Dispersion in Coaxial Cables

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1 Summary

At the LWA "Pre-SRR and Kickoff Meeting" in Albuquerque in September 2008, TAC Member Jack Welch suggested that we consider the possibility that coaxial feedlines would introduce significant dispersion in our frequency range. He provided a memo written in 1990 describing analysis and measurement of dispersion on 1/2-in coax at frequencies in the range 90 MHz to about 1 GHz. I have generalized his analysis and applied it to the special case of RG58-type cables in our frequency range. I find that the excess delay due to dispersion is

$$\tau_d = (4.78 \text{ ns}) \left(\frac{l}{100 \text{ m}}\right) \left(\frac{f}{10 \text{ MHz}}\right)^{-1/2} \tag{1}$$

This is significant; for comparison consider that the free-space light travel time over 4 m (roughly the element spacing in the station array) is about 13 ns and frequency independent. The dispersion can be improved slightly (but probably only at great expense) by using coaxial cables having larger cross-section with inner and outer conductors having higher conductivity. This dispersion will probably need to be taken into account in calibration and observations of any reasonable bandwidth. Formulas are provided for computing the dispersion delay and attenuation of any coaxial cable given the relevant material and geometrical parameters.

2 Theory

In this section I will use essentially the same general approach as Jack but with a different starting point and retaining all the relevant parameters so that results can be generated for a variety of cable types. A reference which contains all the formulas and constants used here is [1] (my personal favorite), but the same information can be found in various textbooks on engineering electromagnetics.

The field inside any transmission line propagates according to the factor $e^{\gamma z}$ where γ is the complex-valued "propagation constant" and z is distance along the line. The propagation constant is given by

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{2}$$

where R, L, G, and C are the series resistance, series inductance, shunt conductance, and shunt capacitance, respectively; $\omega = 2\pi f$ where f is frequency; and $j = \sqrt{-1}$. The shunt conductance describes the leakage of current through the medium separating the inner and outer conductors and is negligible for well-designed transmission lines. For example, for RG59 (which is similar to RG58 and for which I just happen to have these parameters worked out from a past project) C is about 100 pF/m and G is about 200 μS/m, so at 10 MHz the capacitive reactance is $\omega C \approx 6000$ μS/m; i.e., already more than 30 times greater than G and increasing with frequency. Thus we approximate:

$$\gamma \approx \sqrt{(R + j\omega L) j\omega C} = \sqrt{-\omega^2 LC + j\omega RC} \tag{3}$$

At this point is useful to identify and make explicit the frequency dependence. C is given by

$$C = \frac{2\pi\epsilon}{\ln(b/a)} \tag{4}$$

where ϵ is the permittivity of the medium between the inner and outer conductors of the coax, and a and b are the radii of the inner conductor and the facing surface of the outer conductor, respectively. $\epsilon = \epsilon_r \epsilon_0$ where $\epsilon_0 = 8.854 \times 10^{-12}$ F/m and ϵ_r is the relative permittivity, which is a property of the material which is essentially frequency-independent over the range of frequencies of interest. Thus, C is frequency independent. L accounts for two sources of inductance. The ideal inductance is that associated with the magnetic component of the propagating field between the conductors and is given by

$$L_0 = \frac{\mu}{2\pi} \ln \frac{b}{a} \tag{5}$$

where $\mu = 4\pi \times 10^{-7}$ H/m is the permeability of the medium between the inner and outer conductors of the coax; essentially the same as that of free space for any sane choice of spacing material. Note that L_0, like C, is frequency-independent. The other source of series inductance is that associated with the magnetic component of the propagating fields *interior* to the inner and outer conductors, which is possible because they are not perfectly conductive. This inductance is for the inner and outer conductors respectively:

$$L_a = \frac{\mu \delta_a}{4\pi a} \quad \text{and} \tag{6}$$

$$L_b = \frac{\mu \delta_b}{4\pi b} \tag{7}$$

where δ_a and δ_b are the skin depths for the inner and outer conductors respectively. Skin depth is given by

