

*RFI mitigation workshop, Penticton, July 2004*

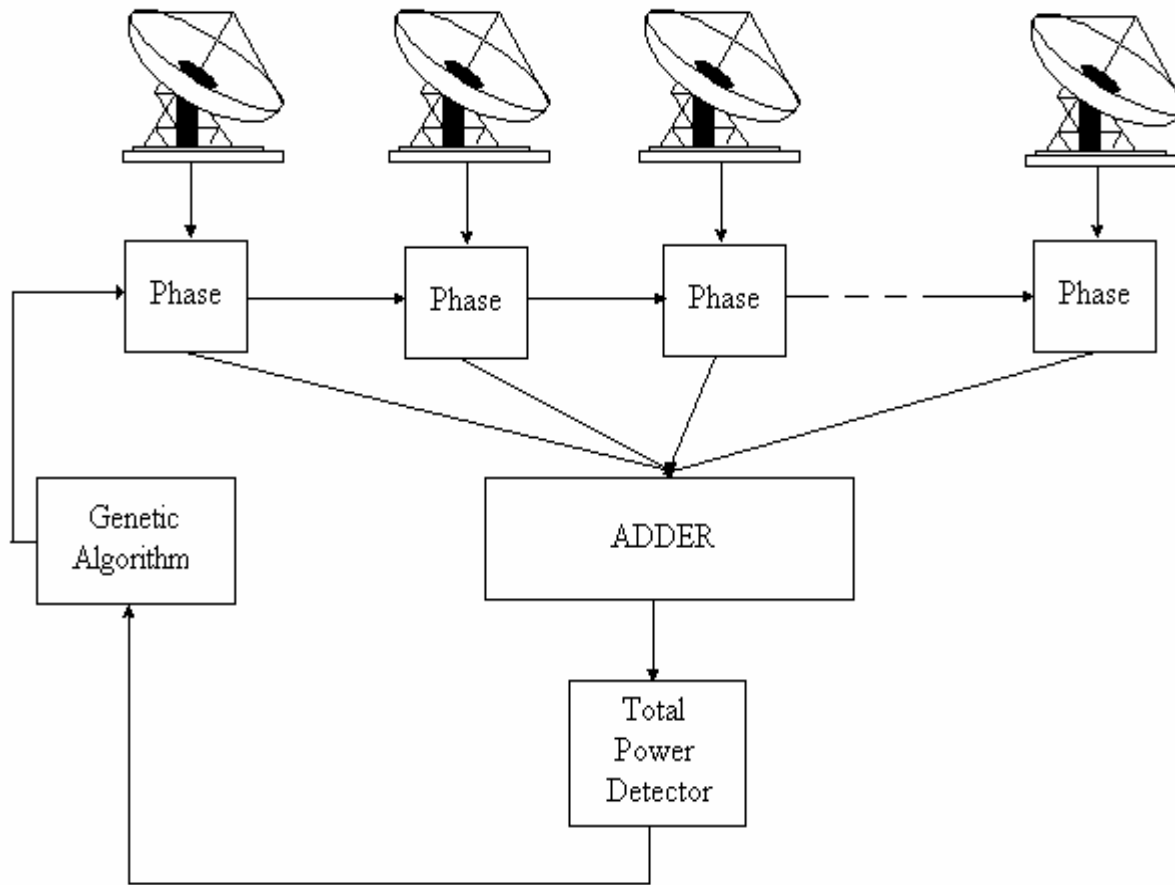
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# RFI mitigation with the phase-only adaptive beamforming

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**Adaptive phase control**

## Stochastic gradient search

Fitness function (calibration) - total power detector output:

$$C(\alpha) = -\int_0^T \left\{ \sum_{m=1}^2 \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{sig})/\lambda + \alpha_m + \beta_m] \right\}^2 dt$$

Algorithm :

$$\alpha_n = \alpha_{n-1} + \mu \cdot \tilde{\nabla}_{\alpha^F} C(\alpha)$$

$$C^+(\alpha, d\alpha) = C^+(\alpha + d\alpha),$$

$$C^-(\alpha, d\alpha) = C^+(\alpha - d\alpha),$$

$$\tilde{\nabla}_{\alpha^F} = \frac{C^-(\alpha, d\alpha) - C^+(\alpha, d\alpha)}{2d\alpha}$$

## Genetic algorithm search

Fitness function (calibration) - total power detector output:

$$C(\alpha) = \int_0^T \left\{ \sum_{m=1}^M \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{sig})/\lambda + \alpha_m + \beta_m] \right\}^2 dt$$

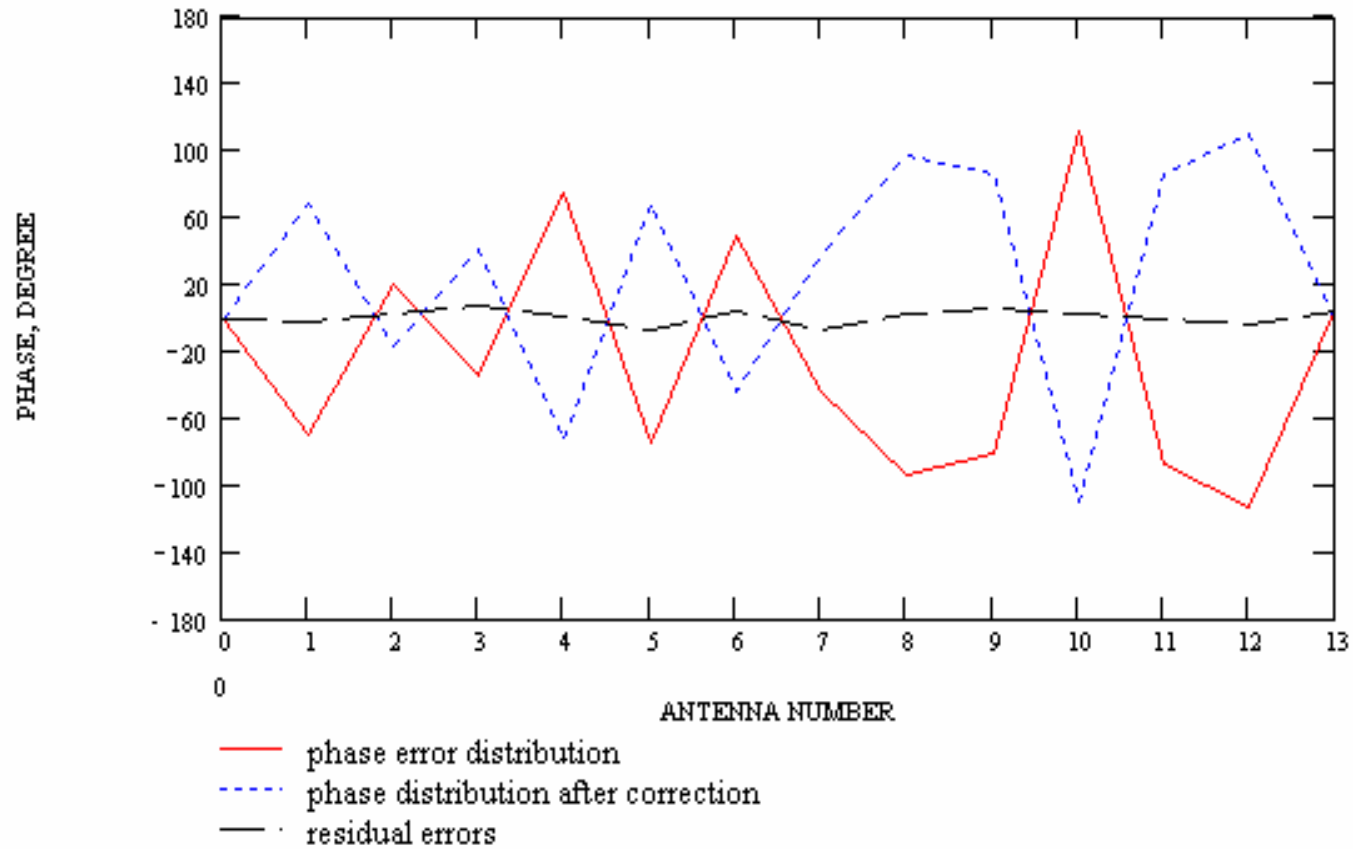
Fitness function (RFI mitigation) - total power detector output:

$$TPD_{RFI_j}(\alpha) = \int_0^T \left\{ \sum_{m=1}^M \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{RFI_j})/\lambda + \alpha_m] \right\}^2 dt, \text{ RFI response}$$

