Collecting Area of Planar Arrays of Thin Straight Dipoles

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1 Introduction

An important problem looming in LWA development is how to accurately determine the collecting area – referred to here as the effective aperture A_e – of an LWA station. In the more familiar case of dishes, A_e is readily estimated as the product of the physical aperture of the dish and various factors which account for the efficiency of the feed and so on. Useful expressions exist also for simple antennas, such as the vanishingly-thin straight half-wave dipole. In contrast, an LWA station as currently conceived consists of relatively complex antenna elements which are electromagnetically coupled in a complex way. As a result, A_e for the station beam is difficult to calculate, and it varies in a complex way as a function of pointing direction and frequency.

In the past, simple approximations have been used to estimate A_e for a station beam; specifically, treating the elements as ideal half-wave dipoles isolated in free space, making adjustments to account for the presence of the ground, and multiplying by the number of dipoles. If done with care, and assuming that the electromagnetic coupling is not too large a factor, then this probably puts us in the right ballpark. Going forward, however, this is not a satisfactory state of affairs. We need to design stations such that we have confidence that the resulting collecting area meets science requirements – specifically, calibratibility requirements [1] – *but no more.* This is so because the cost of a station scales roughly as the number of antenna elements, so excess antennas beyond that needed for system calibration increase station cost without a proportional increase in scientific value. Thus, we wish to improve the accuracy of our estimates of station beam A_e.

The Numerical Electromagnetics Code (NEC) is a compelling tool in this case. NEC version 2 (NEC-2) is a commonly used open-source and freely-available software implementation of the "moment method" technique for analysis of antennas and electromagnetic scattering. NEC-2 is particularly well-suited for modeling antennas which consist of, or which can be modeled as consisting of, wires. We already know NEC-2 is suitable for modeling "LWA type" antennas such as inverted V-shaped dipoles [2] and blades [3]. While NEC-2 is quite simple to use, it notoriously tricky to correctly model problems, and similarly tricky to correctly interpret its output. In a previous report [4], I described a simple example of the use of NEC-2 to calculate the collecting area of a thin straight half-wave dipole, for which an independent check of results is available from well-known theoretical analysis [5]. It was found that the proposed "direct" measurement of the collecting area yielded results within 6% of the theoretical value, whereas the traditional "indirect" method, which relies on a reciprocity argument, yielded results within 1% of the theoretical value. In this report, this work is extended to arrays of such dipoles in various geometries and over various types of ground. In these examples, the indirect method is difficult to apply specifically because it requires the introduction of a very large number of voltage sources, which seems to create problems in my NEC setup. The direct method, in contrast, requires only one voltage source and seems to run very smoothly even for very large numbers of segments; and, in addition, bypasses potential difficulties in the proper application of reciprocity arguments and interpretation of the results.

The scope of this report is limited to thin straight half-wave dipoles, as opposed to any antenna type currently being investigated as a candidate for LWA. The reason for this is that thin straight dipoles are simple to model, lead to relatively rapid computations, and offer better opportunities for comparison to theoretical results and "common sense" verification. Subsequently, this allows some improved insight into the behavior of dipole arrays generally. Such insights may be useful in the design and analysis of arrays consisting of LWA candidate elements. I think the results also have some implications for the design of the elements themselves.

2 Problem Statement

We wish to determine the collecting area of planar arrays of thin straight half-wave dipoles. For simplicity, we consider only one frequency $f = 38$ MHz, for which the free space wavelength $\lambda = 7.895$ m. The dipoles are exactly $\lambda/2$ long. Each dipole is constructed from perfectly-conducting material of circular cross section having a radius of 0.05 mm. The coordinate system is such that the dipoles lie in the $z = \lambda/4$ plane and the surface of the ground lies in the $z = 0$ plane.

A_e is defined as the ratio of P_r, the power successfully received by the array, to the incident power density S^i, typically having units of W/m^2. Here we are interested in the *optimum* A_e, which is achieved when the power collected individually by the elements of the array is coherently combined, i.e., through beamforming. For this analysis it is not necessary that we actually know the optimum beamforming coefficients; it is equivalent to determine the power generated in the load attached to each antenna independently and simply add the results. The power collected by any given element depends on the patterns of the antenna elements; thus A_e will be function of direction. A_e will also depend on quality of the match offered by impedance Z_L of the load attached to the terminals of the antenna elements. We will consider several possible values of Z_L, including the likely value (due to hardware considerations) of $100 + j0\ \Omega$.

We will consider two types of ground: perfectly conducting ground, which can be achieved to a good approximation using a ground screen; and a realistic (somewhat lossy) ground having relative permittivity $\epsilon_r = 13$ and conductivity $\sigma = 5$ mS/m. The latter is not necessarily representative of the conditions at the proposed sites for LWA stations, however this is a commonly used "typical" value [5]. The actual values for LWA candidate sites have not yet been documented.

3 Single Stand Results

To illustrate the analysis and to provide some useful reference cases, we first consider the problem of a single stand; that is, two dipoles with collocated feedpoints but oriented at right angles to each other. A number of scenarios will be considered, but in each case we are interested in the collecting area of the co-polarized dipole. In terms of the coordinate system used in this report, this dipole is parallel to x axis. The *cross*-polarized dipole is parallel to the y axis.

The problem is parameterized in terms of (1) the ground type, (2) the load impedance, and (3) the direction of interest. For simplicity, we will consider only two directions: $\{\theta = 0, \phi = 0\}$ (zenith and broadside) and $\{\theta = 45^\circ, \phi = 0\}$; both lie in the E-plane. Three methods will be considered:

- The direct method, described in [4]. We refer to the collecting area determined in this way as A_e^d.
- The traditional approach, based on reciprocity. In this approach, the load on the dipole of interest is replaced with a voltage source and the transmit-mode gain G_t is calculated. The collecting area in the case of a matched load is then calculated as $A_e = G_t\lambda^2/(4\pi)$ [5]. In the case of a mismatched load, this result is multiplied by a factor of $1 - |\Gamma|^2$, where Γ is the reflection coefficient between the antenna and the load. We refer to the collecting area determined in this way as A_e^i.
- An emperical method, given by the following formula:

$$A_e^e = (0.13\lambda^2)(3.4)(1 - |\Gamma|^2)(\cos^3\theta)L_g\ , \tag{1}$$

  where $0.13\lambda^2$ is the theoretical broadside A_e for a matched half-wave dipole in free space [5], the factor of 3.4 accounts for the presence of a perfectly conducting ground, $(1 - |\Gamma|^2)$ accounts for load impedance mismatch, $\cos^3\theta$ accounts for pattern, and L_g accounts for ground loss, if present. The value of 3.4 for reflection was determined by solving the "image" version of the transmit problem, in which the ground is replaced by a mirror image of the dipole [5]. The values of $L_g = 0.6562$ (-1.8 dB) and the exponent of $\cos\theta$ were determined from the transmit-mode problem by application of reciprocity. The latter is approximate (the nearest integer that seemed to fit the data), and applies only to the E-plane pattern.

