A Design Study Comparing LWA Station Arrays Consisting of Thin Inverted-V Dipoles

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1 Introduction

This memo describes the design and evaluation of two LWA station arrays. In both cases thin inverted-V (a.k.a. "droopy") dipoles are used as the array elements. The primary difference between the arrays is that one is a "sparse" array with "pseudorandom" spacings; whereas the other is a "compact" array with uniform spacings.

In the sparse array design, the first step is to optimize the elements. Then, 256 stands (each consisting of one pair of orthogonally-polarized elements) are situated in a pseudorandom geometry which fills a circular station footprint having a diameter of 100 m [cite memo]. In this approach, it is assumed that the coupling between elements does not significantly change the array performance, or can mitigated by some method yet to be determined.

In the compact array design, the elements are packed relatively closely together, with uniform spacing. The elements and spacings are optimized *in situ*; i.e., including mutual coupling as part of the design process. As will be shown in this memo, this leads to a design consisting of a relatively large number of elements which are relatively small. Despite this, it turns out that to achieve the current (unofficial) LWA station A_e/T_{sys} performance requirements, about the same number of elements are required in either design, and thus both designs seem to be on roughly equal footing in terms of raw performance. However, I will make the case that the compact array approach exhibits significant advantages over the sparse array approach in terms of sidelobe levels and calibratability.

To make this study tractable – in particular, to make it possible to perform the necessary calculations in a reasonable amount of time – several simplifications have been made:

- The elements are assumed to be simple wire inverted-V dipoles, as opposed to some other element that exhibits larger impedance bandwidth in isolation, such as blades [1, 2]. Whereas such elements may enable better overall performance, it is my belief that the general trends identified in this study will probably apply to arrays built from any other element design.
- Only the co-polarized elements from each stand are considered; the cross-polarized elements are neglected. The justification for this is that is well-known that the effect of the cross-polarized elements is insignificant, and thus there is no advantage in including them, which would dramatically increase the time required to make the calculations.
- Only the co-polarized fields in the E-plane of the elements (i.e., the plane in which the element lies) will be considered. The reason for this is again to limit the amount of calculation that must be done, and because one expects that the E-plane performance to be worse than the performance in any other plane [3].
- All calculations assume the ground is perfectly conducting. One reason for this is that there is some indication that this would be a good thing in terms of array performance [3]. A second reason is that this simplifies the analysis (and thereby probably results in more accurate answers) by eliminating the additional complexity of dealing with array performance in the presence of a lossy dielectric material with uncertain characteristics.

2 Design of the Sparse Array

All elements in this study are assumed to be constructed from perfectly conducting wires of circular cross-section having radius $a = 9.5$ mm. This is intended to model braided ground strap material having width $w = 1.5$ in, which has been proposed as a convenient material from which to construct elements [4]. The value of a is determined according to the relationship $w = 4a$ suggested in the antenna textbook by Stutzman & Thiele [5].

Parameter	Symbol	Sparse Array	Compact Array
Length of radiator (both arms)	B	4.00 m	3.54 m
Height above ground	h	2.00 m	1.77 m
Element spacing		pseudorandom,	uniform rectilinear,
		4.00 m min spacing	3.61 m × 3.61 m
Radius of radiator	a	9.50 mm	9.50 mm
Droop angle	θ_b	45°	45°
Ground material		perfect conductor	perfect conductor

Table 1: Summary of design parameters for the sparse and compact arrays.

The remaining element design parameters include the total length of the element (including both arms), B; the height of the feedpoint above ground, h; and the angle at which the arms droop from the horizontal plane, θ_b. As another simplification, we shall simply assume that $\theta_b = 45°$ is a reasonable value.