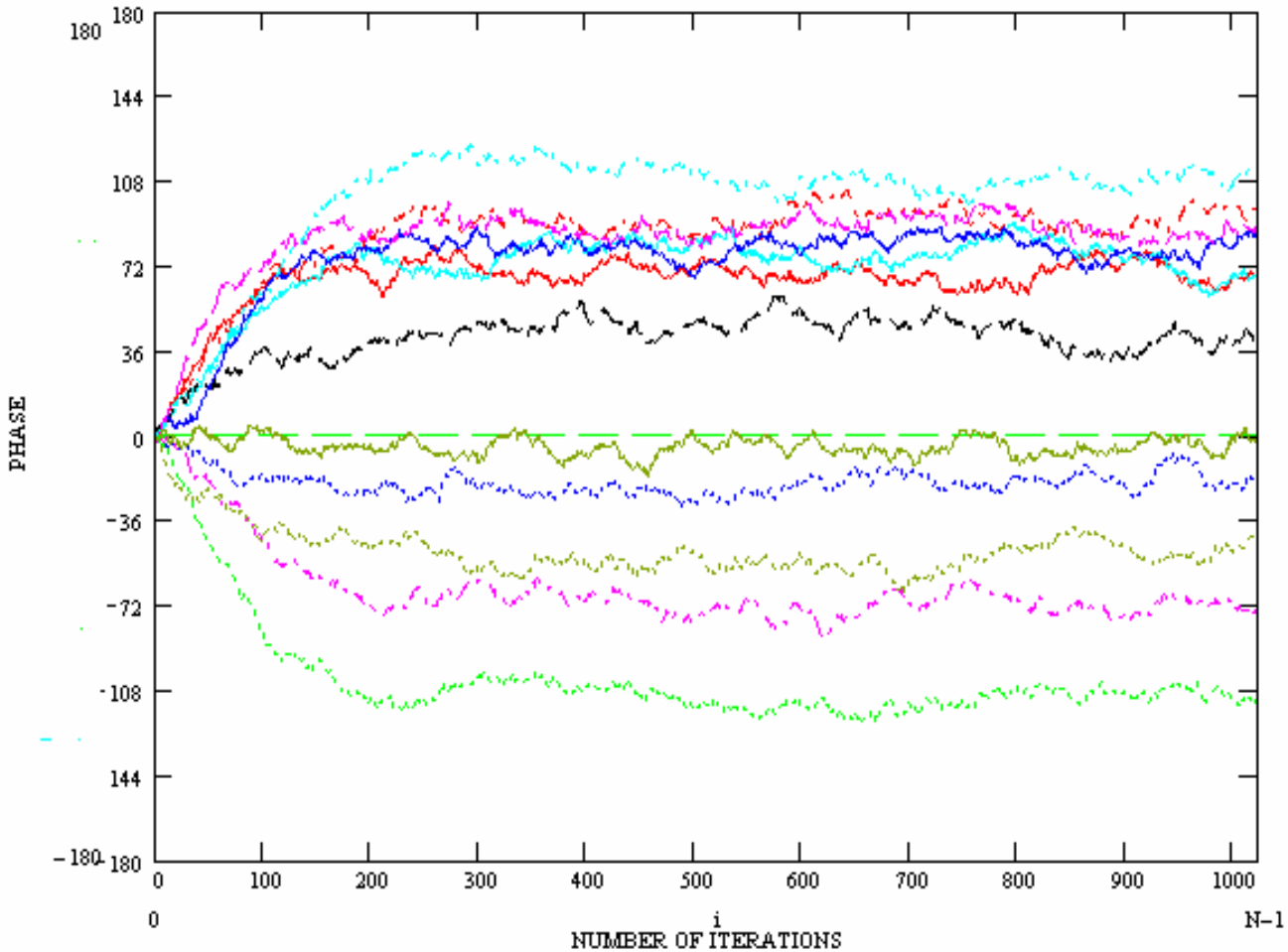
$$TPD_{sig}(\alpha) = \int_0^T \left\{ \sum_{m=1}^M \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{sig})/\lambda + \alpha_m] \right\}^2 dt, \text{ signal response}$$

$$C(\alpha) = \frac{TPD_{sig}(\alpha)}{\sum_{i=1}^N TPD_{RFI_i}(\alpha)}$$

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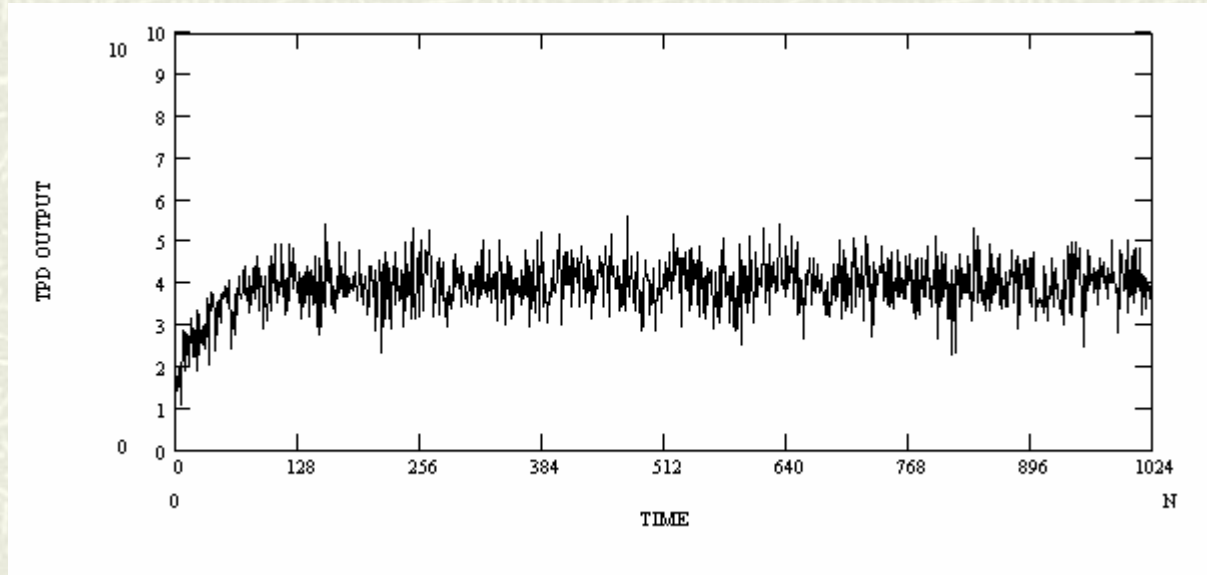


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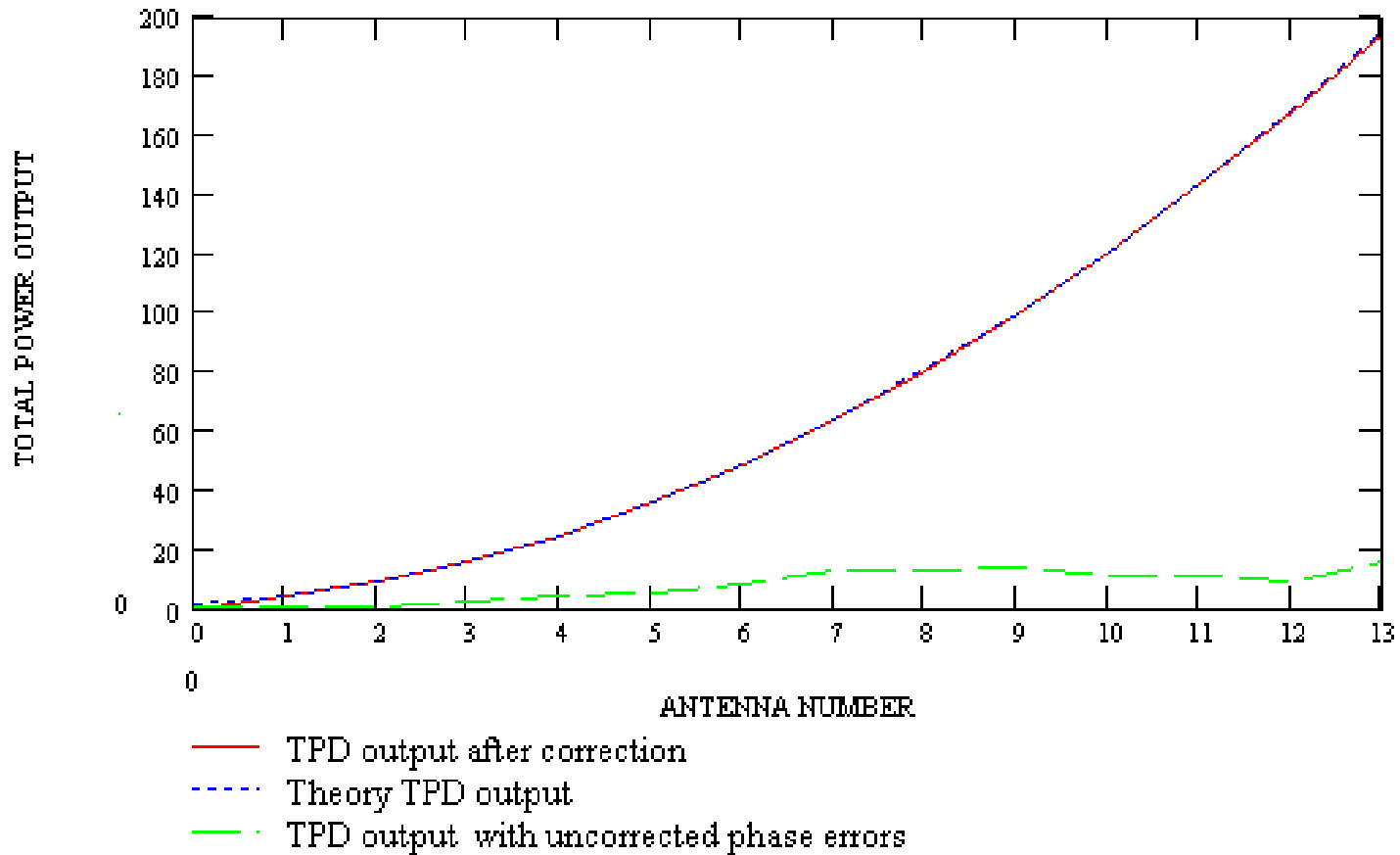
**Process of the phases correction  
each of 14 phases is adjusted  
to compensate  
the initial phase errors.**