Ground	Load	θ	A_e^e	A_e^d	A_e^i
PEC	$(Z_A^{p1})^*$	0°	27.55 m^2	29.52 m^2	27.76 m^2
		45°	9.74 m^2	9.28 m^2	8.72 m^2
PEC	$100 + j0\ \Omega$	0°	23.78 m^2	25.49 m^2	23.97 m^2
		45°	8.41 m^2	8.01 m^2	7.53 m^2
Realistic	$(Z_A^{r1})^*$	0°	18.08 m^2	19.50 m^2	18.22 m^2
		45°	6.39 m^2	5.55 m^2	5.21 m^2
Realistic	$100 + j0\ \Omega$	0°	16.24 m^2	17.41 m^2	15.73 m^2
		45°	5.74 m^2	4.98 m^2	4.49 m^2

Table 1: Predicted A_e for the co-polarized dipole in a single stand.

NEC was used to determine Z_A, the *in situ* antenna terminal impedance, for each of the two ground conditions of interest. The results were $Z_A = Z_A^{p1} = 92.59{+}j76.19\ \Omega$ for perfectly-conducting ground and $Z_A = Z_A^{r1} = 86.05 + j60.70\ \Omega$ for the realistic (lossy) ground. Whenever NEC is used, dipoles are modeled using 11 segments of equal length.

The results are summarized in Table 1. The expected trends are all apparent; i.e., collecting area suffers with increasing ground loss, increasing impedance mismatch, and decreasing elevation angle. Also we see that the direct method gives a reasonably good estimate. A good basis for comparison is with A_e^i for the "PEC"–Z_A^{p1} and "PEC"–Z_A^{r1} cases, for which the indirect approach should be exact to within the limits of numerical error. In both cases, A_e^d is about 6% higher than A_e^i, which was also the finding from [4]. The ratio A_e^d/A_e^i is about the same in all other cases. Finally, we note that the empirical method also yields reasonable answers.

4 Results for a Candidate Station Layout

Next, we consider a complete LWA station consisting of the antenna stands described in the previous section. The layout used is a pseudorandom design with minimum allowed spacing of 4 m, provided by Aaron Cohen (NRL), shown in Figure 1.

From this point forward, only the "direct" method for calculating A_e will be used. The principal reason for doing so is that the straightforward implementation of the "indirect" (reciprocity-based) method requires a voltage source on every co-polarized dipole; i.e., 256 voltage sources, and my NEC-2 setup will not allow so many sources. I have to admit that I do not know why this should be. It seems to me that the work is in inverting the impedance matrix, and that having that the number of sources should not then be an issue. Perhaps NEC-2 is able to exploit advance knowledge that most of the elements of the voltage (source) vector are zero to simplify the calculations. In any event, a way to bypass this would be to use superposition; i.e., apply the voltage sources one at a time, and add the responses. However, this requires that the NEC deck be run 256 times – yuck! Since the direct method avoids these issues and yields reasonable results, the direct method will be used exclusively from this point forward.

The results of the analysis are shown in Table 2. For easy comparison to Table 1, the total collecting area of the co-polarized portion of the array is divided by the number of stands (256) to obtain the mean collecting area per dipole. Also shown is the percentage difference from the analogous result from Table 1. It is immediately apparent that the array collecting area cannot be reliably estimated from the single-stand results.

The large difference between the per-dipole array results and the single stand results is certainly due to mutual coupling. It makes sense that the mutual coupling should be significant, since the 4 m minimum spacing in this array corresponds to about $\lambda/2$; i.e., electrically close. One way to

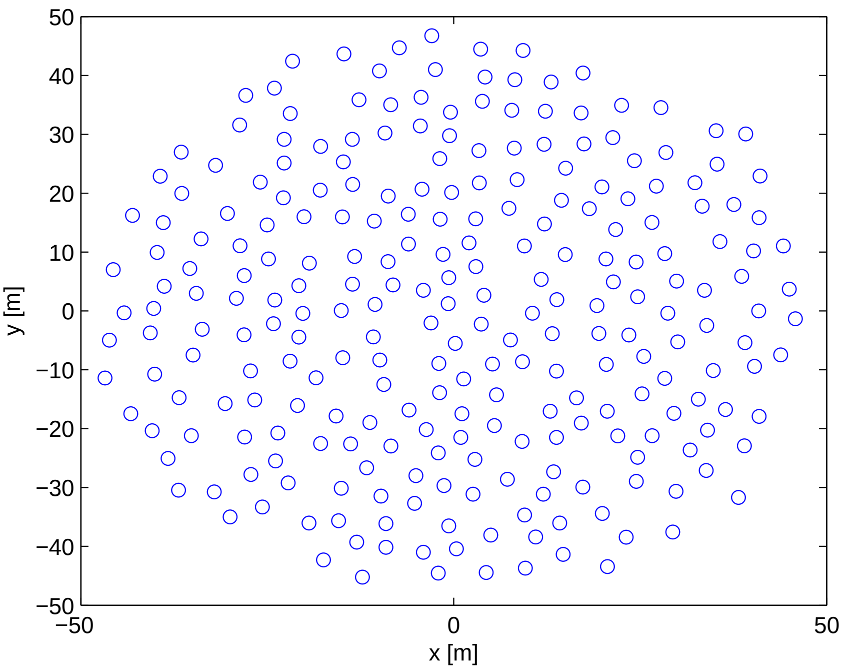

Figure 1: NRL Pseudorandom LWA station geometry with 4 m minimum spacing. 256 stands, consisting of 256 co-polarized dipoles and 256 cross-polarized dipoles.

Ground	Load	θ	$A_e^d/256$	Δ% WRT single-stand
PEC	$(Z_A^{p1})^*$	0°	25.22 m^2	−15%
		45°		
PEC	$100 + j0\ \Omega$	0°	25.96 m^2	+2%
		45°	10.16 m^2	+27%
Realistic	$(Z_A^{r1})^*$	0°	22.08 m^2	+13%
		45°		
Realistic	$100 + j0\ \Omega$	0°	19.08 m^2	+10%
		45°	6.40 m^2	+28%

Table 2: Predicted mean (per-dipole) A_e for the co-polarized dipoles in the NRL pseudorandom station layout, obtained by dividing the total A_e for optimum combining of co-polarized dipoles by 256. The right column is difference with respect to the single stand results (Table 1). Blank entries are scenarios which were not computed.

