

Radio Frequency Interference Mitigation for Detection of Extended Sources with an Interferometer

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Radio frequency interference (RFI) is a significant problem for current and future radio telescopes. We describe here a method for post-correlation cancellation of RFI for the special case of an extended source observed with an interferometer that spatially resolves the astronomical signal. In this circumstance, the astronomical signal is detected through the auto-correlations of each antenna but is not present in the cross-correlations between antennas. We assume that the RFI is detected in both auto- and cross-correlations, which is true for many cases. The large number of cross-correlations can provide a very high interference to noise ratio reference signal which can be adaptively subtracted from the autocorrelation signals. The residual signal is free of interference to significant levels. We discuss the application of this technique for detection of the spin-flip transition of interstellar deuterium with the Allen Telescope Array. The technique may also be of use for epoch of reionization experiments and with multi-beam feeds on single dish telescopes.

1. Introduction

Radio frequency interference (RFI) mitigation is fast becoming a necessary aspect of radio astronomy as terrestrial and space-based radio transmitters become more widespread and more powerful, as radio telescopes become more sensitive and search for ever fainter signals, and as radio astronomers seek to observe outside of the protected radio astronomy bands. A variety of RFI mitigation methods have been developed that rely on a range of signal processing techniques [e.g., Leshem et al., 2000]. Many of these techniques are generic in their application in the sense that they are not specific to the scientific goals and the types of interferers. There is much to be gained, however, from techniques which are specific to a given type of astronomical observation or a given type of interferer [e.g., Ellingson et al., 2001].

We describe here a technique developed for detection of a diffuse astronomical source with a high resolution interferometer in the presence of point-like or partially-resolved interference. The technique exploits the fact that the astronomical signal is detectable only through the auto-correlation of each antenna's signal, while the interference is detected in

both the auto- and cross-correlations of the signals. For an interferometer with even a modest number of elements, the number of cross-correlations is significantly greater than the number of auto-correlations. Thus, the problem becomes similar to a reference antenna method with multiple reference antennas.

In §2, we describe the technique and show results of computer simulations which demonstrate its performance characteristics. In §3, we discuss possible applications of this technique. These include detection of the interstellar deuterium spin-flip transition with an interferometer with baselines on the order of 100 meters, and detection of the cosmological epoch of reionization signature. The method is also applicable for multi-beam feeds on single-dish radio telescopes.

2. Cross-correlation Subtraction

We consider our method for an interferometer with N_a elements configured in a non-regular pattern. Each element receives signal from the sky and converts the signal to baseband. The baseband signals are processed by a correlator which produces auto-correlation and cross-correlation power spectra with N_{ch} channels. These spectra are computed from N_s digital samples.

We characterize the signal received at each antenna i as the sum of the astronomical signal, S_i ,

the receiver noise, N_i , and the interference, I_i :

$$E_i = S_i + N_i + I_i. \quad (1)$$

The auto- and cross-correlation power spectra are then

$$\langle E_i E_i^* \rangle = \langle S_i S_i^* \rangle + \langle I_i I_i^* \rangle + \langle N_i N_i^* \rangle, \quad (2)$$

$$\langle E_i E_j^* \rangle = \langle S_i S_j^* \rangle + \langle I_i I_j^* \rangle + \langle N_i N_j^* \rangle. \quad (3)$$

We assume that the source term in the cross-correlation function $\langle S_i S_j^* \rangle$ goes to zero while the source term in the auto-correlation function $\langle S_i S_i^* \rangle$ is non-zero. This assumption holds for the case of an extended, smoothly distributed brightness distribution observed with a high resolution interferometer. In particular, this condition holds for baselines with a fringe spacing much smaller than the angle subtended by the source on the sky. The difference between the auto- and cross-correlation functions is then:

$$\langle E_i E_i^* \rangle - \langle E_i E_j^* \rangle = \langle S_i S_i^* \rangle + \langle I_i I_i^* \rangle - \langle I_i I_j^* \rangle + \langle N_i N_i^* \rangle + \langle N_i N_j^* \rangle \quad (4)$$

