Another algorithm for phased array null formation.

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Abstract

Recently several algorithms for creating null at the phased array beam pattern were designed. See for example ATA memos of Geofrey Bower ([1]) and G.R. Harp ([2]). The phased array beam pattern null towards the RFI source can mitigate effect of the RFI. Another algorithm for creating null at the phased array beam pattern is described at this memo. This algorithm is based on modification of my algorithm of minimizing side lobes of an array ([3]). We tested this algorithm for possible SKA antenna station as a phased array of 19 antennas and for VLA-D working as a phased array for VLBI experiments. The results are very similar to the simulation described by G. Bower and G.R Harp. The null can be created at one direction and at the square around the given direction. The phase and amplitude correction to create the null has to be given with the very high accuracy which may be not practical. Rounding phase correction to 1 degree aggravates the null from $> 100db$ to $\sim 50db$. The narrow null ($\sim 0.1\frac{\lambda}{D}$) at the level > 100db can be easily created at the cost of loosing signal to noise ratio at the main direction at the level of 3%. Widening the null to $0.5\frac{\lambda}{D}$ leads to null level $\sim 40db$ and loosing 13% of signal to noise ratio. The depth of the null decreases dramatically when the required phase/weight correction is rounded to 1 degrees at phase and 0.01 at weight.

1 Description of the algorithm

The beam power pattern of a phased array can be described by the following expressions:

$$BEAM(\vec{e}) = \left|\sum_{n=1}^{N} w_n \exp j(2\pi\vec{r}_k(\vec{e}-\vec{e}_0)+\phi_n)\right|^2 = \sum_{i=1}^{N}\sum_{k=1}^{N} w_i w_k cos(2\pi(\vec{r}_i-\vec{r}_k)(\vec{e}-\vec{e}_0)+\phi_i-\phi_k) \quad (1)$$

where $\vec{e}$ is vector directed to the point at the sky;
$\vec{e}_0$ is vector towards the target source;
$\vec{r}_n$ are vectors determining the position of antennas n;
N is number of antennas in the array;
w_n, ϕ_n are the weights and phases we want to add for each antenna
to create the null at the vicinity of $\vec{e}$.

It is supposed in the equation 1 that the array is ideally phased at the direction of vector $\vec{e}_0$. So the value of $BEAM(\vec{e})$ at the direction of the target source ($\vec{e}=\vec{e}_0$) depends only on the added weights and phases w_n, ϕ_n. Positions of the antennas are given at portion of the array maximum baseline **D** and the component of vector $\vec{e}$ at the array plane is given at $\frac{\lambda}{D}$.

The derivatives of BEAM by phase ϕ_k and by w_k are given by the following expressions:

$$\frac{dBEAM}{d\phi_k} = \sum_{i=1}^{N} w_i w_k \; sin(2\pi(\vec{r}_i-\vec{r}_k)(\vec{e}-\vec{e}_0)+\phi_i-\phi_k) \quad (2)$$

$$\frac{dBEAM}{dw_k} = \sum_{i=1}^{N} w_i \; cos(2\pi(\vec{r}_i-\vec{r}_k)(\vec{e}-\vec{e}_0)+\phi_i-\phi_k) \quad (3)$$

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If we want to minimize the value of BEAM at the direction $\vec{e}$ we need to change the value of ϕ_k and by w_k proportionally to the relevant derivatives. We used this statement for optimization of an array configuration minimizing side lobes ([3]). Now the same statement is used to find the correction the phase and weight of each antenna to create the null at the vicinity of the given direction in the sky. The creating of null is multi iteration process. At each iteration the algorithm changes the phase and(or) weight on a small portion (GAIN controlled by user) of the recommended value. The solution requires minimum of time(seconds on my SUN station). The task (written in AIPS) can create two or one null but it is possible to increase the number of nulls.
It is clear that creating the nulls by adding phases and weights to each antenna decreases the signal to noise ratio observing the target source by the main lobe of the array (direction $\vec{e}_0$). The reduction of the signal to noise ratio (SNR) can be calculated by the following expression:

$$SNR = \frac{\left|\sum_{k=1}^{N} w_k\ cos(\phi_k)\right|^2}{N\ \sum_{k=1}^{N} w_k^2} \tag{4}$$

If $w_k = 1$ for all k and $\phi_k = 0$ for all k, then SNR=1; else $SNR < 1$.
Position of the null and its width is determined (at this algorithm) by the number of the synthesize beams - ratios $\frac{\lambda}{D}$, where D is the array size (maximum base line). So if the position of the null and its width is determined as $\theta = \alpha\frac{\lambda}{D}, \Delta\theta = \Delta\alpha\frac{\lambda}{D}$, then the width of the null at the frequency axis is determined as $\Delta\theta = \alpha\frac{\Delta\lambda}{D}$. Therefore the bandwidth of the null can be found as:

$$\frac{\Delta\alpha}{\alpha} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta\ f}{f} \tag{5}$$

It is clear from the equation (5) that the far null area locates of the main beam the lower bandwidth is.