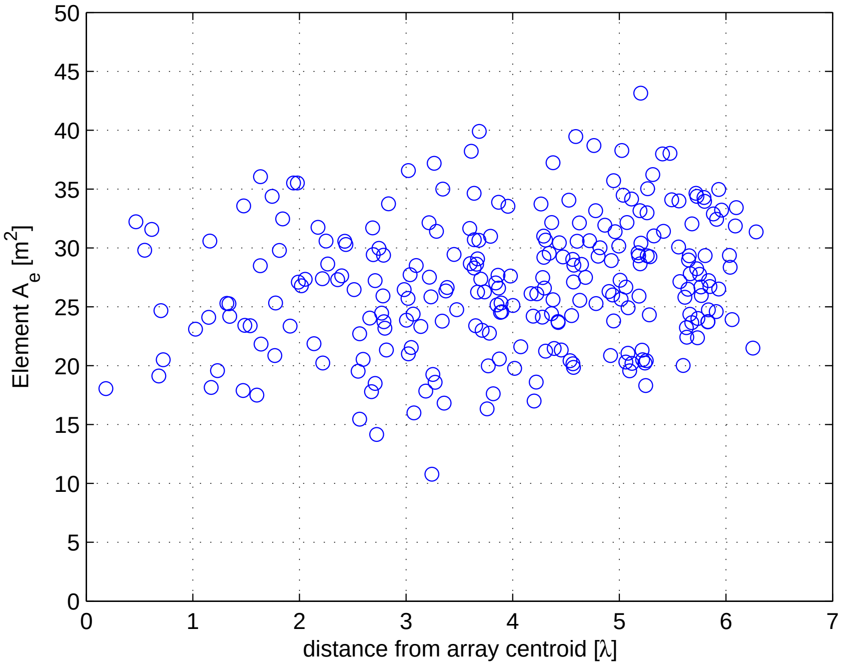

Figure 2: A_e for each element in the NRL 4 m array, assuming 100Ω loads, perfectly-conducting ground, and broadside ($\theta = 0$) illumination.

see the issue more clearly is to consider the collecting area calculated on a per-element basis for one of the cases in Table 2, as shown in Figure 2. Note that this quantity varies considerably and in a disorderly way.

It is interesting also to consider the phase of terminal currents, as shown in Figure 3. In this scenario, the illumination is from the zenith, so that in the absence of mutual coupling we would expect these phases to be equal. Note however that the phases "jitter" over a range of $\sim 20°$ degrees. Thus, the optimal beamforming coefficients are not equal; in fact, using the coefficients obtained from purely geometrical considerations (i.e., neglecting mutual coupling) would actually result in a *decrease* in collecting area because the element voltages would not add coherently.

As a "sanity check" to make sure that mutual coupling is actually the culprit, let us consider a modification to this scenario in which the array layout is scaled up by a factor of 10 (in the horizontal plane only) so as to greatly increase the spacing between elements. The resulting spacing between elements is now on the order of 5λ, thus we would expect the mutual coupling to be greatly reduced. All other parameters remain unchanged. The results are shown in Figure 4 and 5. As anticipated, the per-element A_e is now equal across the array. The mean of the per-element A_e's is 26.51 m^2, which is very close to the single-stand A_e^d (see Table 1). The phase variation shown in Figure 5 is now "jitter free" and exactly as expected from geometrical considerations,[1] and thus indicates negligible effects from mutual coupling.

Finally, let us return to the orginal array and consider illumination from $\theta = 45°$. The results are shown in Figure 6 and 7. The thing to note here is that the per element phase "errors" are different from the broadside illumination case. Thus, the coupling effects are direction-dependent,

[1]Note that this array is now so large that it is not completely in the far field of the source dipole, hence the phase gradient.

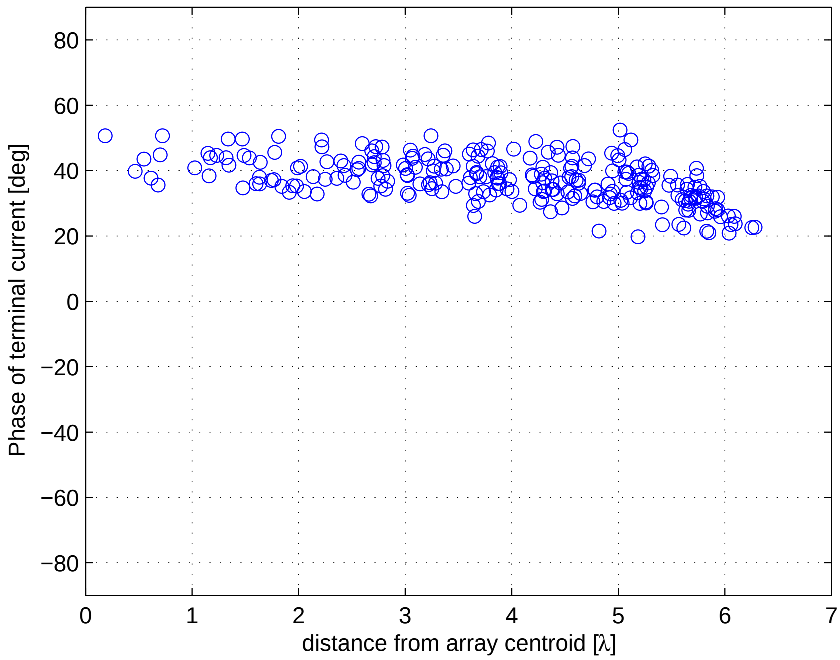

Figure 3: Phase of the terminal current for each element in the NRL pseudorandom array, assuming 100Ω loads, perfectly-conducting ground, and broadside ($\theta = 0$) illumination.