$$\delta = (\pi \mu \sigma f)^{-1/2} \tag{8}$$

where σ is conductivity. It should be noted that Equations 6 and 7 are valid only for the case that $\delta \ll$ material thickness; however this is the case in the present problem over the range of frequencies of interest. The total series inductance L is the sum of L_0, L_a, and L_b. After substitutions and some algebra, we obtain:

$$L = L_0 + L_{s0} f^{-1/2} \text{ , where} \tag{9}$$

$$L_{s0} = \frac{\mu^{1/2}}{4\pi^{3/2}} \left(\frac{\sigma_a^{-1/2}}{a} + \frac{\sigma_b^{-1/2}}{b} \right) \tag{10}$$

The resistance R arises from the exact same current associated with L_a and L_b. For good conductors it is known that real and imaginary parts of wave impedance are equal; thus $R = \omega(L_a + L_b)$ which can be written:

$$R = 2\pi L_{s0} f^{1/2} \tag{11}$$

It is noted that any frequency dependence arises from non-zero L_{s0}, which manifests as non-zero R and frequency-dependent L.

After substituting Equations 9 and 11 into Equation 3 and some algebra, we find:

$$\gamma = j\beta_0 \sqrt{1 + (1-j)\frac{L_{s0}}{L_0} f^{-1/2}} \text{ , where} \tag{12}$$

$$\beta_0 = \omega \sqrt{L_0 C} \tag{13}$$

Note that β_0 is the wavenumber for an ideal transmission line; that is, one with perfectly-conducting materials such that $R = L_{s0} = 0$ and thus is frequency-independent. Note that

$$\frac{L_{s0}}{L_0} = \frac{1}{2\sqrt{\pi\mu}} \left(\frac{\sigma_a^{-1/2}}{a} + \frac{\sigma_b^{-1/2}}{b} \right) \left(\ln \frac{b}{a} \right)^{-1} \tag{14}$$

The second term under the radical in Equation 12 is small compared to 1. For example, using the parameters determined for RG58 in the next section, we find that for $f = 10$ MHz:

$$\frac{L_{s0}}{L_0} f^{-1/2} = 0.04 \quad \text{(Estimated for RG58 at 10 MHz)} \tag{15}$$

Thus it is reasonable to apply the "small x" approximation $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ to Equation 12. We obtain:

$$\gamma = \beta_0 \frac{1}{2} \frac{L_{s0}}{L_0} f^{-1/2} + j\beta_0 \left(1 + \frac{1}{2} \frac{L_{s0}}{L_0} f^{-1/2} \right) \tag{16}$$

Recall that the fields propagate along the transmission line according to the factor $e^{\gamma z}$. Separating γ into real and imaginary parts $\gamma = \alpha + j\beta$, we have $e^{\alpha x} e^{j\beta x}$ where the first and second factors describe attenuation and phase, respectively, along the line. In our case we have

$$\beta = \text{Im}\{\gamma\} = \beta_0 \left(1 + \frac{1}{2}\frac{L_{s0}}{L_0} f^{-1/2}\right) \tag{17}$$

The phase ϕ at x assuming $\phi = 0$ at $x = 0$ is $\phi = \beta x$. Let $x = ct_0$, where c is the speed of light in free space. We then have:

$$\phi = \beta_0 c t_0 \left(1 + \frac{1}{2}\frac{L_{s0}}{L_0} f^{-1/2}\right) = 2\pi t_0 \left(f + \frac{1}{2}\frac{L_{s0}}{L_0} f^{1/2}\right) \tag{18}$$

since $\beta_0 c = 2\pi f$. The associated delay τ is given by:

$$\tau = \frac{1}{2\pi}\frac{d\phi}{df} = t_0 \left(1 + \frac{1}{4}\frac{L_{s0}}{L_0} f^{-1/2}\right) \tag{19}$$