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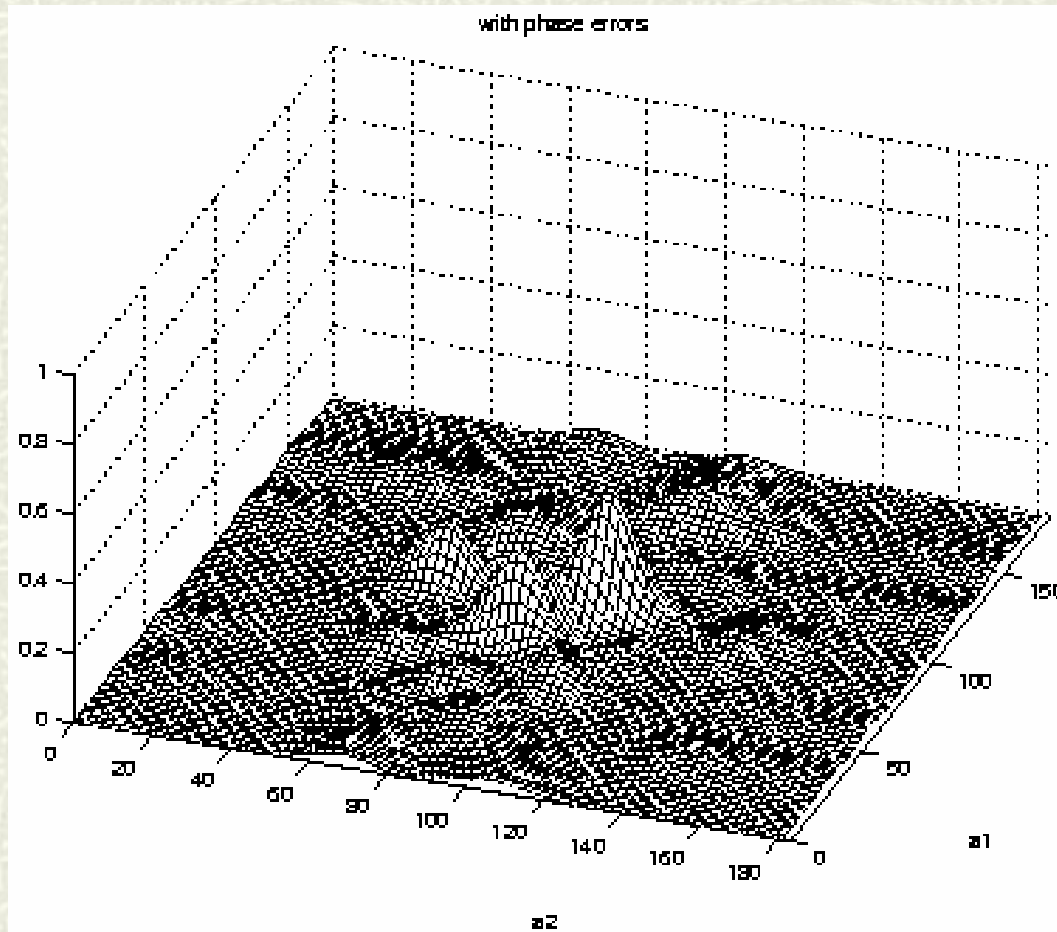
**Total power detector output during the process of the phases correction: each of 14 phases is adjusted to compensate the initial phase errors.**

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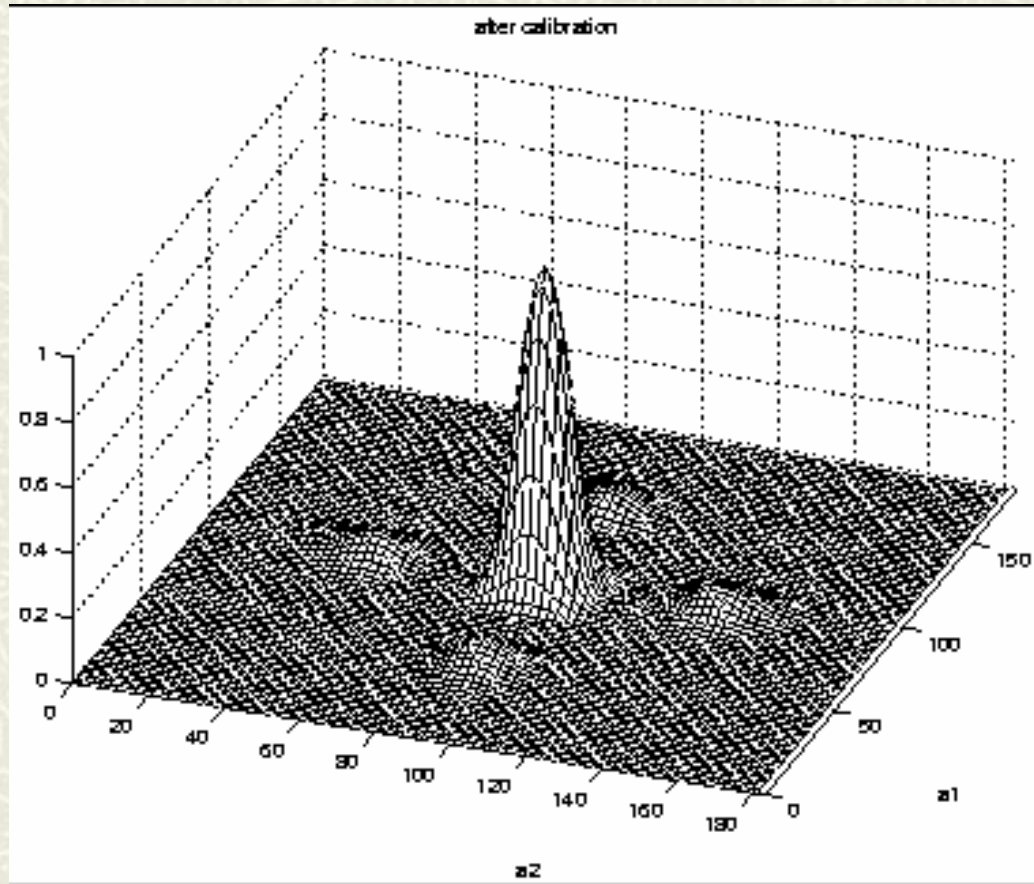