2 SKA antenna station

We tested this algorithm for possible SKA antenna station as a phased array of 19 antennas, the array size $\sim 100m$ (Figure 2). We found that the depth of the null decreases dramatically when the implemented phase/weight correction is rounded to 1 degrees at phase and 0.01 at weight. The synthesized beam of the SKA antenna station is 410" ($\lambda = 20cm$, D=100m). Consider the null is located at 10 degrees from the main lobe ($\alpha = 90$) and its width at some level is equal 200" ($\Delta\alpha = 0.5$). Then bandwidth at the central frequency 1440 MHz should be equal (equation 5): $\Delta f = f \cdot \frac{\Delta\alpha}{\alpha} = 1440 \cdot 0.5/90 = 9MHz$. The table 2 shows the parameters of the array pattern at the created null area.

Table 1: Parameters of the null at 10 degrees of the main lobe (SKA example).

	null depth db	null width asec(30db)	SNR	Δ f MHz
narrow null	124	65	0.984	2.3
phase, weight rounded	47	70	0.985	2
wide null	45	200	0.915	6

Phase/weight correction is rounded to 1 degrees at phase and 0.01 at weight for the second row of the table. f = 1440MHz; $\frac{\lambda}{D}$=400"

Figure 1 shows the effect of rounding of weights and phase on the beam pattern near the null centered at 40 degrees apart of the main lobe. Phases/weights are rounded to 1 degrees and 0.01 respectively. The depth of the null drops down from 123db to 49db when rounding. The width of the null at the level of 30db stays the same $\sim$ 40arcsec.

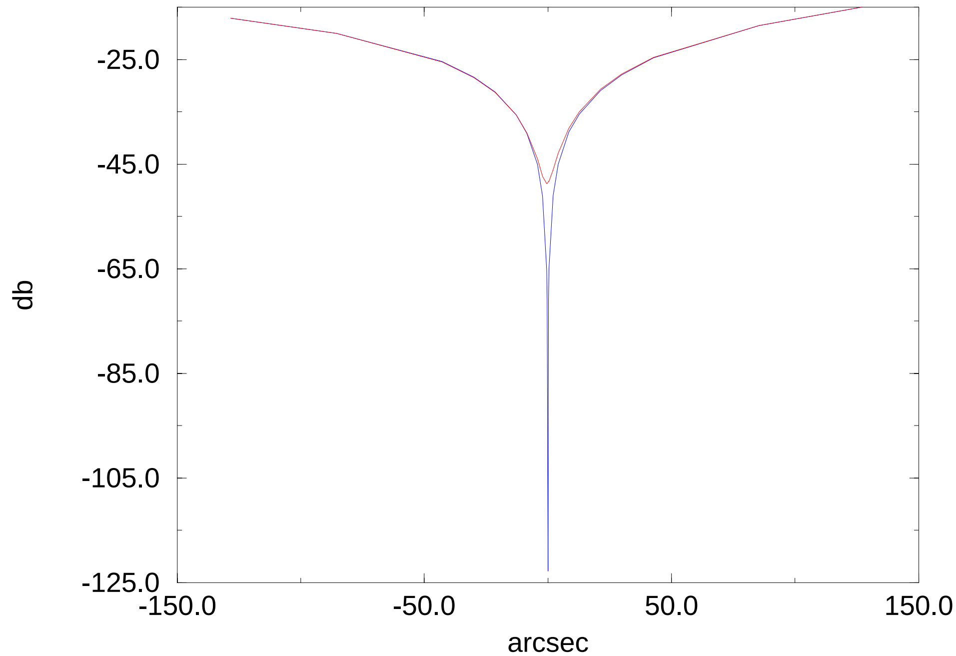

Figure 1: Cross section of the two dimensional narrow null area. The null center is 40 degrees apart of the main lobe. The blue plot corresponds to the weight and phase solution with six digits after the decimal point. Phases/weights are rounded to 1 degrees and 0.01 respectively for the red curve. Calculations were performed at $\lambda = 20cm$