The excess delay due to dispersion is therefore

$$\tau_d = t_0 \frac{1}{4}\frac{L_{s0}}{L_0} f^{-1/2} \tag{20}$$

or, written in terms of the constitutive parameters:

$$\tau_d = t_0 \, \frac{1}{8\sqrt{\pi\mu}} \left(\frac{\sigma_a^{-1/2}}{a} + \frac{\sigma_b^{-1/2}}{b}\right) \left(\ln\frac{b}{a}\right)^{-1} f^{-1/2} \tag{21}$$

This is the primary result of this analysis. However, it is also possible to obtain the attenuation constant α, which is useful for checking the result. One finds:

$$\alpha = \text{Re}\{\gamma\} = \beta_0 \frac{1}{2}\frac{L_{s0}}{L_0} f^{-1/2} \tag{22}$$

After substitutions and some algebra, we obtain:

$$\alpha = \sqrt{\frac{\pi\epsilon}{4}} \left(\frac{\sigma_a^{-1/2}}{a} + \frac{\sigma_b^{-1/2}}{b}\right) \left(\ln\frac{b}{a}\right)^{-1} f^{-1/2} \tag{23}$$

The attenuation is then given by

$$A = -20 \log_{10} e^{\alpha} \quad [\text{dB/m}] \tag{24}$$

assuming α is expressed in m^{-1}.

3 Comparison to Welch's Result

In his memo, Jack describes Cablewave 1/2-in coax having a copper inner conductor with $2a = 0.168$ in, an aluminum outer conductor having $2b = 0.480$ in, and a spacing material with $\epsilon_r = 1.508$ (He gives the index of refraction n as 1.228, and $n = \sqrt{\epsilon_r}$). I used values of $\sigma_a = 5.8 \times 10^7$ S/m and $\sigma_b = 3.82 \times 10^7$ S/m from [1]. From Equations 23 and 24 I obtain 0.047 dB/m, whereas Jack obtains 0.050 dB/m; probably OK given that we may have used slightly different conductivities (Jack used skin impedance as opposed to conductivity). For τ_d I obtain $5.28 t_0 f^{-1/2}$ whereas Jack obtains $7.4 t_0 f^{-1/2}$. Jack's result includes an empirical correction based on measurements of the coax which increased the value of his constant; I believe this accounts for most of the difference between his result and mine.

4 Findings for RG58 at LWA Frequencies

To the best of my ability to determine, RG58 has the following characteristics:

- Inner conductor (twisted tinned copper strands): $a = 0.42$ mm, $\sigma_a = 2.9 \times 10^7$ S/m
- Outer conductor (braided tinned copper strands): $b = 1.75$ mm, $\sigma_b = 5.8 \times 10^6$ S/m
- Spacing material (polyethylene): $\epsilon_r = 2.26$.

The conductivities σ_a and σ_b are a bit tricky to determine, because the inner and outer conductors are not simple homogeneous media. I used the following procedure: First, I assumed $\sigma_a = \sigma_b = 5.8 \times 10^6$ S/m, which is the value given for copper in [1]. From this, I obtained $A = 0.093$ dB/m at 100 MHz. The correct value should be more like 0.164 dB/m (i.e., 5 dB/(100 ft)). I then guessed that the use of braid in the outer conductor reduces the conductivity by an order of magnitude, and that the use of strands in the inner conductor reduces the conductivity by half, giving the values of listed above. Using these new values I obtain very nearly the desired value of 0.164 dB/m, and I therefore assume these to be appropriate.

For τ_d I obtain $(29.95 \text{ Hz}^{1/2}) t_0 f^{-1/2}$. This can be translated into a more meaningful expression by letting t_0 be $l/(0.66c)$, the travel time along a dispersion-free cable of length l, where c is the speed of light in free space and the factor of 0.66 is the well-known "velocity factor" for RG58. After substitutions and some math, we obtain:

$$\tau_d = (4.78 \text{ ns}) \left(\frac{l}{100 \text{ m}} \right) \left(\frac{f}{10 \text{ MHz}} \right)^{-1/2} . \qquad (25)$$

References

[1] W.H. Hayt, Jr., *Engineering Electromagnetics*, 4th Ed., McGraw-Hill, 1981.