To determine the remaining parameters, B and h, the following approach is used. It is well known that the antenna temperature seen by a single low-gain element is approximately

$$(2000 \text{ K})(74/\nu_{\text{MHz}})^{2.6} \ , \tag{1}$$

where ν_{MHz} is frequency in MHz. Assuming the instrument is also Galactic noise-dominated (assumed throughout this memo), this is a reasonable approximation of the system temperature T_{sys}. It is also known that the collecting area A_e of an element is proportional to the power delivered to the load attached to the element, which is equal to

$$\frac{1}{2}|I_L|^2 \text{Re}\{Z_L\} \tag{2}$$

where I_L is the current through the load impedance Z_L. Thus, if we wish for the element to exhibit constant sensitivity over some frequency range, we should require A_e/T_{sys} to be a constant over that range. This is equivalent to requiring that the square of the ratio of $|I_L|$ measured at the low end of the frequency range ν_L, to $|I_L|$ measured at the high end of the frequency range ν_H, be equal to $(\nu_H/\nu_L)^{2.6}$. The current (unofficial) specification for the frequency range of LWA is 20 MHz to 80 MHz; however, it is probably not reasonable to require that the performance be uniformly good over this range. Instead, $\nu_L = 38$ MHz and $\nu_H = 74$ MHz are chosen, resulting in $(\nu_H/\nu_L)^{2.6} = 5.66$.

An additional consideration that can be used to constrain the design of the element is to require $B = 2h$. The justification for this is that $B = 2h = \lambda/2$ (where λ is wavelength) is a well-known design for a straight dipole over perfectly conducting ground. Thus, we are down to 1 parameter to optimize, B. B was determined by trial and error using a NEC model of a single element over a ground plane, illuminated by a plane wave incident from $\theta = 74°$, where θ is the angle measured from the zenith. This value of θ was chosen because it represents the lowest elevation at which the current (unofficial) LWA station performance requirements are expected to apply [6]. It was found that $B = 4$ m results in the squared ratio of load currents being about 5.22, which was deemed close enough to 5.66. Thus, $h = 2$ m.

The layout used for the sparse array is a pseudorandom design with minimum allowed spacing of 4 m, provided by Aaron Cohen (NRL), shown in Figure 1. A summary of the design parameters is given in Table 1.

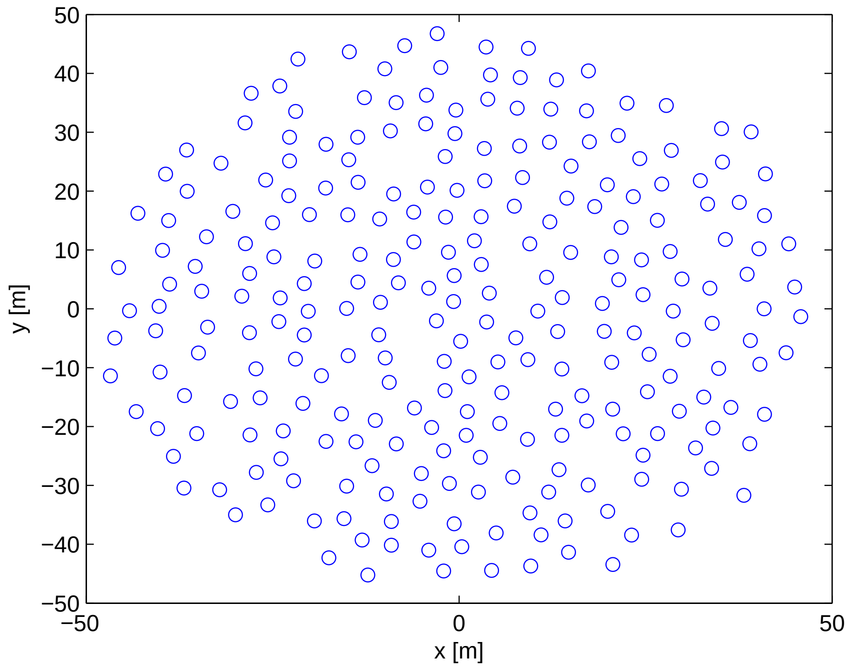


Figure 1: Sparse array geometry. 256 stands, consisting of 256 co-polarized dipoles and no cross-polarized dipoles.