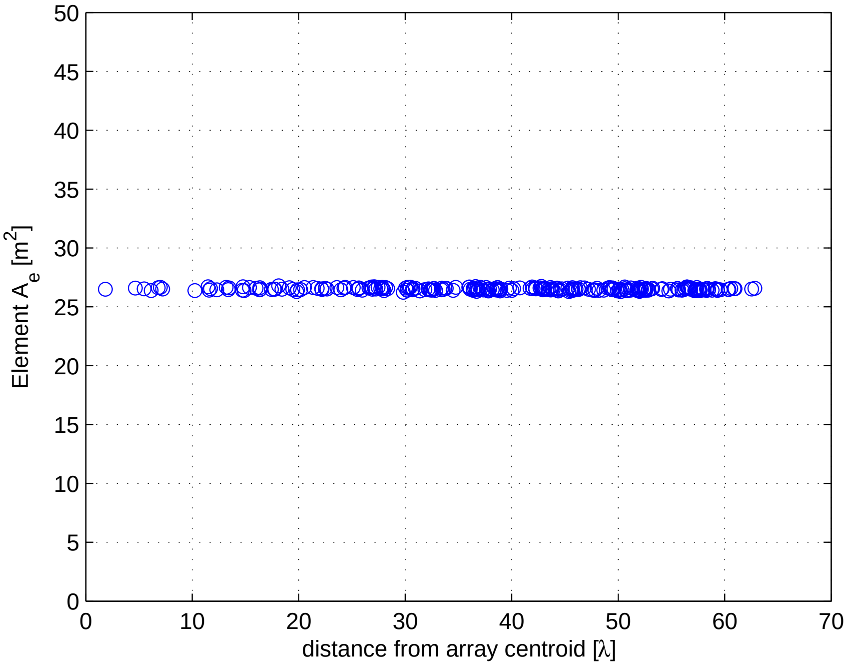

Figure 4: Same as Figure 2, except for an array layout scaled up (in the horizontal plane only) by a factor of 10.

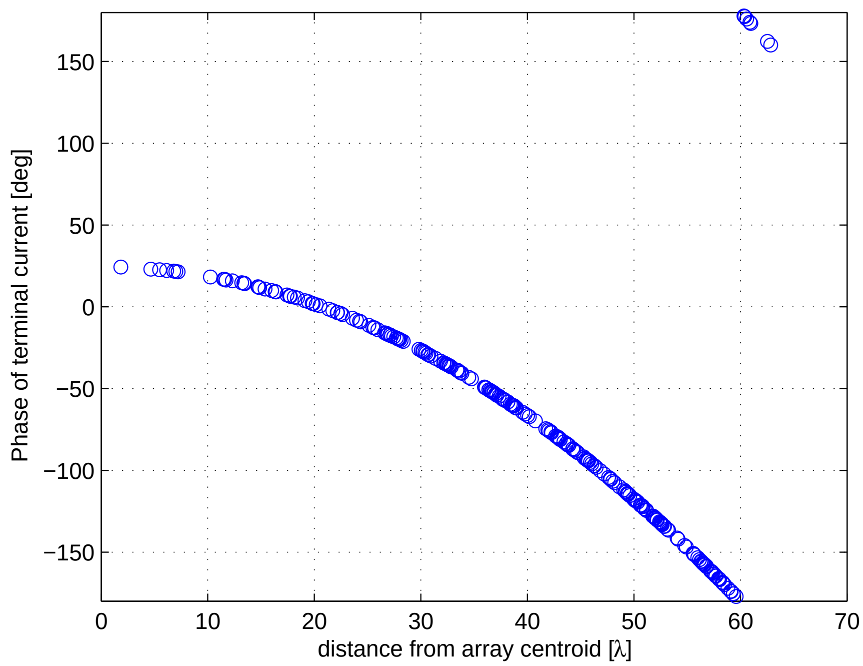

Figure 5: Same as Figure 3, except for an array layout scaled up (in the horizontal plane only) by a factor of 10.

which will make calibration of this array a real challenge.

The above analysis was repeated to consider the effect of replacing the perfectly-conducting ground with "realistic" (lossy) ground. The observed coupling-induced phase "errors" were different, but similar in magnitude to those observed above.

5 Is This Consistent With LWDA Results?

From the results of the previous section it seems likely that the effects of mutual coupling will be quite large and problematic for an LWA station using the type of layout shown in Figure 1. However, the 16-stand LWDA uses a pseudorandom geometry with similar spacings, and yet is producing beautiful images of the sky with presumably no accounting for mutual coupling effects. Does this makes sense?

To answer this question, consider an array using the LWDA layout, except scaled up by a factor of (74 MHz)/(38 MHz), as shown in Figure 8, with constant height equal to that used in the examples above. All parameters other than number of stands and layout are identical to the scenarios considered in previous sections, including frequency and the height and design of the antennas. "Realistic" ground conditions, as described above, are used since it is assumed this is closer to the real situation than assuming perfectly-conducting ground.

The results are shown in Figure 9 through 12. Note that all of the same behaviors exhibited for the larger array are present in these results as well, however the coupling-induced phase "errors" are somewhat less – just $\sim 13^\circ$ peak-to-peak. I would speculate that such errors are sufficiently small so as not to noticeably affect the sky images obtained from LWDA, due to the limited spatial resolution resulting from the small physical aperture. In contrast, similar phase errors in higher

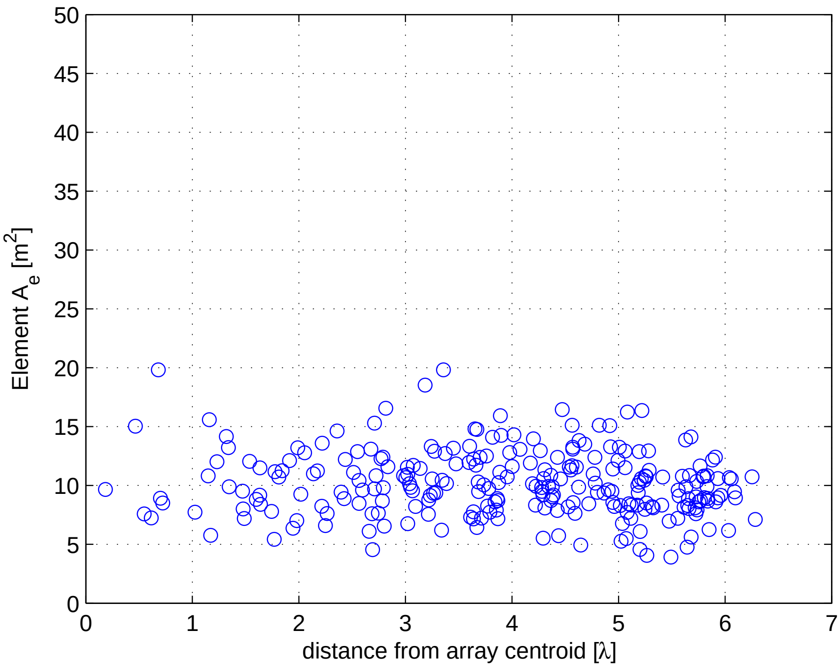

Figure 6: Same as Figure 2, except illumination from $\theta = 45°$.

