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Nullforming for Large Arrays
A Subspace-Tacking Approach to
Introduction
Conversely, eigenstructure-based techniques offer some advantages:

- M/VA is not especially good on any of these.

Until post-processing (e.g., via {
\varepsilon} 
\text{scal}) post-processing (e.g., via {
\varepsilon} 
\text{scal})

Independent of array calibration, because useful calibration may not be available leave noise alone (\text{This is where the signals are})

- Form that "sector" nulls to deal with fast-moving interferences protect main beam from do not allow for dealing with interferences that are undetectable in short timeframes (\text{INR} \gg 1)
- Deal with interferences that are undetectable in short timeframes (\text{INR} \gg 1) deal with interferences that are undetectable in short timeframes (\text{INR} \gg 1)
- Ability to add, delete, or modify nulls – necessary because we desire to:

\text{Interference Suppression} \times \frac{\text{SNR}}{\text{INR}} \text{ or better, as opposed to } \frac{\text{SNR}}{\text{INR}}

\text{Interference Suppression Algorithm for Radio Astronomy}

Desirable Features of a Real-Time Spatial...
the „rank“ $\eta >> N$

This is powerful because for large arrays we usually find that linear combination of the $\eta$ columns of $U$, e.g., the interference subspace $U^n$ (columns of $U$ associated with $\eta$ noise eigenvectors), $U^n + U^n U^n$.

Subspace Partition: $R = U^n U^n + U^n U^n$

Eigenvalues associated with noise should be weakest and nearly equal — In short integration times, noise should be without structure — Sum of the eigenvalues = total power captured by array

The eigenvalues are simply power estimates:

$U$: diagonal matrix of eigenvalues associated with columns of $U$

$U$: matrix of eigenvectors of $R$, stacked in columns

$U$: Eigenstructure: $R = U^n U^n + U^n U^n$

Coercance matrix: $R = U^n U^n + U^n U^n$

Eigenstructure-Based Strategy
nulling is good for all beams

Extension to multi-beam systems is straightforward

\[ \varphi_H(x_H) = w \]

• Or we can just redefine the beamformer as \( \mathbf{d} \), where

\[ \mathbf{d}^T \mathbf{p} \]

projects out "interference subspace estimate \( \mathbf{H} \), that's nulling!

\[ \begin{pmatrix} N \times N \end{pmatrix} \mathbf{H}^T \mathbf{H} - \mathbf{I} = \mathbf{d}^T \mathbf{d} \]

\( \mathbf{d} \) is the vector of weights for desired beam ("steering vector")

Consider beamformer \( \mathbf{d} \) before beamforming: \( \mathbf{x}_t \)

Nulling Using Interference Subspace Estimate
Dramatically more efficient than MV for large-\( N \) arrays

- Computational cost \( \propto N^3 \), where \( I \) can be \( \gg I \)

(allation)

- Allows selection of maximum interference subspace rank \( k_{\text{max}} \) to consider

- Not necessary to compute \( R \)

- Allows sample-by-sample iterative eigensubspace estimate

- Use projection-approximation subspace tracking (PAST) technique (Yang, 1995)

Workaround:

- Rank \( k \) of interference subspace tricky to select

- Direct eigendecomposition of \( R \) is typically \( \propto N^3 \) FLOPS

- (BTW, this typically dominates cost of MV)

- \( R \) is expensive; compute: FLOPS \( \propto N^2 T \) (\( T \)=number of samples)

Computationally Efficient Subspace Estimation
Subspace-Tracking Spatial Projections (Single Beam)
Double- and half-wavelength spacing, respectively. Equivalent to 2 rows of 8 elements with 64 elements reduced to \( N = 16 \) instrumented outputs by analog beamforming.

NRAOS OSMA system
Subspace Identification Verification

- Interference subspace correctly identified within 60 samples ($\approx 64s$)
- $\text{PASD}$ with $K_{\text{max}} = 5$
- Single signal incident with $INR = +32$ dB
- Measurement in NFRA's anechoic chamber
Interference subspace correctly identified within 300 samples (384s)

\[ \text{PASTD with } k_{\text{max}} = 5 \]

Simulation of same scenario with INR=0 dB

INR=1 Simulation
• Solid line – STSP, dashed line – MV

• Same scenario, INR=0 dB, signal incident from 20°
especially issue of detection for \textit{INF} the

interference subspace rank estimation is a problem that needs more attention.

$N \log_2 N$

$N$ beams with good nulling and computational complexity only a little greater than

OSU/NEPA effort underway to develop FFT-based extension of STSF that yields

for time-domain cancelling

Subspace tracking also shows promise as technique for obtaining reference signals

systems with large numbers of broadbeam elements

STSF shows strong potential for use as an FRI preprocessor and calibration aid for

array pattern (for example) – toxicity is under user control

in STSF, user decides tradeoff between algorithm aggressiveness and stability of

known (exploit the interference)

Thus, STSF can also be used as a calibration aid if positions of interferers are

Note that array calibration is not required to null interference using STSF

Closing Remarks