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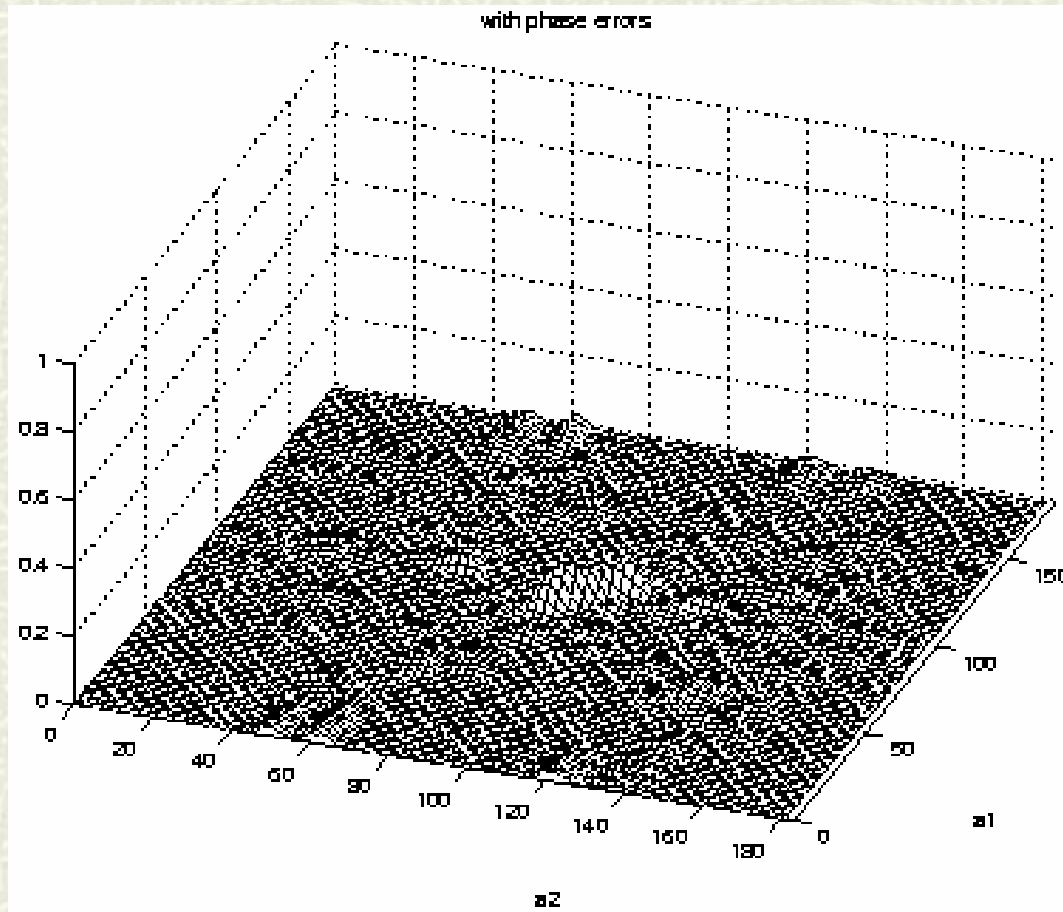
**Array pattern  
with phase errors**

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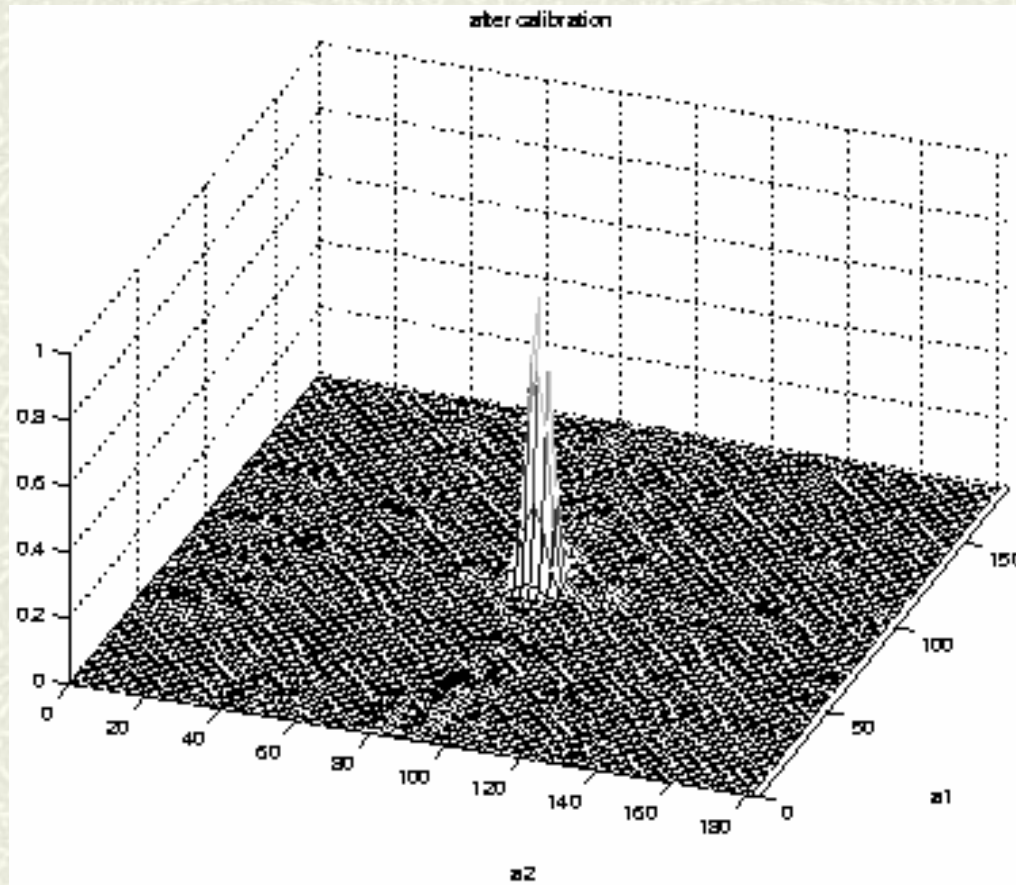
**Array pattern  
after calibration**

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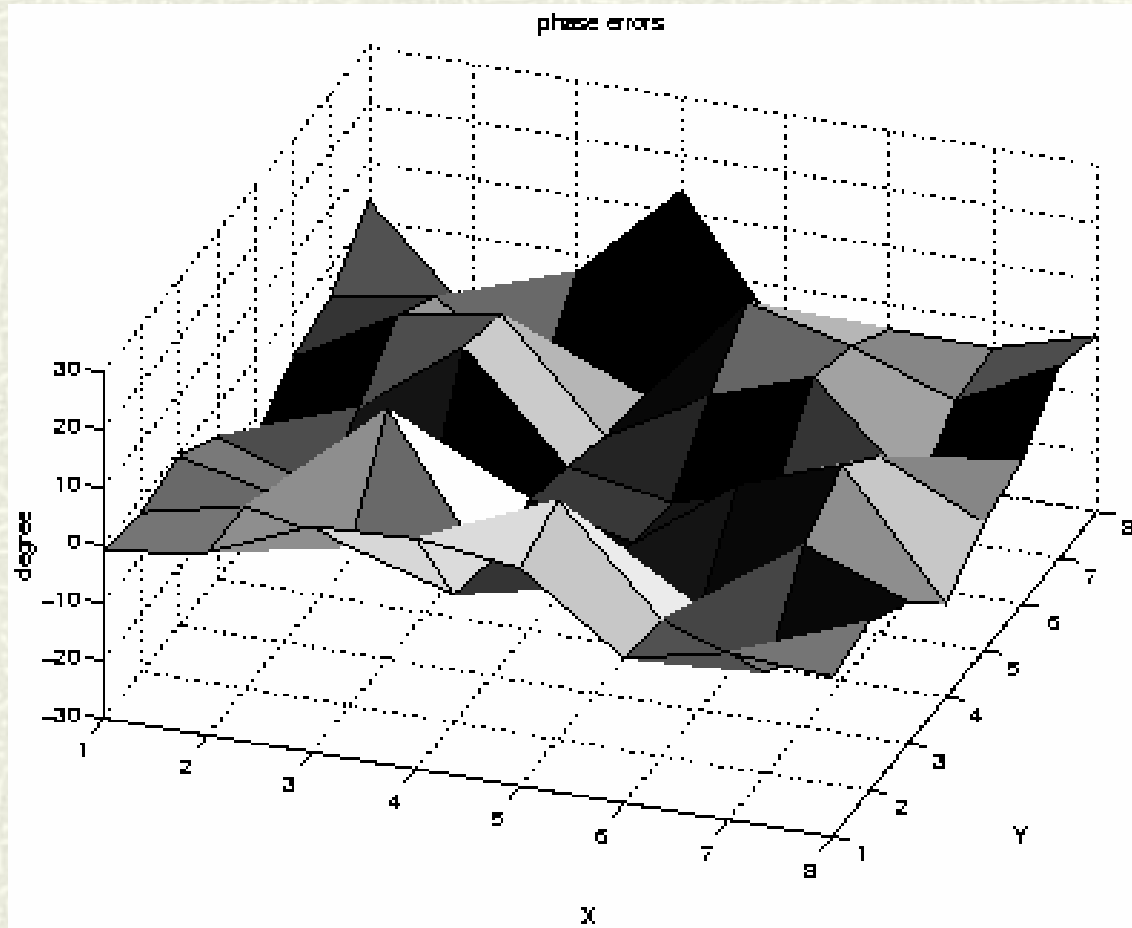
**Array pattern  
with phase errors**

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**Array pattern  
after calibration**

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**Phase errors**

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The complex amplitude of a signal  $x(t) = X(\theta)e^{j2\pi f_0 t}$ , consists of the following components :

$$X(\theta) = S(\theta_0) + \sum_{n=1}^N RFI_n(\theta_n) + N_{\text{sys}}, \quad (1)$$

$$S(\theta_0) = [1, e^{-i\varphi_0}, \dots, e^{-i(M-1)\varphi_0}]^T, \quad (2)$$

where the phase shift  $\varphi_0 = (2\pi d/\lambda) \sin(\theta_0)$ ,  $d$  is the spacing between array's elements,  $\lambda$  is the wavelength. The phase of the first antenna is chosen equal to 0.