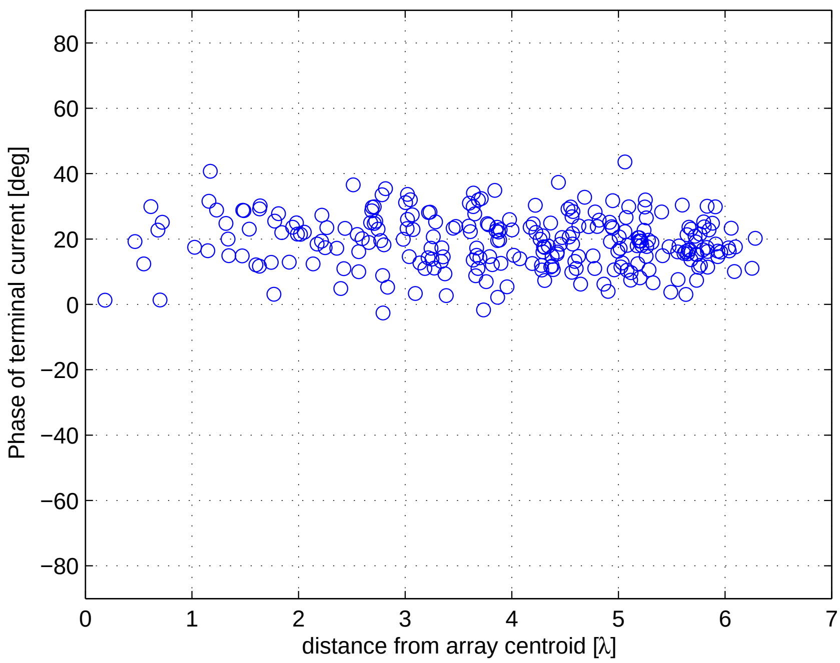

Figure 7: Same as Figure 3, except illumination from $\theta = 45°$. Geometry-induced phase is subtracted out so that differences from constant value are attributable entirely to mutual coupling.

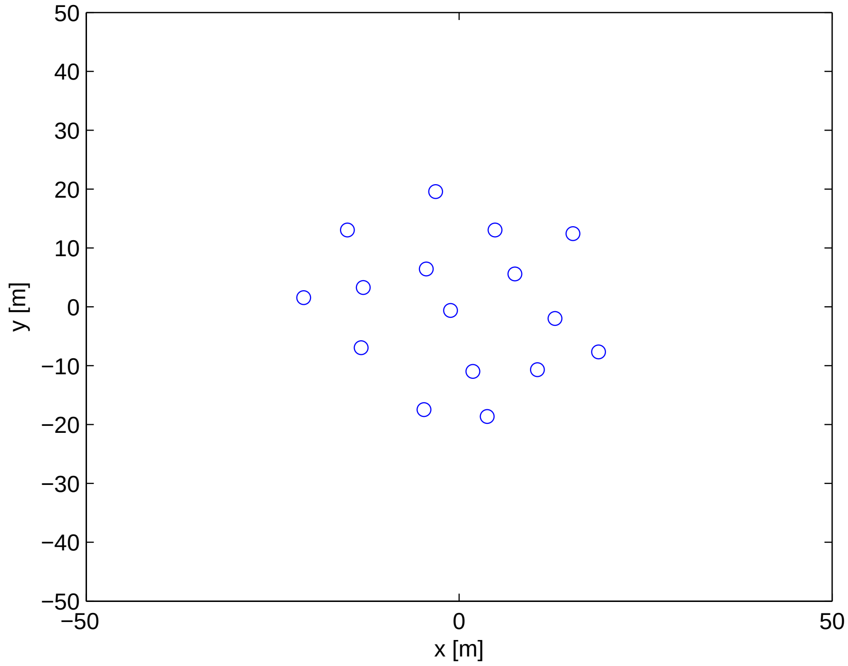

Figure 8: LWDA layout, scaled up by (74 MHz)/(38 MHz) for use in this analysis.

resolution images (i.e., from a large array) probably would have a noticeable "smearing" effect, and certainly phase errors comparable to those predicted for the larger array considered in the previous section would be noticeable.

A better way to detect the kinds of errors predicted here in LWDA data might be to observe the fringes from a strong source such as Cas A. Since the effect is predicted to be direction-dependent, there should be a noticeable and repeatable difference from the fringes predicted on a purely geometrical basis.

6 Conclusions

The conclusions of this study are summarized below.

- Mutual coupling plays a significant role in the determining the collecting area and optimum beamforming coefficients for geometries of the type considered here (e.g., the NRL pseudo-random array with 4-m minimum spacings). Whereas it is possible to estimate the collecting area of single stands from empirical formulas, it is found that estimates derived from simple multiplication of these values by the number of elements may be in error by as much as 28%. Interestingly, the collecting area estimates for the full array including coupling seem often to be larger than that expected from simple scaling up from the single stand results.
- It was found that mutual coupling has a significant direction-dependent "jittering" effect on the magnitude and phase of the signals received at each element. As a result, I am now somewhat skeptical that there is much value in trying to optimize antenna designs and array geometries unless this coupling is taken into account. More to the point, I think any such optimization probably only makes sense in a "co-design" paradigm, where the antenna elements and geometry are optimized jointly and *in situ* with the expected ground parameters.

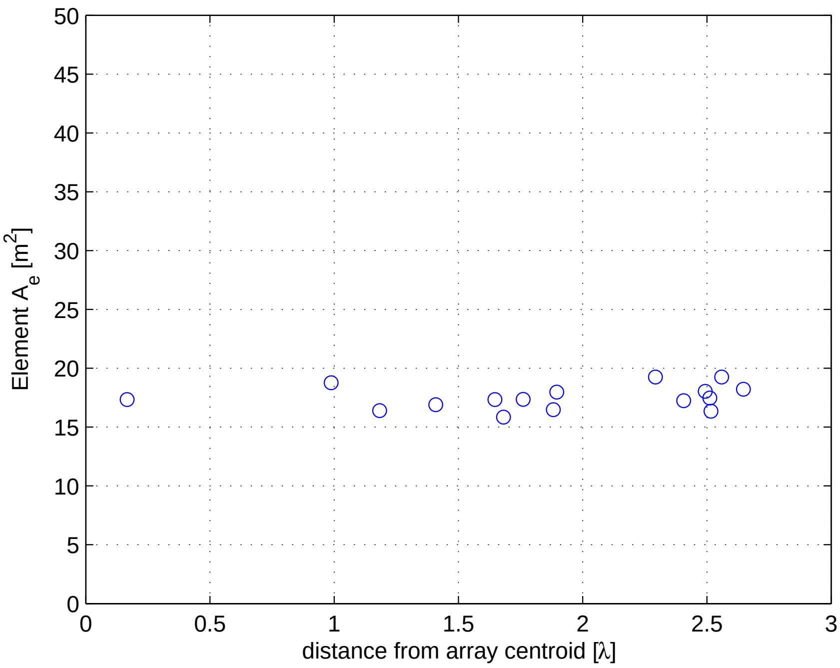

Figure 9: A_e for each element in the scaled-up LWDA, assuming 100Ω loads, "realistic" ground, and broadside ($\theta = 0$) illumination.