3 Design of the Compact Array

The starting point for the design of the compact array was a Ph.D. dissertation by Lin [7]. Lin studied the scan impedance of infinite arrays of uniformly-spaced singly-polarized inverted-V dipoles to determine the effect of the element design parameters B, θ_b, h, and a; as well as array design parameters D_x and D_y, the spacings between elements in those two orthogonal directions parallel to the ground. His work assumed a perfectly conducting ground but considered much smaller, higher frequency arrays suitable for operation between 4 GHz and 7.5 GHz. In his work, he shows how the scan impedance evolves as a function of frequency and θ in the principal planes, for many trial values of the design parameters. However, the work is not directly relevant because he always assumes very thick elements ($\sim$ 10 cm radius when scaled to our frequency range of interest), does not consider the effect of truncation (i.e., the fact that the array is finite), and never considers the actual power transfer to loads, only the scan impedance to which one would attempt to match.

Nevertheless, for a starting point, I chose one of Lin's designs that seemed likley to produce a good match to a $100+j0\ \Omega$ load, scaled this to a frequency in the middle of the LWA range, and then set $\theta_b = 45°$ and replaced the radiators with the same $a = 9.5$ mm radius used for the sparse array elements. From that point, I experimented by fixing the ratio of B to h, D_x, and D_y and scaling the resulting single parameter up and down, keeping all other parameters fixed. As this parameter is changed, more or fewer elements are required to fill a circular station footprint of 100 m diameter. The goal of the optimization was the similar to that used in the design of the single element for the sparse array: I illuminated a trial array with a co-polarized plane wave incident from $\theta_s = 74°$ in the E-plane and computed the effective aperture for the optimum station beam using the same procedure described in [8] at two frequencies, 38 MHz and 74 MHz. I then sought the value of the design parameter which resulted in A_e/T_{sys} being equal at these frequencies. The resulting design had $B = 3.54$ m, $h = 1.81$ m, and $D_x = D_y = 3.61$ m, and required 593 stands to fill the 100 m diameter circular station footprint, as shown in Figure 2. A summary of the design parameters is

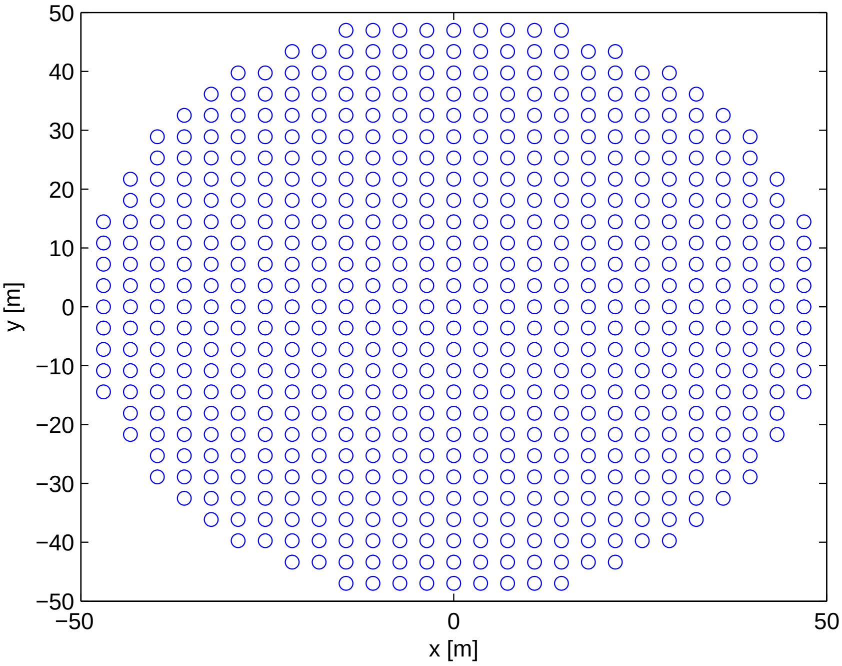


Figure 2: Compact array geomety. 593 stands, consisting of 593 co-polarized dipoles and no cross-polarized dipoles.

given in Table 1.

4 Results

Both the sparse and the compact array were evaluated using the procedure described in [8], in which we illuminate the station with an incident plane wave, compute the terminal currents using NEC, from this compute the effective aperture per element, and then add these up to compute the effective aperture A_e of the optimum-gain station beam. Because we are dealing with a range of frequencies, it is more convenient to represent the results in terms of A_e/T_{sys}, since then the results are proportional to sensitivity. To do this, I used the expression for T_{sys} given in Section 2. This is not exactly appropriate since T_{sys} may vary significantly depending on where in the sky the beam is pointed; however, since it has the right frequency dependence it nevertheless provides a useful basis for comparison. A further caveat is that the actual sensitivity may be confusion-limited, either because the beam is too wide or the sidelobes are too large. Again, the value in the A_e/T_{sys} metric is primarily as a basis for comparison as opposed to a prediction of achievable sensitivity.