Under the assumption that the interference is not resolved by the interferometer (i.e., that it is a point source), the second and third terms on the right hand side of the equation sum to zero. We are then left with a signal that only includes the effects of the astronomical source and the system noise. The system noise term will be dominated by the noise auto-correlation and we are left with a residual signal:

$$\langle E_i E_i^* \rangle - \langle E_i E_j^* \rangle = \langle S_i S_i^* \rangle + \langle N_i N_i^* \rangle \quad (5)$$

which is simply the auto-correlation signal in the absence of interference.

The cancellation of interference that takes place between the auto- and cross-correlation signals as described above requires an idealized system. Under realistic conditions, a number of effects will limit the difference between the auto- and cross-correlation of the interference. These include multiple sources of interference, multi-path propagation of interference, and variable antenna gain between two different array elements in the direction of the interferer or interferers. Adaptive cancellation methods introduce new degrees of freedom that make the realistic problem tractable. We suggest a method that minimizes the residual ϵ through the adjustment of the cross-correlation gains μ_{ij} , where

$$\epsilon = \left(\sum_i \langle E_i E_i^* \rangle - \sum_i \sum_{j>i} \mu_{ij} \langle E_i E_j^* \rangle \right)^2. \quad (6)$$

This method is equivalent to a reference antenna adaptive cancellation method in which the astronomy signal is the sum of the auto-correlations and the reference signal is the weighted sum of the cross-correlations. We can use the results for the reference antenna adaptive cancellation technique to estimate the effectiveness of this methodology [Barnbaum & Bradley, 1998]. The attenuation of interference in the astronomy signal goes as

$$A = (\text{INR}_{\text{ref}} + 1)^2, \quad (7)$$

where INR_{ref} is the interference power to noise power ratio (INR) in the combined reference signal. Thus, the residual astronomy signal will have interference at the level of

$$\text{INR}_{\text{resid}} = A^{-1} \text{INR}_{\text{ast}}, \quad (8)$$

where INR_{ast} is the initial INR in the sum of the auto-correlations.

We can determine the INRs for the reference and astronomy signals as follows. We assume that the signal is Nyquist-sampled at a rate f for N_s samples. The INR in the autocorrelation of a single sample is INR_0 . In the case of the astronomy signal, the astronomy signal grows coherently with number of antennas and number of samples while the noise grows incoherently leading to:

$$\text{INR}_{\text{ast}} = \sqrt{N_a N_s} \text{INR}_0. \quad (9)$$

In the reference signal, the interference and noise signals grow similarly; however, the number of signals contributing to each sum grows as $N_a(N_a - 1)$. This leads to

$$\text{INR}_{\text{ref}} = \sqrt{N_a(N_a - 1)/2N_s} \text{INR}_0. \quad (10)$$

Substituting these results into Equation 8, we find

$$\text{INR}_{\text{resid}} = \frac{\sqrt{N_a N_s} \text{INR}_0}{(\sqrt{N_a(N_a - 1)/2N_s} \text{INR}_0 + 1)^2}. \quad (11)$$

In the limit of $\sqrt{N_a(N_a - 1)/2N_s} \text{INR}_0 \gg 1$ and $N_a \gg 1$, we find that $\text{INR}_{\text{resid}} \approx N_a^{-3/2} N_s^{-1/2} \text{INR}_0^{-1}$.

We can also compute the ratio of signal to interference (SIR). The signal to noise ratio in the auto-correlation sum is

$$\text{SNR}_{\text{ast}} = \sqrt{N_a N_s} \text{SNR}_0, \quad (12)$$

where SNR_0 is the signal to noise ratio of a single auto-correlation sample. Thus, SIR is the ratio of Equations 12 to 11. In the high INR_0 limit discussed above, $\text{SIR} \approx N_a^2 N_s \text{SNR}_0 \text{INR}_0$.

