t) = 0, U_i) \\
+ \lambda_{01} h(Y_1 \mid S_1(t) = 0, S_2(t) = 1, U_i) \\
+ (\lambda_{01} + 2\lambda_{10} + 2\lambda_{11}) \log(P_T) \right] \\
= \sum_{t=1}^{n} \left[ \lambda_{00} h(H_1(t)X(t) + N_1(t) \mid S_1(t) = 0, S_2(t) = 0, U_i) \\
+ \lambda_{00} h(H_2(t)X(t) + N_2(t) \mid S_1(t) = 0, S_2(t) = 0, U_i) \\
+ \lambda_{00} h(H_1(t)X(t) + N_1(t) \mid S_1(t) = 0, S_2(t) = 1, U_i) \\
+ (\lambda_{01} + 2\lambda_{10} + 2\lambda_{11}) \log(P_T) \right] \\
= 2\lambda_{00} \sum_{t=1}^{n} \eta_{t}^{(00)} + \lambda_{01} \sum_{t=1}^{n} \eta_{t}^{(01)} + n(\lambda_{01} + 2\lambda_{10} + 2\lambda_{11}) \log(P_T). \\
\end{align*}
\] (191)

Substituting (193) in (187), we get

\[
nR_2 \leq 2\lambda_{00} \sum_{t=1}^{n} \eta_{t}^{(00)} + \lambda_{01} \sum_{t=1}^{n} \eta_{t}^{(01)} + n\lambda_{10} \log(P_T) + n\epsilon_n. \\
\] (194)

Collectively, from (186) and (194), we can then write:

\[
\begin{align*}
nR_1 &\leq n(\lambda_{00} + \lambda_{01}) \log(P_T) - \lambda_{00} \sum_{t=1}^{n} \eta_{t}^{(00)} - \lambda_{01} \sum_{t=1}^{n} \eta_{t}^{(01)} + n\epsilon_n \\
nR_2 &\leq 2\lambda_{00} \sum_{t=1}^{n} \eta_{t}^{(00)} + \lambda_{01} \sum_{t=1}^{n} \eta_{t}^{(01)} + n\lambda_{10} \log(P_T) + n\epsilon_n \\
\end{align*}
\] (195) (196)

Taking \(2 \times (195) + (196)\), we obtain:

\[
n(2R_1 + R_2) \leq n(2\lambda_{00} + 2\lambda_{01} + \lambda_{10}) \log(P_T) - \lambda_{01} \sum_{t=1}^{n} \eta_{t}^{(01)} + n\epsilon_n \]
\[
\leq n(2\lambda_{00} + 2\lambda_{01} + \lambda_{10}) \log(P_T) + n\epsilon_n, \\
\] (197) (198)

where we used the fact that \(\eta_{t}^{(00)} \geq 0\) for all \(t\). Normalizing by \(n \log(P_T)\), and taking the limits \(n \to \infty\), and then \(P_T \to \infty\), we obtain

\[
2d_1 + d_2 \leq 2\lambda_{00} + 2\lambda_{01} + \lambda_{10}. \\
\] (199)

Reversing the roles of receivers 1 and 2, we can obtain the other bound:

\[
d_1 + 2d_2 \leq 2\lambda_{00} + 2\lambda_{10} + \lambda_{01}. \\
\] (200)
7.4 Converse Proof for Theorem 5

Here, we consider the configuration in which there is no CSIT and no JSIT i.e., NN configuration and prove the bound:

\[
\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} \leq 1. \tag{201}
\]

To this end, we recall a classical result [27], which states that for memoryless broadcast channels without feedback, the capacity region only depends on marginal distributions \( p(y_k|x) \), for \( k = 1, 2 \). This implies for the problem at hand, in which the jammer’s strategy is memoryless, and there is no CSIT and no JSIT, the capacity region only depends on the marginal probabilities \( \lambda_1 \) and \( \lambda_2 \), i.e., the probabilities with which each of the receiver is not jammed. Without loss of generality, assume that \( \lambda_1 \geq \lambda_2 \), i.e., receiver 2 is jammed with higher probability than receiver 1.

We will now show that this MISO BC falls in the class of stochastically degraded broadcast channels. We first recall that a broadcast channel (defined by \( p(y_1, y_2|x) \)) is stochastically degraded [28] if there exists a random variable \( Y_1' \) such that

1. \( Y_1'|\{X = x\} \sim p_{Y_1|X}(y_1'|x) \), i.e., \( Y_1' \) has the same conditional distribution as \( Y_1 \) (given \( X \)), and
2. \( X \rightarrow Y_1' \rightarrow Y_2 \) form a Markov chain.

Hence, in order to show that the MISO BC with no CSIT and no JSIT is stochastically degraded, we will show the existence of a random variable \( Y_1' \) such that \( Y_1' \) has the same conditional pdf as \( Y_1 \) and \( X \rightarrow Y_1' \rightarrow Y_2 \) form a Markov chain. We first note that the channel outputs for the original BC at time \( t \) are:

\[
Y_1(t) = H_1(t)x(t) + S_1(t)G_1(t)j(t) + N_1(t) \tag{202}
\]
\[
Y_2(t) = H_2(t)x(t) + S_2(t)G_2(t)j(t) + N_2(t). \tag{203}
\]

Next, we create an artificial output \( Y_1' \), defined at time \( t \) as:

\[
Y_1'(t) = H_2(t)x(t) + \tilde{S}(t)S_2(t)G_2(t)j(t) + N_2(t), \tag{204}
\]

where the random variable \( \tilde{S}(t) \) is distributed i.i.d. as follows:

\[
\tilde{S}(t) = \begin{cases} 
0, & \text{w.p. } \frac{\lambda_1 - \lambda_2}{\lambda_1 - \lambda_2}, \\
1, & \text{w.p. } \frac{1 - \lambda_1}{\lambda_1 - \lambda_2}. 
\end{cases} \tag{205}
\]

Furthermore, \( \tilde{S}(t) \) is independent of all other random variables.