The results are summarized in Table 2. It is quite apparent that the cost (as determined by the number of stands and combined length of radiators) of the compact array would be about twice that of the sparse array, but also that the performance is considerably better as well. At this point, it is useful to consider what A_e/T_{sys} is actually required. By reverse-engineering some arguments made in LWA Memos 52 [9] and 70 [6], I come up with a value of ~ 0.32 m^2/K, which varies only very slowly with frequency. Using this criterion, we see that both designs fall quite short, especially for low elevation pointings.

	Sparse	Compact	ratio
Diameter of station	100 m	100 m	1.0
Number of stands	256	593	2.3
Combined length of radiators	1024 m	2099 m	2.1
A_e/T_{sys} for $\theta = 0°$, 20 MHz	0.00335 m^2/K	0.00381 m^2/K	1.1
A_e/T_{sys} for $\theta = 0°$, 38 MHz	0.47 m^2/K	0.64 m^2/K	1.4
A_e/T_{sys} for $\theta = 0°$, 74 MHz	0.11 m^2/K	0.31 m^2/K	2.8
A_e/T_{sys} for $\theta = 74°$, 20 MHz	0.00050 m^2/K	0.00052 m^2/K	1.0
A_e/T_{sys} for $\theta = 74°$, 38 MHz	0.08 m^2/K	0.17 m^2/K	2.1
A_e/T_{sys} for $\theta = 74°$, 74 MHz	0.10 m^2/K	0.18 m^2/K	1.8

Table 2: Summary and comparison of the results for the sparse and compact arrays.

	Sparse	Compact	ratio
Diameter of station	200 m	140 m	0.7
Number of stands	1024	1127	1.1
Combined length of radiators	4096 m	3988 m	1.0
A_e/T_{sys} for $\theta = 0°$, 20 MHz	0.01340 m^2/K	0.00724 m^2/K	0.5
A_e/T_{sys} for $\theta = 0°$, 38 MHz	1.88 m^2/K	1.22 m^2/K	0.6
A_e/T_{sys} for $\theta = 0°$, 74 MHz	0.44 m^2/K	0.59 m^2/K	1.3
A_e/T_{sys} for $\theta = 74°$, 20 MHz	0.00200 m^2/K	0.00099 m^2/K	0.5
A_e/T_{sys} for $\theta = 74°$, 38 MHz	0.32 m^2/K	0.32 m^2/K	1.0
A_e/T_{sys} for $\theta = 74°$, 74 MHz	0.40 m^2/K	0.34 m^2/K	0.8

Table 3: Summary and comparison of the results for the *revised* sparse and compact arrays, scaled up to achieve minimum $A_e/T_{sys} = 0.32$ m^2/K for frequencies ≥ 38 MHz .

At this point let us consider how much bigger these arrays would have to be in order to meet the requirement suggested above. To avoid repeating all the calculations, one can obtain the desired result in an approximate way simply by scaling the diameter of the station footprint upward and assuming all tabulated values of A_e/T_{sys} scale in proportion to the area of the station. Requiring a minimum $A_e/T_{sys} = 0.32$ m^2/K for frequencies ≥ 38 MHz we obtain the new designs shown in Table 3. It is remarkable that these new designs contain almost the same number of elements. However, now the compact array seems less attractive in terms of A_e/T_{sys} except in a few cases.

5 Discussion

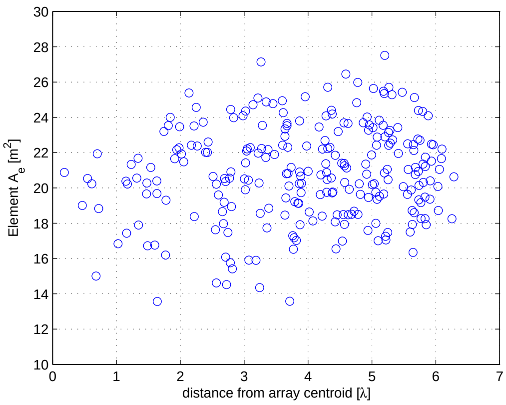
These results have many interesting implications, and in this section I shall attempt to identify and address these.