We plot the SIR in Figure 1 for two cases. We assume $\text{SNR}_0 = -50$ dB. In both cases the SIR increases dramatically for very large INR_0 . That is, powerful interferers are relatively easy to detect and subtract. Both cases also show an inflection point at which the SIR reaches a minimum. Below this point, the interference is too weak to be detected and cannot be cancelled. If the minimum occurs at $\text{SIR} > 0$, then the technique has succeeded in protecting the signal against all interferers. In our example, this holds for the case with large N_a and N_s . For a smaller N_a and N_s , the SIR dips below zero, leaving the observations potentially corrupted by weak interference.

2.1. Simulations and Practical Considerations

We describe a method for solving for the parameters μ_{ij} described in Equation 6. We use a Wiener method to calculate the best values for μ_{ij} but other methods are available [Vaseghi, 2000]. The Wiener method has the advantage that it directly calculates the values; however, it is computationally expensive and may be replaced in practice by an iterative method [Ellingson et al., 2001].

We recast the problem into a single astronomical signal $\hat{\mathbf{s}}$, which is the sum of the auto-correlation functions above, and $N_b = N_a(N_a - 1)/2$ reference antennas $\hat{\mathbf{R}}$. Both $\hat{\mathbf{s}}$ and $\hat{\mathbf{R}}$ are functions of channel number. We also add an additional reference signal which is a constant as a function of channel number to remove the effects of the bandpass. For more complex bandpasses, multiple template functions such as a tilt or a ripple can be included. It is important that $N_{ch} > N_{ref}$, where N_{ref} is the number of reference signals. If not, the solution is over-determined and all signal will be removed by the algorithm. The Wiener solution is then

$$\hat{\boldsymbol{\mu}} = (\hat{\mathbf{R}}\hat{\mathbf{R}}')^{-1}\hat{\mathbf{R}}\hat{\mathbf{s}}. \quad (13)$$

In Figure 2, we show the result of a simulation of the cross-correlation method. We used an 8-element array with 65 spectral channels. The interference was a frequency comb with a spacing of 8 channels and a width of 2 channels in each peak. The gain of each antenna was modulated in each iteration separately for the interferer and for the source. The interference power was set to 20% of the noise power. The signal was characterized as a Gaussian with width 16 channels with a peak power that is 1% of the noise power. Each iteration consisted of 100 samples. We

iterated for 2000 iterations. For 10 kHz of channel bandwidth, this would correspond to a 0.3 second integration. The interference is cleanly removed and the weaker signal is readily apparent in the residual spectrum with an amplitude of 1% of the noise power. The expected attenuation of the interference in the residual signal is 21 dB. The actual reduction of interference is ~ 20 dB, comparable to that expected.

There are a few drawbacks and additional considerations to the method outlined here. The first problem is that the mean power off source of the residual signal is reduced from the no-interference case. This may lead to calibration uncertainty. This effect results from the fact that all of the reference signals are positive quantities. The second problem is that in cases where the astronomical source signal is strong, the interference is over-subtracted. Since the algorithm is attempting to minimize the total spectrum, the presence of an astronomical signal biases the subtraction. The algorithm will also remove strong astronomical point sources from the residual signal. Whether this is a problem is a function of the particular experiment being undertaken. Additionally, the effects of partial resolution of the astronomical source of interest have not been fully explored. In what SNR regime does the remaining component of the source serve to mitigate the source itself? Finally, interferers in the near-field as well as those undergoing significant multi-path propagation may be resolved by the interferometer. It may not be possible to remove interferers of this kind from the astronomy signal.

Alternative methods of performing the subtraction may avoid some of these problems. Knowledge of the noise power spectrum obtained through careful calibration, for instance, would allow one to give each spectrum zero mean. One could also generalize the two-reference antenna method described in Briggs et al. [2000] for this multiple reference antenna case. It is also worthwhile to explore the effect of using subsets of baselines to generate the reference signal.