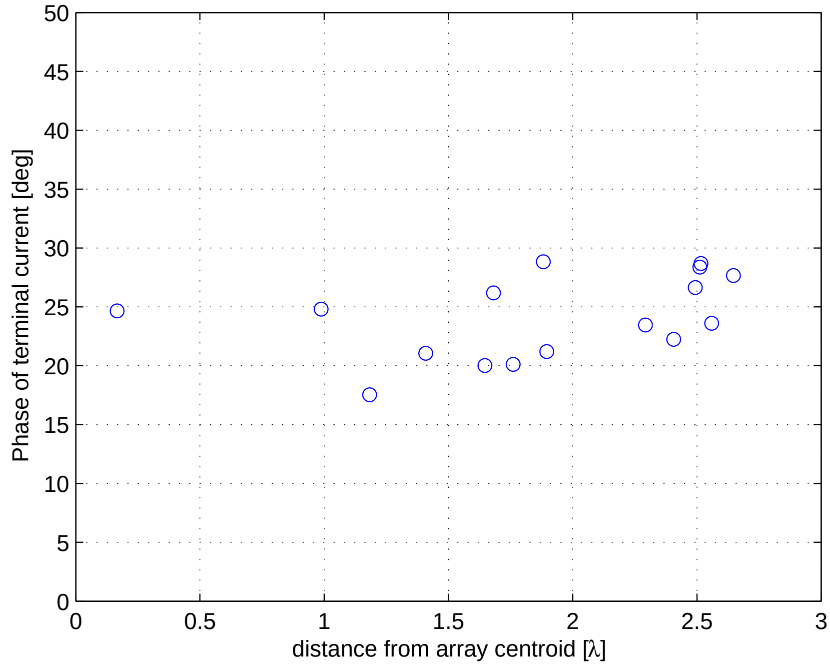

Figure 10: Phase of the terminal current for each element in the scaled-up LWDA, assuming 100Ω loads, "realistic" ground, and broadside ($\theta = 0$) illumination.

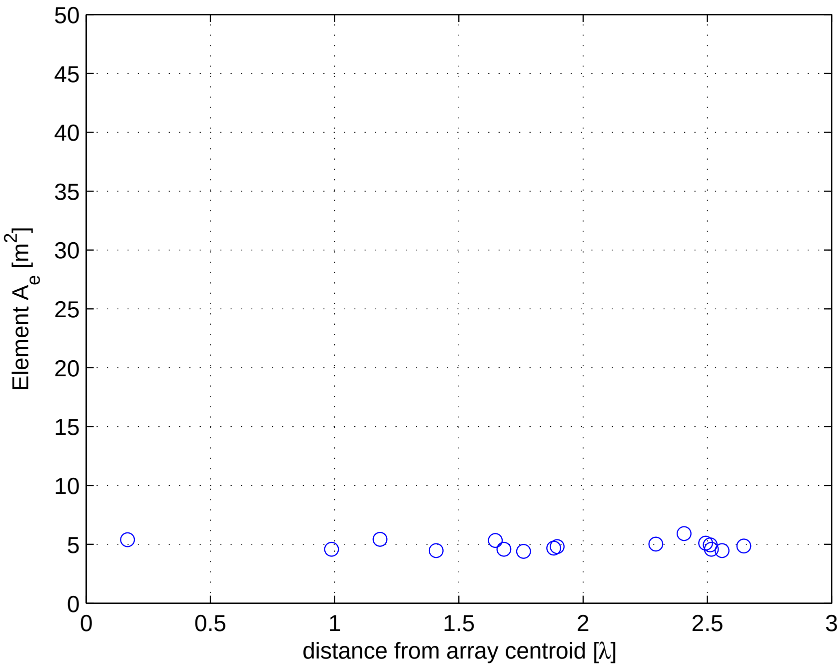

Figure 11: Same as Figure 9, except illumination from $\theta = 45°$.

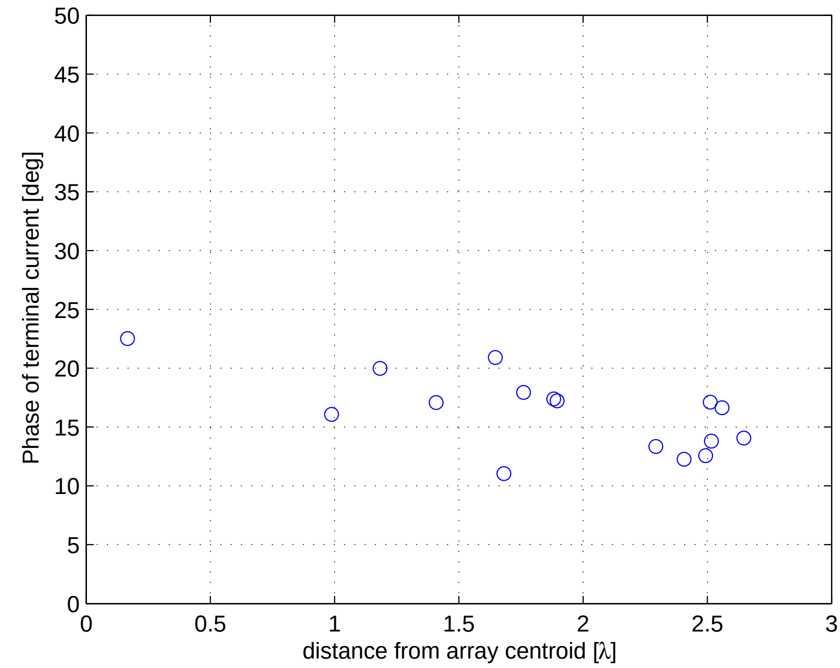

Figure 12: Same as Figure 10, except illumination from $\theta = 45°$. Geometry-induced phase is subtracted out so that differences from constant value are attributable entirely to mutual coupling.