It is straightforward to verify that \( Y_1' \) and \( Y_1 \) have the same marginal distribution: since \( H_1(t) \) and \( H_2(t) \) are identically distributed, \( G_1(t) \) and \( G_2(t) \) are identically distributed, \( N_1(t) \) and \( N_2(t) \) are identically distributed, and most importantly, the random variables \( \tilde{S}(t)S_2(t) \) and \( S_1(t) \) are identically distributed. Furthermore, note that when \( \tilde{S}(t) = 0 \), then \( Y_2(t) = Y_1'(t) + G_2(t)j(t) + N_2(t) \), and when \( \tilde{S}(t) = 0 \), then we have \( Y_2(t) = Y_1'(t) \), i.e., \( X(t) \rightarrow Y_1'(t) \rightarrow Y_2(t) \) forms a Markov chain.

This argument proves that the original MISO broadcast channel with no CSIT falls in the class of stochastically degraded broadcast channels, for which the capacity region is given by the set of rate pairs \((R_1, R_2)\).
where $U \rightarrow X \rightarrow (Y_1, Y_2, S_1, S_2)$ forms a Markov chain. Using this, we can write

$$R_2 \leq h(Y_2|H, S_1, S_2) - h(Y_2|U, H, S_1, S_2) \leq \log(P_T) - (1 - \lambda_2) \log(P_T) - \lambda_2 h(H_2 X + N_2|U, H) + o(\log(P_T)).$$  

(208)

Similarly, the other bound can be written as:

$$R_1 \leq h(Y_1|U, H, S_1, S_2) - h(Y_1|X, U, H, S_1, S_2) = (1 - \lambda_1) \log(P_T) + \lambda_1 h(H_1 X + N_1|U, H) + o(\log(P_T)).$$  

(211)

where (214) follows from the statistically equivalence property (as stated in the previous section). Combining (210) and (214), we obtain:

$$\frac{R_1}{\lambda_1} + \frac{R_2}{\lambda_2} \leq \log(P_T) + o(\log(P_T)).$$  

(215)

Normalizing by $\log(P_T)$ and taking the limit $P_T \rightarrow \infty$, we have the proof for

$$\frac{d_1}{\lambda_1} + \frac{d_2}{\lambda_2} \leq 1.$$  

(216)

### 7.5 Converse Proof for Theorem 4

Here, we consider the configuration in which there is no CSIT and perfect JSIT i.e., NP configuration and prove the bound:

$$d_1 + d_2 \leq \lambda_{00} + \lambda_{01}.$$  

(217)

Let $\Omega = (S_1^n, S_2^n)$ denote the global JSIT for the entire block length $n$. We have the following sequence of bounds

$$n(R_1 + R_2) = H(W_1) + H(W_2)$$  

(218)

$$= H(W_1, W_2)$$  

(219)

$$= H(W_1, W_2|\Omega)$$  

(220)

$$= I(W_1, W_2; Y_1^n, Y_2^n|\Omega) + H(W_1, W_2|Y_1^n, Y_2^n, \Omega)$$  

(221)

$$\leq I(W_1, W_2; Y_1^n, Y_2^n|\Omega) + n\epsilon_n$$  

(222)

$$= h(Y_1^n, Y_2^n|\Omega) - h(Y_1^n, Y_2^n|\Omega, W_1, W_2) + n\epsilon_n.$$  

(223)

Note here that the two receivers are statistically equivalent when they are not jammed with a probability $\lambda_{00}$. In such a scenario, the transmitter can send information to only one receiver as there is no CSIT available.
Using this, we have the following
\[
 n(R_1 + R_2) \leq h(Y^n_1, Y^n_2 | \Omega) - h(Y^n_1, Y^n_2 | \Omega, W_1, W_2) + n \epsilon_n
\]  
(224)
\[
\leq n(\lambda_{00} \log P_T + \lambda_{01} 2 \log(P_T) + \lambda_{10} 2 \log(P_T) + \lambda_{11} 2 \log(P_T))
\]  
(225)
\[
- n(\lambda_{01} \log(P_T) + \lambda_{10} \log(P_T) + \lambda_{11} \log(P_T))
\]  
(226)
\[
n(R_1 + R_2) = n(\lambda_{00} \log(P_T) + \lambda_{01} \log(P_T) + \lambda_{10} \log(P_T))
\]  
(227)

Normalizing by \( n \log(P_T) \) and then \( n \to \infty \) and \( P_T \to \infty \) we obtain the bound
\[
d_1 + d_2 \leq (\lambda_{00} + \lambda_{01} + \lambda_{10}).
\]  
(228)

This completes the converse proof for Theorem 5.

References