3. Applications

3.1. Interstellar Deuterium

The 92 cm deuterium spin flip transition (DI) is one of the most important radio spectroscopic lines not yet detected [Weinreb, 1962; Anantharamaiah & Radhakrishnan, 1979; Blitz & Heiles, 1987; Chen-

galur et al., 1997]. The transition is the deuterium analog of the hydrogen 21 cm line (HI) and arises from a flip in the spin direction of the electron with respect to the nuclear spin. The deuterium to hydrogen ratio ($N(D)/N(H)$) which can be determined from detection of the DI line is an important constraint on cosmic nucleosynthesis.

The signal is known to be quite weak. Blitz & Heiles [1987] place an upper limit of $N(D)/N(H) < 3 \times 10^{-5}$, which is consistent with ultra-violet detections at 2×10^{-5} . This ratio implies a line strength towards the Galactic anti-center on the order of 1 mK, on the order of -50 dB times the system temperature of a radio telescope observing in the Galactic plane at these wavelengths. Since the DI emission traces the HI emission, the line width is expected to be quite narrow (~ 10 km/s) towards the anti-center.

We have simulated DI observations with the 32 element configuration of the Allen Telescope Array [DeBoer et al., 2004]. The ATA-32 employs 6.1m paraboloids distributed in two dimensions with baseline lengths that range from 8m to ~ 100 m. We simulated a snap shot observation towards the Galactic anti-Center using MIRIAD software. The sky model assumes that the DI traces HI as observed by the Leiden Dwingeloo survey [Hartmann & Burton, 1997]. We determined the correlated amplitude as a function of baseline length (Figure 3). The source is substantially resolved on baselines longer than about 20m. These baselines comprise more than 90% of all ATA-32 baselines. Thus, the signal will primarily be detected in the auto-correlations from each antenna and will be absent from the cross-correlations. The weakness of the signal, its spatially diffuse nature, and the difficult interference environment at 327 MHz make this experiment an excellent candidate for the cross-correlation subtraction method.

3.2. Epoch of Reionization

The Universe went through a phase transition at a redshift $z > 6$ from neutral gas to ionized gas corresponding to the appearance of the first stars, quasars or other objects which produced ultraviolet photons [Sunyaev & Zeldovich, 1972]. Prior to this epoch of reionization (EOR), all hydrogen was in an atomic or molecular state. The spin flip transition of atomic hydrogen (HI) is expected to produce emission that is essentially continuous at frequencies less than $\nu_{\text{HI}}/z_{\text{EOR}}$, where ν_{HI} is the rest frequency of the HI transition and z_{EOR} is the redshift of the EOR.

At frequencies above this critical frequency, the signal disappears due to the ionization of all atomic hydrogen. The expected amplitude of the signal is on the order of 10 mK, or > 30 dB below foreground noise. The redshift of the EOR places the signature at frequencies where interference is a critical problem for radio telescopes.

The emission is expected to be globally distributed with a characteristic scale length of ~ 10 arcmin [Zaldarriaga et al., 2004]. Detection of the global signature is the first step towards characterization of the EOR. The cross-correlation subtraction method can be applied to this problem provided that baselines of the interferometer are sufficiently long to resolve out the global EOR signal. In practice, this implies baselines of 3 km for an attempted detection at a frequency of 100 MHz ($z_{\text{EOR}} \sim 13$). Integration times must be less than 10 seconds to prevent decorrelation of the interfering signal on the long baselines.

Since such an experiment may have baselines on many intermediate baselines, it may be desirable to determine the astronomical signal from more than just the auto-correlation functions. A set of cross-correlations from short-baselines may also be included in the summed astronomical signal or have a separate interference signal removed from them.

3.3. Focal Plane Array Feeds

Focal plane array feeds are increasingly important tools for radio astronomy [Staveley-Smith et al., 1996]. These feeds place multiple receivers in the focal plane of a single antenna. Typically, they are designed only to produce auto-correlations for each receiver. If designed to produce the full set or a partial set of cross-correlations, however, these array feeds could make use of the cross-correlation subtraction method to mitigate interference. The technique may also be of use in eliminating bandpass ripples due to solar interference.