- The directional dependence of the coupling-induced phase perturbations significantly complicates the process of station calibration. Unlike a dish array, where it is possible to assume that the instrumental phases are independent such that a single solution applies to all pointing directions, here we have the situation that any one solution for the instrumental phases applies only to the direction for which it was obtained. On the other hand, the range of phase errors was observed to be on the order of 10 or 20 degrees, so useful "starting values" can probably be obtained from purely geometrical considerations and then refined through a series of observations in various directions.

- Perfectly-conducting ground leads to collecting area which is dramatically better than that obtained in scenarios with "realistic" ($\epsilon_r = 13$, $\sigma = 5$ mS/m) ground. The improvement was roughly the same for single-stand and array scenarios: on the order of 40% for zenith pointing (pretty close to the value one might predict given L_g) but considerably greater – about 60% – for $\theta = 45°$.

- The above findings indicate that it is useful to know the true ground properties at each of the candidate station sites. This is urgent, in fact, since it may in part determine the number of stands required and may also enter into the design of the station geometry.

In closing, one should recall that all of these results were obtained assuming arrays of thin, straight half-wave dipoles. Certainly the results could be somewhat different for the types of broadband elements currently being considered for LWA, and will certainly be different at frequencies far from resonance. However the general conclusions – that mutual coupling and ground conditions are important and must be taken into account – will probably remain unchanged, however.

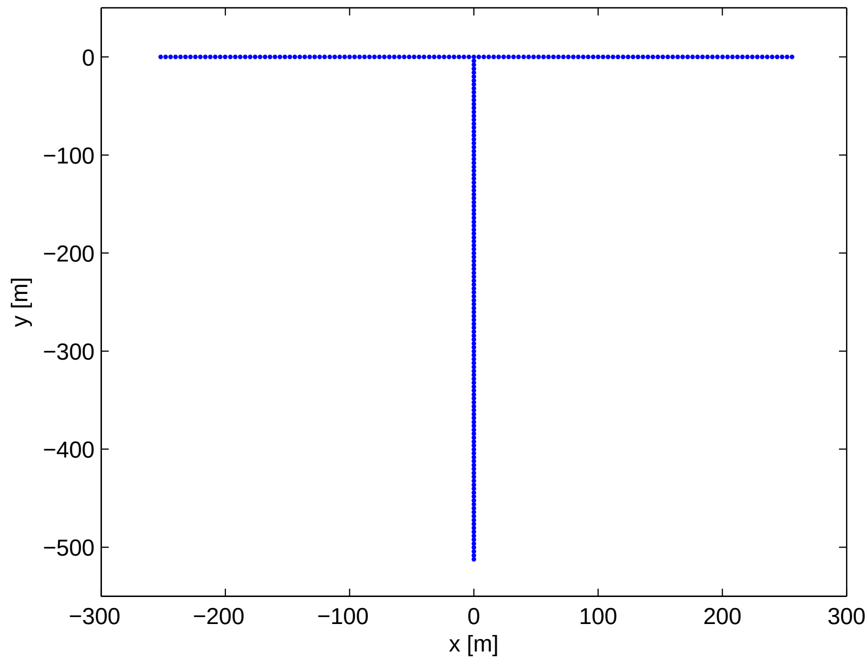

Figure 13: "T" array, consisting of 128 stands along the vertical "stem" and 128 stands along the horizontal "bar".

A Epilogue: A "T" array

After completing this memo, it struck me that doing this analysis for a "T"-shaped array, similar to the Clark Lake geometry, might be informative. Figure 13 shows the array considered. The stands are exactly 4 m apart; otherwise there are no changes from the arrays considered in previous sections.

The results for broadside incidence, "realistic" ground, and 100 Ω termination are shown in Figures 14 and 15. Note the striking difference between the "Stem" (in this case, parallel-aligned) and "Bar" (collinearly aligned) dipoles; clearly dipoles in parallel are doing much better in this case. Also striking is the how constant the values are, except for the clearly pronounced oscillation near the ends and near the junction. The mean collecting area per dipole is 15.0 m^2, which is simply the average of the "Stem" and "Bar" dipoles.

In the phase results, we similarly see no "jitter" (note the smooth variation here is geometrical, due to the large size of this array), yet the results for the stem and bar dipoles are different. It is clear that significant coupling must exist, since neither the "stem" nor "bar" dipoles behave like single stands. Comparing to Figure 5, it is tempting to say the bar dipoles experience less coupling; however in terms of A_e per dipole the stem dipoles are much closer to (but still greater than) the single stand values.

The results for illumination from $\theta = 45°$ are shown in Figures 16 and 17. Clearly, the coupling is giving rise to so are some interesting behavior. Understanding these behaviors may provide useful clues as to how to properly design an LWA station array.

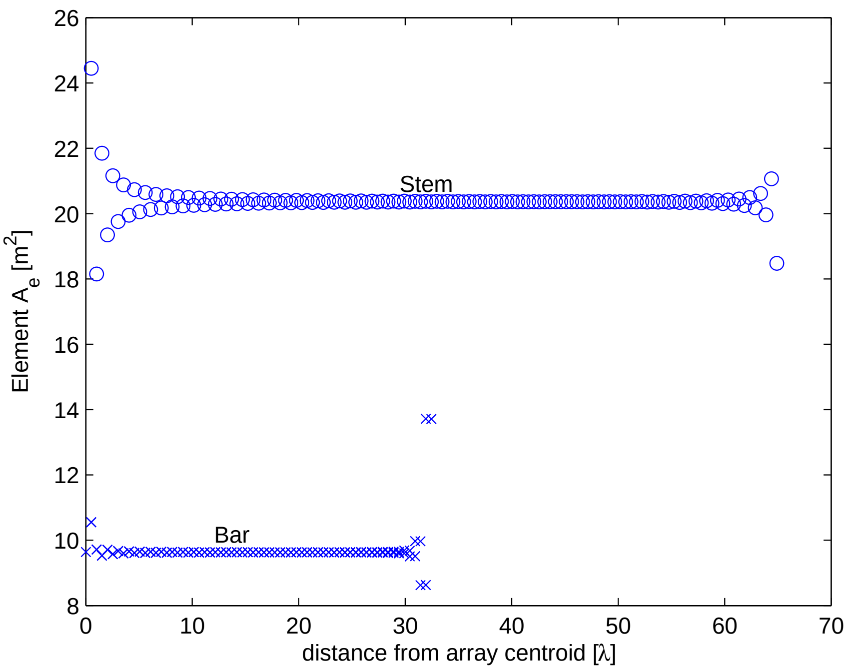

Figure 14: A_e for each element in the "T" array, assuming 100Ω loads, "realistic" ground, and broadside ($\theta = 0$) illumination. "Stem" refers to the constant-x portion of the array, whereas "Bar" refers to the constant-y portion.