3 VLA-D phased array

There is a potential source of the RFI for VLA located at the top of Sandia crest at Albuquerque (130miles of VLA). The group of several communication antennas are located there. We tested the current algorithm for VLA-D used as element of a VLBI experiment, when all 27 VLA antenna are phased together. The array size $\sim 1000m$ (Figure 3). The table 3 shows the parameter of the array pattern at the created null area. The synthesized beam of the VLA-D as an antenna station is 41" ($\lambda = 20cm$, D=1000m). Consider the Sandia crest is located at 10 degrees from the observed source($\alpha = 900$) and its width at level 30db is equal 8" ($\Delta\alpha = 0.2$). Then bandwidth at the central frequency 1440 MHz should be equal (equation 5): $\Delta f = f \cdot \frac{\Delta\alpha}{\alpha} = 1440 \cdot 0.2/900 \sim 0.3MHz$.

4 Conclusion

1. Too high accuracy of the required phase/weight to get the deep null.

2. The difference in atmosphere delay for different array antennas can exceed the descret of the adding phase and as the result destroy the null

3. The null deeper than 100db may not be required because of the individual antenna receiver saturation.

Table 2: Parameters of the null at 10 degrees of the main lobe (VLA-D example).

	null depth db	null width asec(30db)	SNR	Δ f MHz
narrow null	132	8	0.99	0.3
phase, weight rounded	56	8	0.97	0.3
wide null	59	20	0.91	0.8
wide null	43	40	0.90	1.6

Phase correction is rounded to 1 degree and weight $\equiv$ 1 for the second row of the table. f = 1440MHz; $\frac{\lambda}{D}$=400".

References

[1] Bower, Geofrey C., Simulations of Narrow-Band Phased-Array Null Formation for the ATA, ATA memo #37, 2001

[2] Harp, G.R. Customized Beam Forming at the Allen Telescope Array, ATA memo #51, 2002

[3] L. Kogan, Mathematic basis of an array configuration optimization minimizing side lobes, EVLA memo #71, 2004

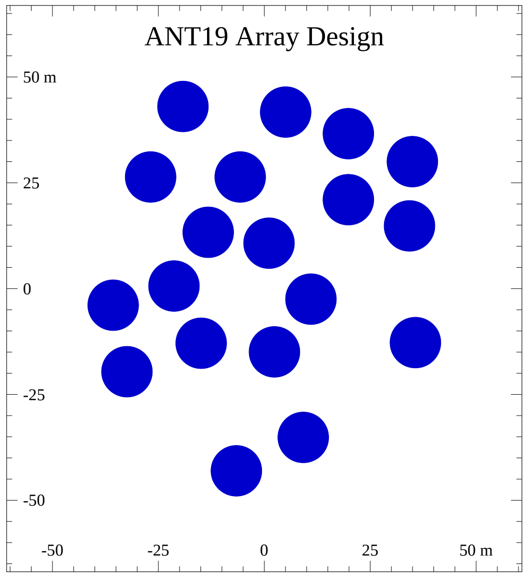

Figure 2: A possible SKA antenna station with 19 antennas.

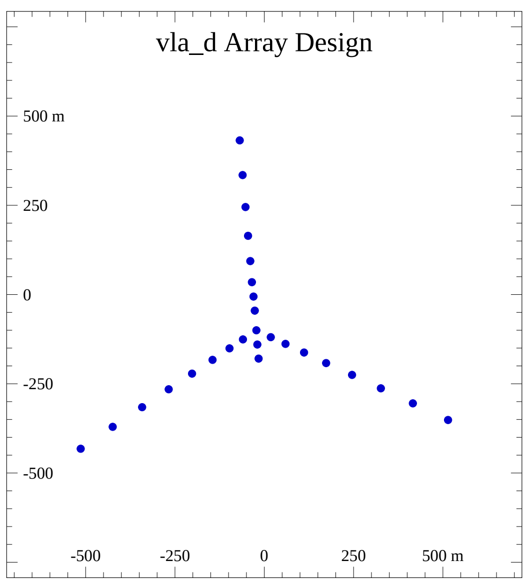

Figure 3: VLA-D configuration with 27 antennas.