4. Conclusions

We have described a new method for cancellation of interference in the power domain for the specific case in which the astronomy signal is apparent only in the auto-correlation signal. Provided that the RFI is not resolved by interferometer, cross-correlation signals are used to determine a high interference to noise ratio reference signal. Analytical results suggest that the method could prove powerful at remov-

ing very weak interference for arrays with N_a greater than a few. Simulations demonstrate the basic performance of the algorithm. There are a few issues for further research. Most important among these is testing the effect of small source contributions to the cross-correlations. This method highlights a critical aspect of RFI mitigation research: the techniques that work best are those that are best-suited to the interferer and to the scientific goal.

References

- Anantharamaiah, K. R. & Radhakrishnan, V. 1979, *Astron. Astrophys.*, , 79, L9
- Barnbaum, C. & Bradley, R. F. 1998, *Astron. J.*, , 116, 2598
- Blitz, L. & Heiles, C. 1987, *Astrophys. J.*, , 313, L95
- Briggs, F. H., Bell, J. F., & Kesteven, M. J. 2000, *Astron. J.*, , 120, 3351
- Chengalur, J. N., Braun, R., & Butler Burton, W. 1997, *Astron. Astrophys.*, , 318, L35
- DeBoer, D. R., Welch, W. J., Dreher, J., Tarter, J., Blitz, L., Davis, M., Fleming, M., Bock, D., Bower, G., Lugten, J., Girmay-Keleta, G., D'Addario, L. R., Harp, G. R., Ackermann, R., Weinreb, S., Engargiola, G., Thornton, D., & Wadefalk, N. 2004, in Proceedings of the SPIE, Volume 5489, pp. 1021-1028 (2004), 1021–1028
- Ellingson, S. W., Bunton, J. D., & Bell, J. F. 2001, *Astrophys. J. (Supp.)*, , 135, 87
- Hartmann, D. & Burton, W. B. 1997, Atlas of galactic neutral hydrogen (Cambridge; New York: Cambridge University Press, ISBN 0521471117)
- Leshem, A., van der Veen, A., & Boonstra, A. 2000, *Astrophys. J. (Supp.)*, , 131, 355
- Staveley-Smith, L., Wilson, W. E., Bird, T. S., Disney, M. J., Ekers, R. D., Freeman, K. C., Haynes, R. F., Sinclair, M. W., Vaile, R. A., Webster, R. L., & Wright, A. E. 1996, Publications of the Astronomical Society of Australia, 13, 243
- Sunyaev, R. A. & Zeldovich, Y. B. 1972, *Astron. Astrophys.*, , 20, 189
- Vaseghi, S. 2000, Advanced Digital Signal Processing and Noise Reduction (2nd Ed. New York : Wiley & Sons)
- Weinreb, S. 1962, *Astrophys. J.*, , 136, 1149
- Zaldarriaga, M., Furlanetto, S. R., & Hernquist, L. 2004, *Astrophys. J.*, , 608, 622

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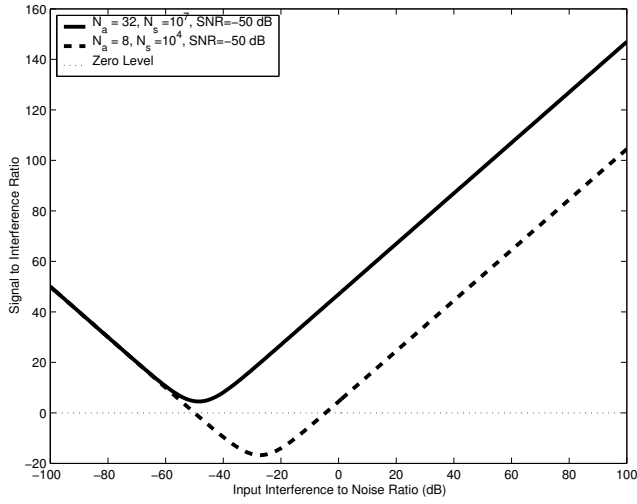