No doubt the data in Table 3 are apoplectic to those among us who must deal with cost and funding, as it seems to suggest that the cost of a station could be about four times greater than the expected value previous to this study. However, the actual situation may not be so bad. First, we have restricted our attention to the inverted-V with $a = 9.5$ mm and $\theta_b = 45°$, so there has been little attempt in either design to optimize the element for broadband operation at low elevation angles. Further modifications to the element design may significantly improve the situation. Second, it is not for certain that the 0.32 m^2/K requirement is the appropriate value, or that it should necessarily apply down to $\theta = 74°$ over the entire tuning range. Because dipoles patterns tend to roll off as something like $\cos^3\theta$ in the E-plane, I suspect that relaxing this (as yet unofficial) requirement would result in a dramatic reduction in the required number of elements. Third, we could consider an analog beamforming stage to combine, for example, groups of four nearby co-polarized elements into one signal. This would reduce the number of receivers to approximately the originally-conceived value of 256. It would also increase the per-stand cost due to the need for 4-to-1 analog beamforming hardware; however this additional expense in fact could be almost insignificant as it could be a relatively simple and inexpensive addition to the functionality of the analog receivers. The main disadvantage of analog beamforming is probably that it restricts field of view, such that the pointing of beams formed digitally is restricted to the beamwidth of the beam formed by the analog beamformer. Here, the compact array has a significant advantage, as any four adjacent dipoles form a uniform footprint 7.22 m across, which translates to a worst-case (narrowest) beamwidth of about 32° for zenith pointing; whereas the sparse array produces an irregular footprint that is often much larger, such that the resulting beams are both non-uniform and narrower on average.

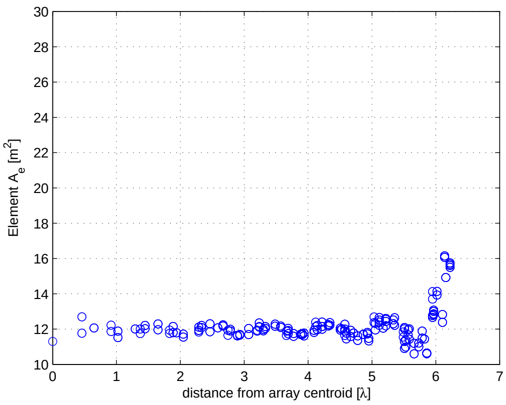
A second issue is the awful performance at 20 MHz. The design procedures described in Sections 2 and 3 are mostly to blame for this, since neither included 20 MHz performance as a consideration. However, doing so would almost certainly result in a deleterious effect on 74 MHz performance. Future work should of course attempt to strike a balance between the performance goals at the low and high extremes, which will result in different designs. However, I believe the general trends – in particular, the relative characteristics of sparse vs. compact arrays – are likely to remain the same. 20 MHz performance could also be significantly impacted by modifications in element design. In particular, it is noted that the wire radius $a = 9.5$ mm used in this study is much less than that used in the designs described by Lin, which seem to achieve somewhat larger scan impedance bandwidth. It could also be that the use of the fork or blade antennas significantly improves the results in this respect. Another approach could be to employ two types of elements; i.e., reduce the number of stands in the arrays described here and add back stands around the periphery that use larger elements better suited to 20 MHz operation.