The beamformer, in general, consists of the number of complex weights  $w_m e^{i\phi_m}$ ,  $m = 1 \dots M$ , which form the beamformer vector  $W$ :

$$W = [1, w_2 e^{i\phi_1}, \dots, w_M e^{i\phi_{M-1}}]^T. \quad (3)$$

The output of the phased array is

$$Y = W^H X. \quad (4)$$

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The beamformer must satisfy the two requirements :

a) *steering capability*, the SOI is protected ( $W^H S = g$ ), that is for a prescribed direction  $\theta_0$ , the response of the array is maintained constant, no matter what values are assigned to the weights  $W$ . This is a *linearly constrained minimum variance* (LCMV) beamforming (Frost 1972) ;

b) effects of RFI should be minimized .

The *minimum-variance distortionless response* (MVDR) beamforming algorithm, subject to this constraint when  $g = 1$ , was proposed to *minimize the variance* of the beamformer output (Capon 1969). The solution for  $W$  in this case is

$$W_{MDVR} = R^{-1} S(\theta_0) [S(\theta_0)^H R^{-1} S(\theta_0)]^{-1}, \quad (5)$$

where  $R$  is the correlation matrix of  $X$ .

WSRT and other large radio astronomy arrays cannot use this algorithm in real time with the **existing** equipment because there is no amplitude control facilities . That is the "RFI nulling" can be performed with the **phase-only** control.

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Phase-only weights can be found as a solution of the following system of nonlinear equations :

$$\operatorname{Re} \left\{ \sum_{m=1}^M e^{i\phi_m} S_m \right\} = M, \quad (6)$$

$$\operatorname{Im} \left\{ \sum_{m=1}^M e^{i\phi_m} S_m \right\} = 0,$$

$$\operatorname{Re} \left\{ \sum_{m=1}^M e^{i\phi_m} RFI_m \right\} = 0,$$

$$\operatorname{Im} \left\{ \sum_{m=1}^M e^{i\phi_m} RFI_m \right\} = 0.$$



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The array total power detector (TPD) output is

$$TPD_{TA} = \left\langle \int_0^T [x(t)]^2 dt \right\rangle = TPD_{sig} + TPD_{RFI} + TPD_N, \quad (7)$$

The mean value of the signal component is

$$TPD_{sig}(\phi_m) = \left\langle \int_0^T \left\{ \sum_{m=1}^M \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_0)/\lambda + \phi_m]^2 dt \right\} \right\rangle, \quad (8)$$

the mean value of the n-th RFI component is

$$TPD_{RFI_n}(\phi_m) = \left\langle \int_0^T \left\{ \sum_{m=1}^M A_{RFI_n} \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{RFI_n})/\lambda + \phi_m]^2 dt \right\} \right\rangle, \quad (9)$$

Considering the TPD output as a function of  $M$  variables  $\phi_m$ , the following criterium for a "good" vector  $\phi = [\phi_1 \dots \phi_M]^T$  can be proposed:

$$C(\phi) = \frac{TPD_{sig}(\phi)}{\sum_{n=1}^N TPD_{RFI_n}(\phi) + TPD_N} \rightarrow \max. \quad (10)$$

## Genetic algorithm search

Fitness function (calibration) - total power detector output:

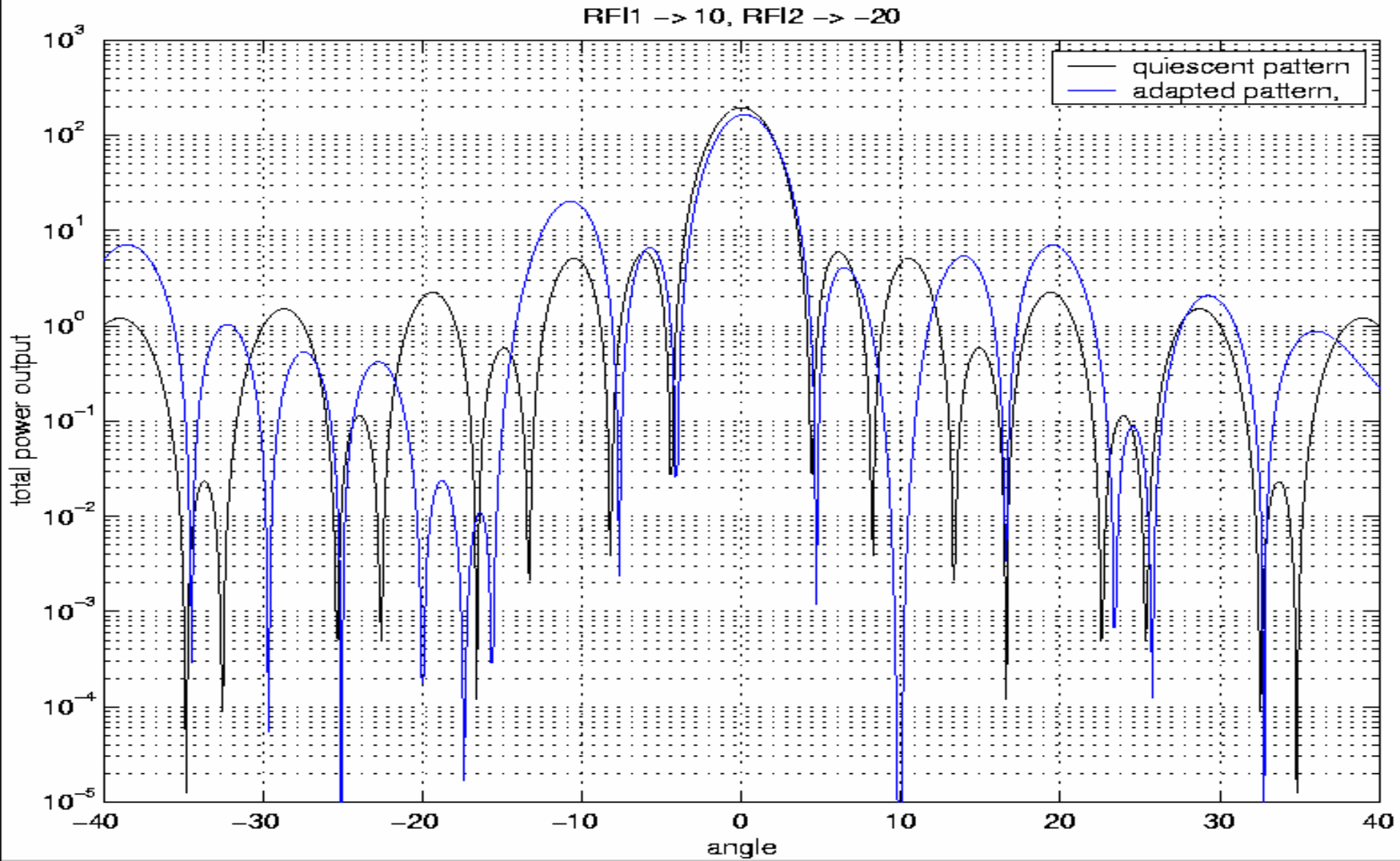
$$C(\alpha) = \int_0^T \left\{ \sum_{m=1}^M \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{sig})/\lambda + \alpha_m + \beta_m] \right\}^2 dt$$

Fitness function (RFI mitigation) - total power detector output:

$$TPD_{RFI_j}(\alpha) = \int_0^T \left\{ \sum_{m=1}^M \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{RFI_j})/\lambda + \alpha_m] \right\}^2 dt, \text{ RFI response}$$