Figure 1. Signal to interference ratio in the residual astronomy signal as a function of initial interference to noise ratio for two cases. In both cases, the signal is assumed to be 50 dB below the noise. In case 1 (solid line), with 32 antennas and 10^7 samples per iteration, the signal is always greater than the interference in the residual. In case 2 (dashed line), with 8 antennas and 10^4 samples per iteration, weak interference can corrupt the signal.

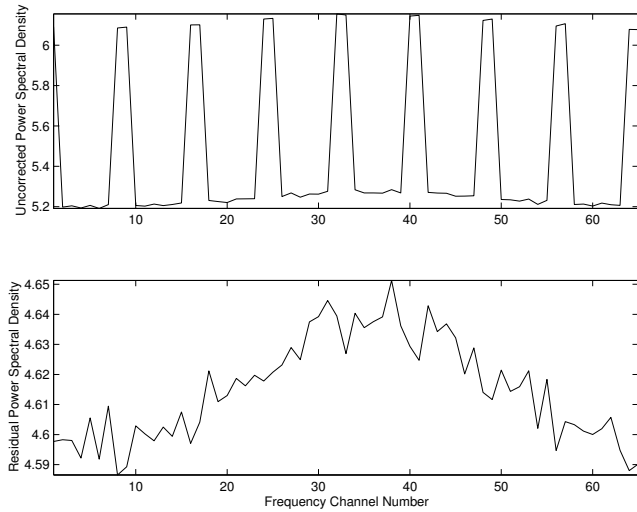


Figure 2. Results of a simulation of the cross-correlation subtraction technique. The top panel shows the astronomy signal without interference reduction. The bottom panel shows the astronomy signal with interference reduction. Details of the simulation are given in the text.

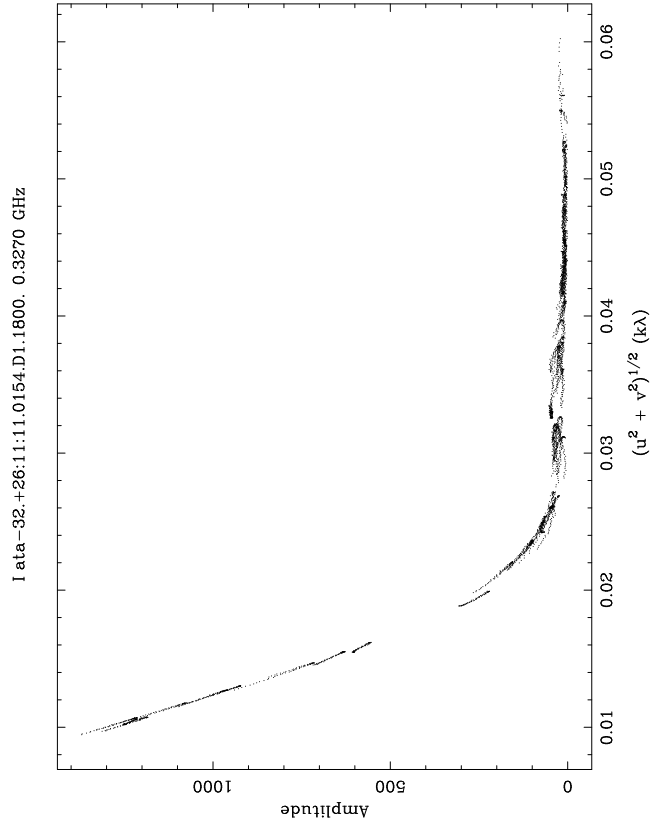


Figure 3. Correlation amplitude as a function of baseline length for deuterium as observed with the ATA-32. The signal drops off sharply with increasing baseline length making this problem well-suited to the cross-correlation subtraction method.