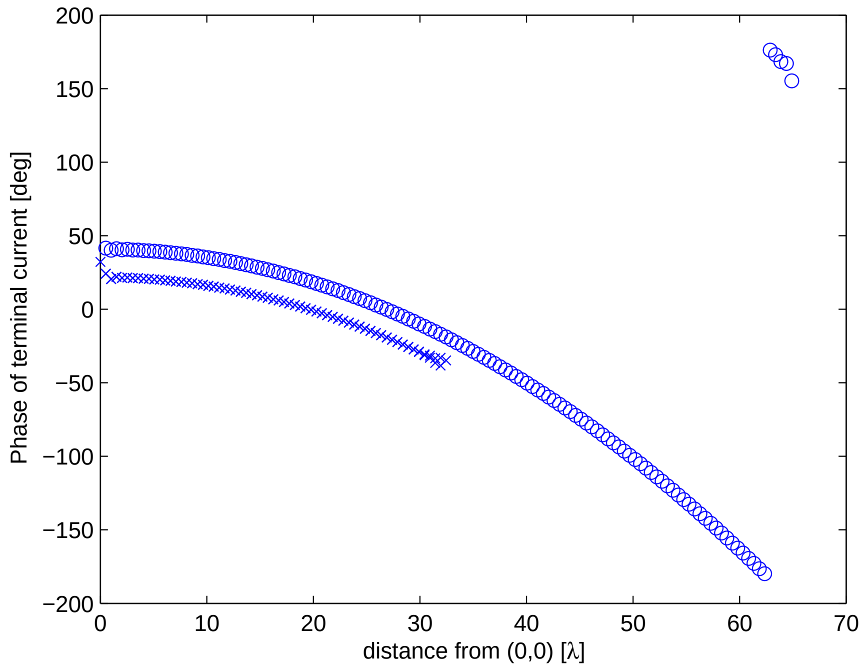

Figure 15: Phase of the terminal current for each element in the "T" array, assuming 100Ω loads, "realistic" ground, and broadside ($\theta = 0$) illumination.

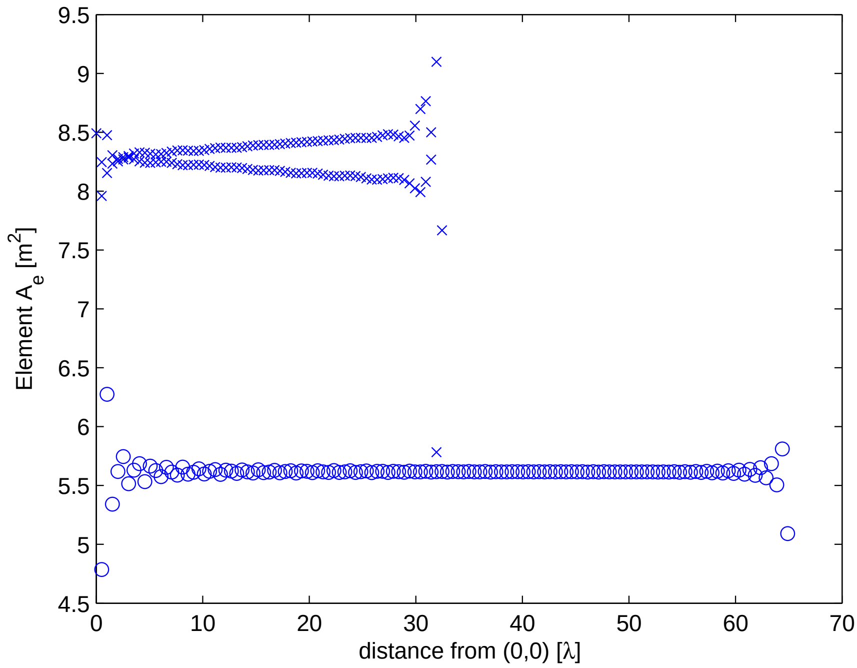

Figure 16: Same as Figure 14, except illumination from $\theta = 45°$.

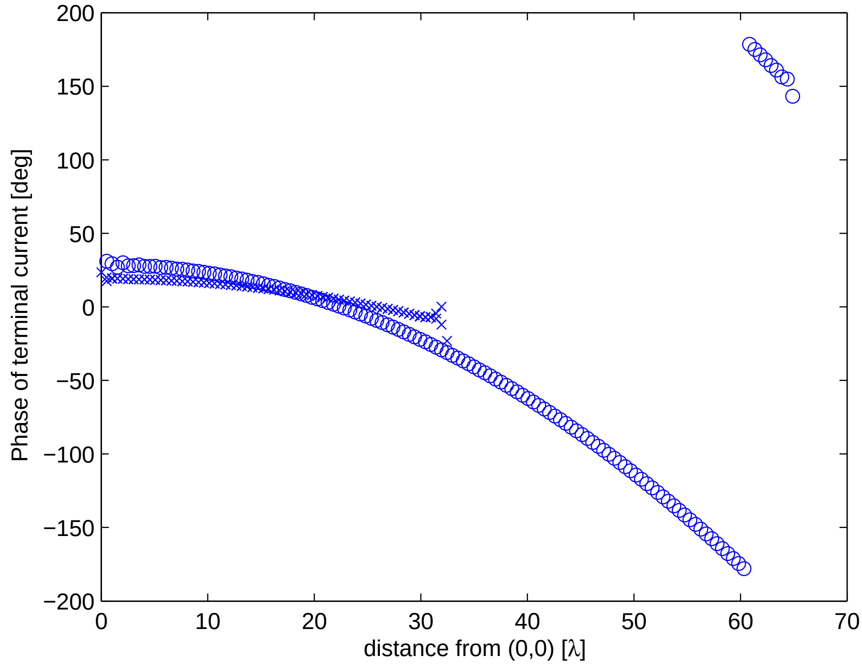

Figure 17: Same as Figure 15, except illumination from $\theta = 45°$. Geometry-induced phase is subtracted out so that differences from constant value are attributable entirely to mutual coupling.

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