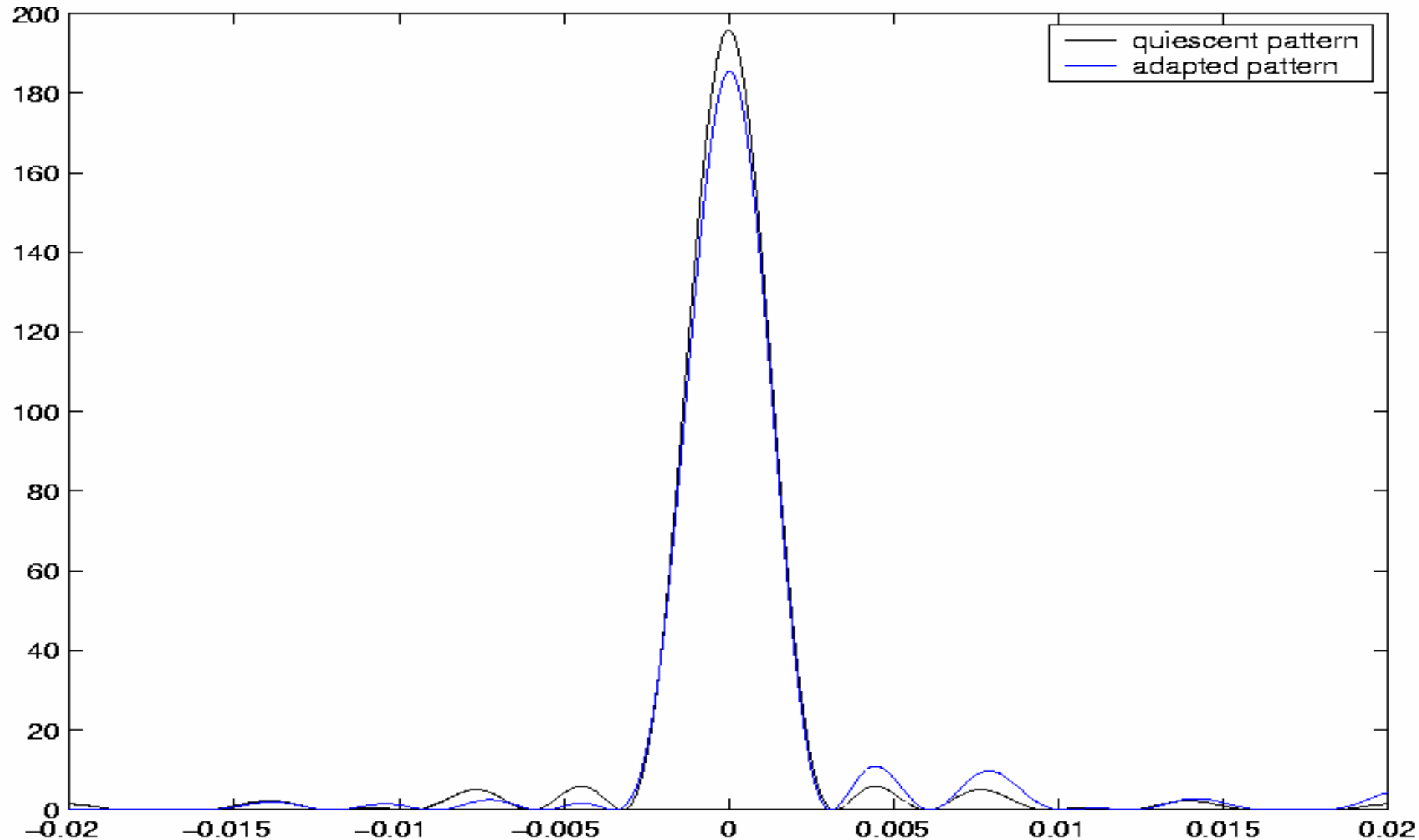
$$TPD_{sig}(\alpha) = \int_0^T \left\{ \sum_{m=1}^M \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{sig})/\lambda + \alpha_m] \right\}^2 dt, \text{ signal response}$$

$$C(\alpha) = \frac{TPD_{sig}(\alpha)}{\sum_{i=1}^N TPD_{RFI_i}(\alpha)}$$

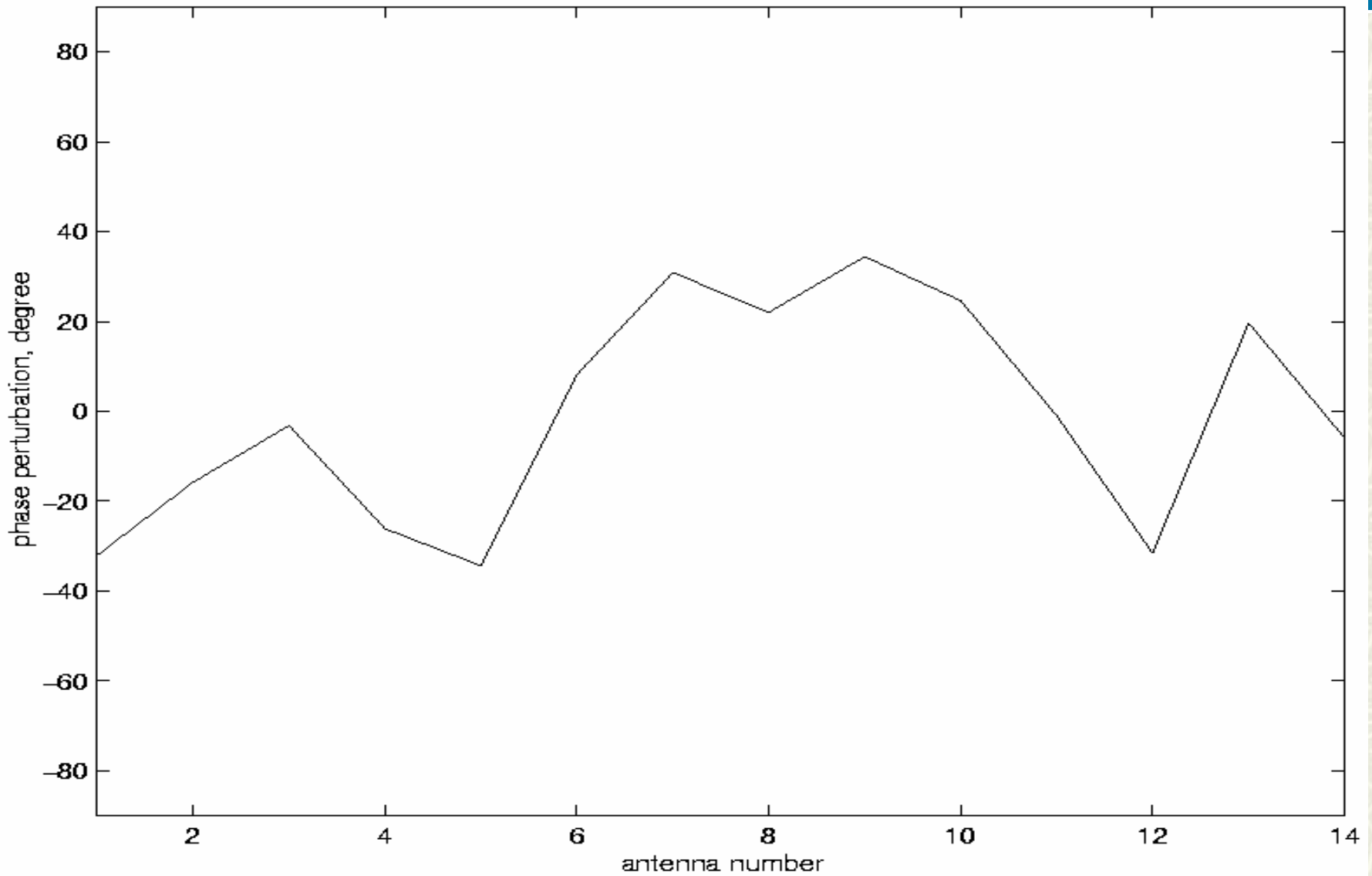


**Quiescent (red line) and adapted (blue line) 14-element half-wavelength array pattern, logarithmic scale; two nulls in the adapted pattern at  $-20\text{dg}$  and  $+10\text{dg}$ .**