With respect to the choice between sparse and compact designs, one might be tempted to conclude from Table 3 that the sparse array is a better choice because, for the same cost, it yields a larger station footprint (allowing a narrower beam when desired) and produces significantly higher A_e/T_{sys} in many cases. However, this is not the whole story. First, one should consider the sidelobe levels that are likely to result in either case. The spacings in the compact array are 3.61 m, and thus the aperture is oversampled with no possibility of aliasing for frequencies below about 80 MHz. Since spacings for the sparse array are considerably larger and non-uniform, we can expect aliasing in form of higher sidelobe levels (not true aliases since the spacings are non-uniform), especially at higher frequencies in the 20–80 MHz range of interest. Further study is required to determine quantitatively the difference in sidelobe performance for the two arrays. The second reason the sparse array in Table 3 may not be as attractive as it first appears has to do with calibratability. To demonstrate the point, consider Figures 3 and 4. Note that the magnitudes and phases of the terminal currents for the compact array are extraordinarily uniform, whereas the magnitudes and currents of the terminal currents for the sparse array exhibit considerable "jitter". It is important to note that the actual values for the sparse array also jitter, effectively independently, as a function of frequency and angle of incidence, and that *this is true independently of how the station beam is pointed*. The implications for calibration of the station array are profound. For the compact array, we can ignore the jitter to get first-order estimates of the terminal currents and phases to accuracies on the order of degrees. For the sparse array, the same first-order estimates are only accurate to within 10 degrees or so, change wildly from element to element, and (worst of all!) change as a function of direction of incidence with a different dependence from element to element. In fact, this behavior is pretty strong indication that the sidelobe structure of the beam will also be particularly obnoxious.

For similar reasons, the compact array will be much easier to design than the sparse array. The reason is that for the compact array, any optimization that yields good per-element performance for any element near the center will lead to globally-good performance, with degradation only near the edges. For the sparse array, an optimization that yields good performance for any one element

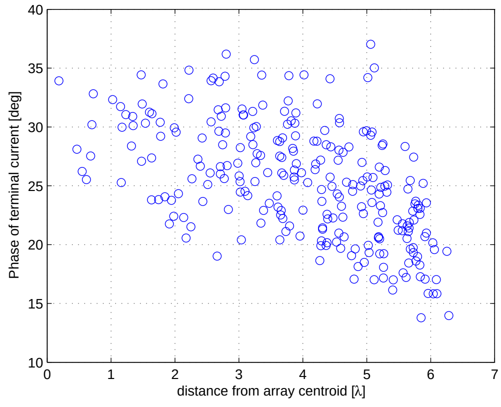


(a) sparse array

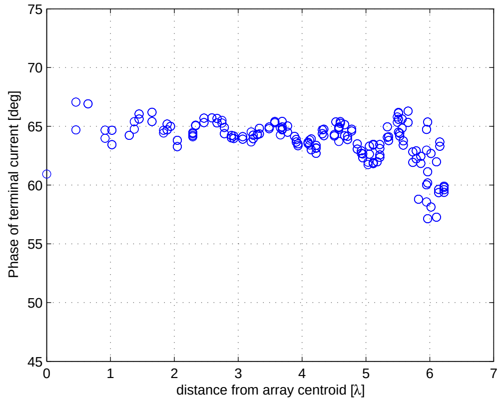


(b) compact array

Figure 3: Comparison of A_e per element for the two (original 100 m) array designs, shown as a scatter plot. This is for plane wave illumination from the zenith at 38 MHz. Note that the per-element A_e is proportional to the square of the magnitude of the terminal current (in fact, this is how it is calculated), so this can also be interpreted as a scatter plot of squared terminal currents.



(a) sparse array



(b) compact array

Figure 4: Comparison of the phase of the terminal currents for the two (original 100 m) array designs, shown as a scatter plot. This is for plane wave illumination from the zenith at 38 MHz.

may result in better, worse, or the same performance from surrounding elements. In other words, optimization of the sparse array requires optimization of the performance of many elements simultaneously. In fact, one way to look at this is to imagine that *every* element in the sparse array is an "edge element"!

6 Conclusions

Within the constraints imposed on this study, it is my opinion that the 140 m-diameter compact array described in Tables 1 and 3 are probably the best fit assuming the unofficial LWA system requirements stated in previous sections of this memo. This array would be considerably more expensive than has previously been assumed; however there are a number of reasons why, given information presented here, we might want to consider relaxing some requirements that lead to a larger number of elements here, in particular that for station beam A_e/T_{sys} for low elevation pointings. Relaxing those specifications can be expected to have a dramatic impact on the number of stands required. It may also be possible to make some additional headway using other element designs and repeating this work with greater effort on optimization.

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