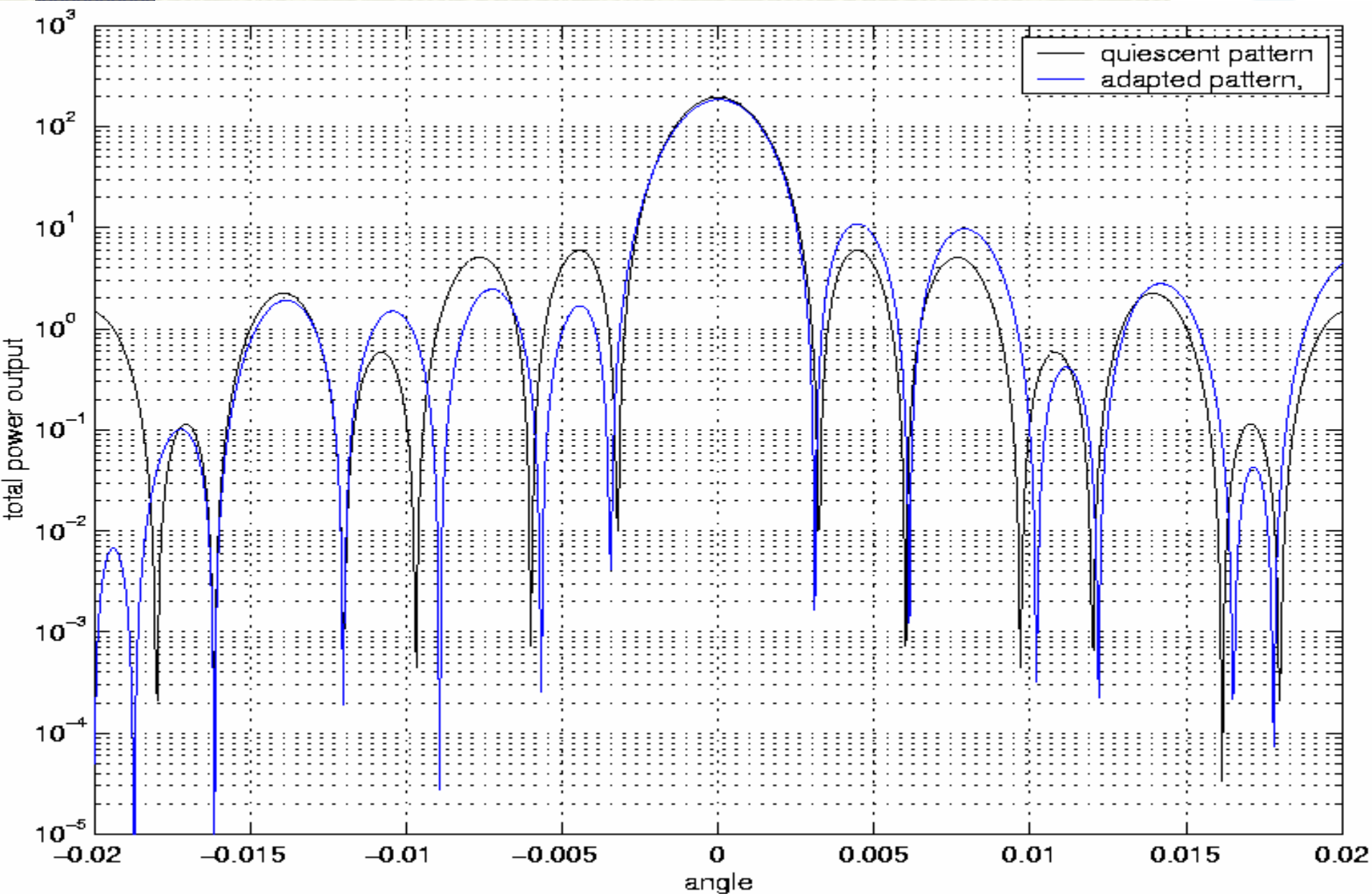
desired direction



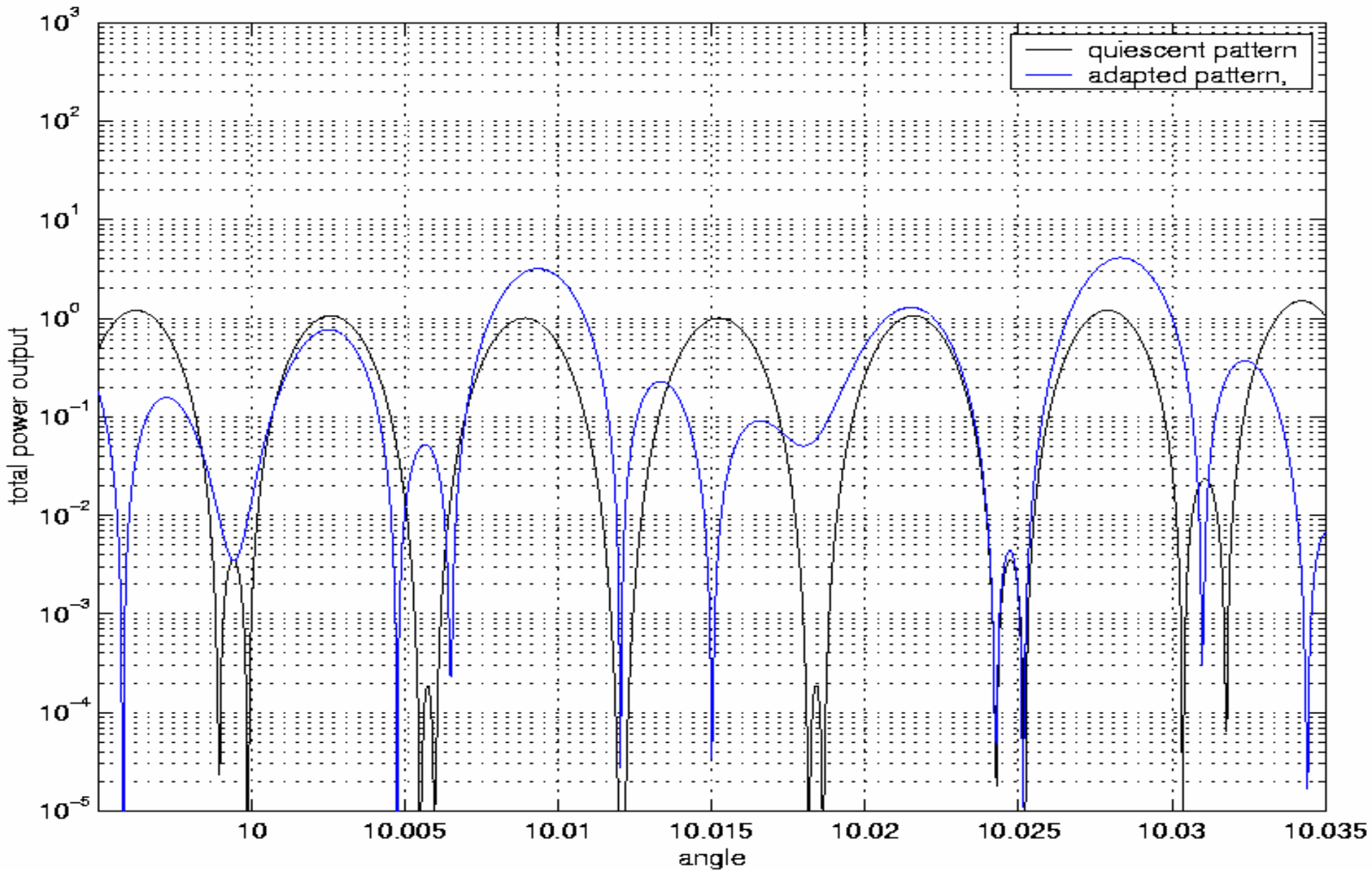
**Quiescent (red line) and adapted (blue line) 14-element half-wavelength array pattern, linear scale**



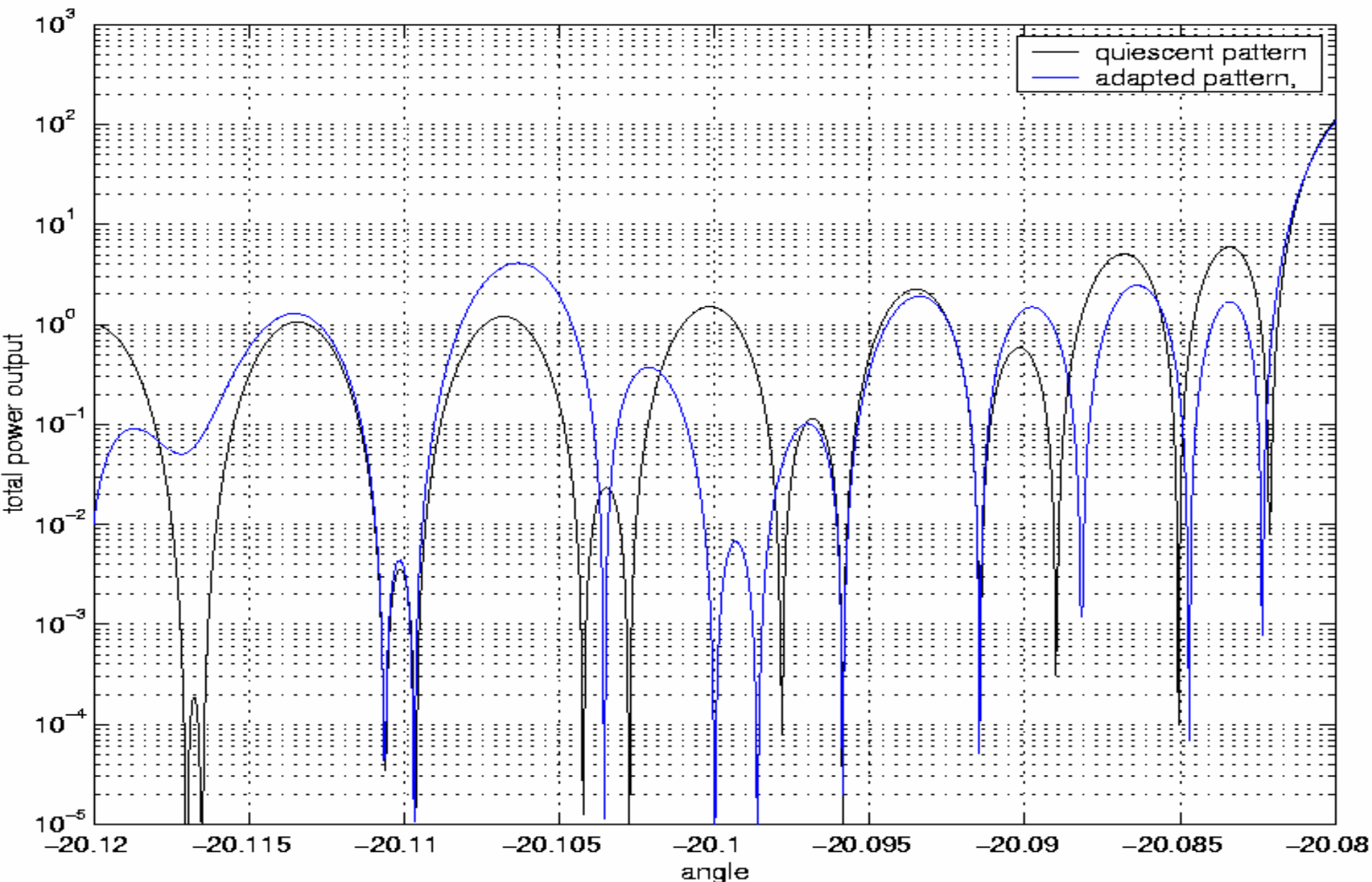
**Phase perturbations corresponding to the adapted array pattern.**



Quiescent (red line) and adapted (blue line) 14-element array pattern, spacing=144m, central frequency=1420MHz, main beam, logarithmic scale.

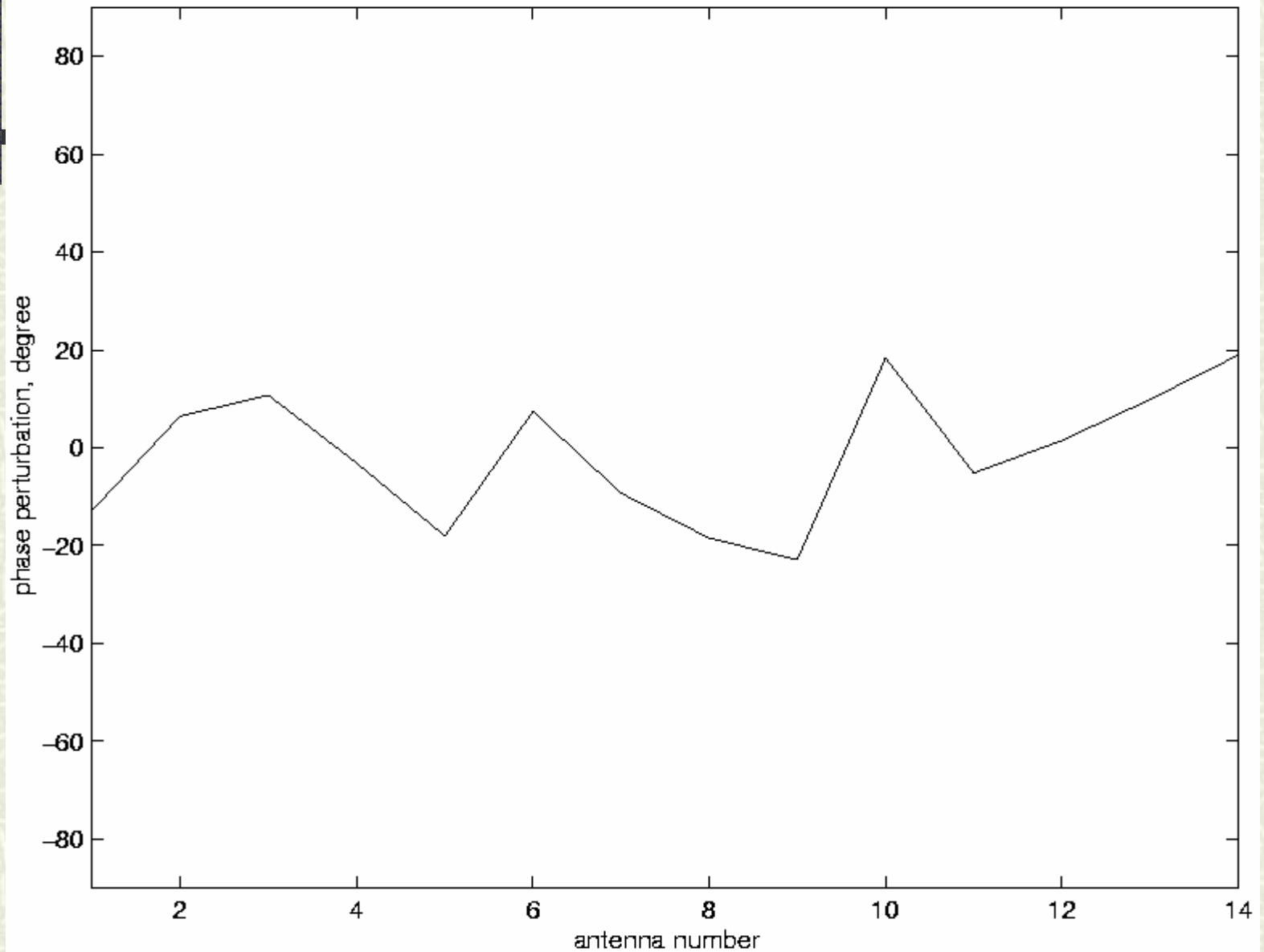


**Quiescent (red line) and adapted (blue line) 14-element array pattern, spacing=144m, central frequency=1420MHz, direction +10.015dg, logarithmic scale.**

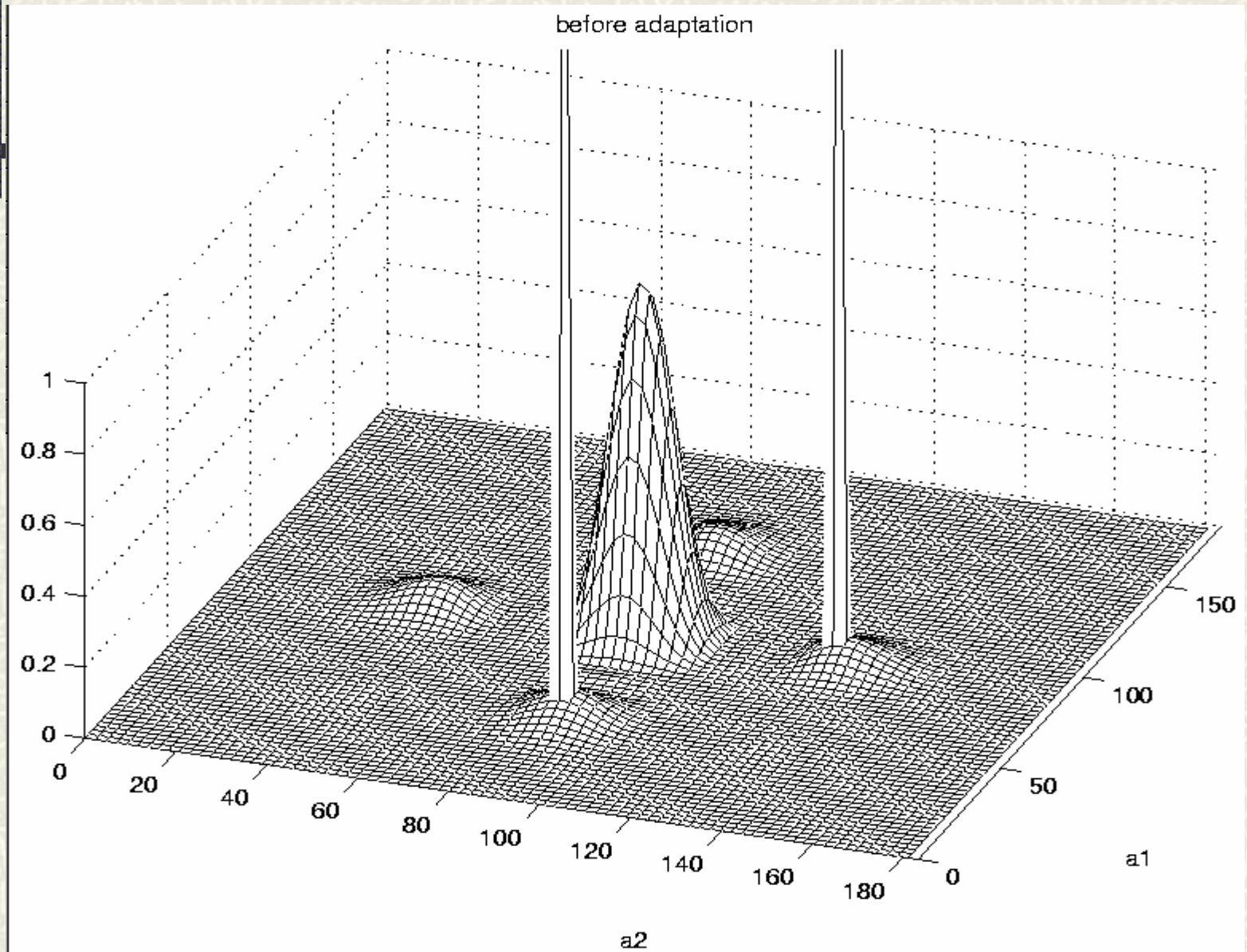


Quiescent (red line) and adapted (blue line) 14-element array pattern, spacing=144m, central frequency=1420MHz, direction -20.1dg, logarithmic scale.

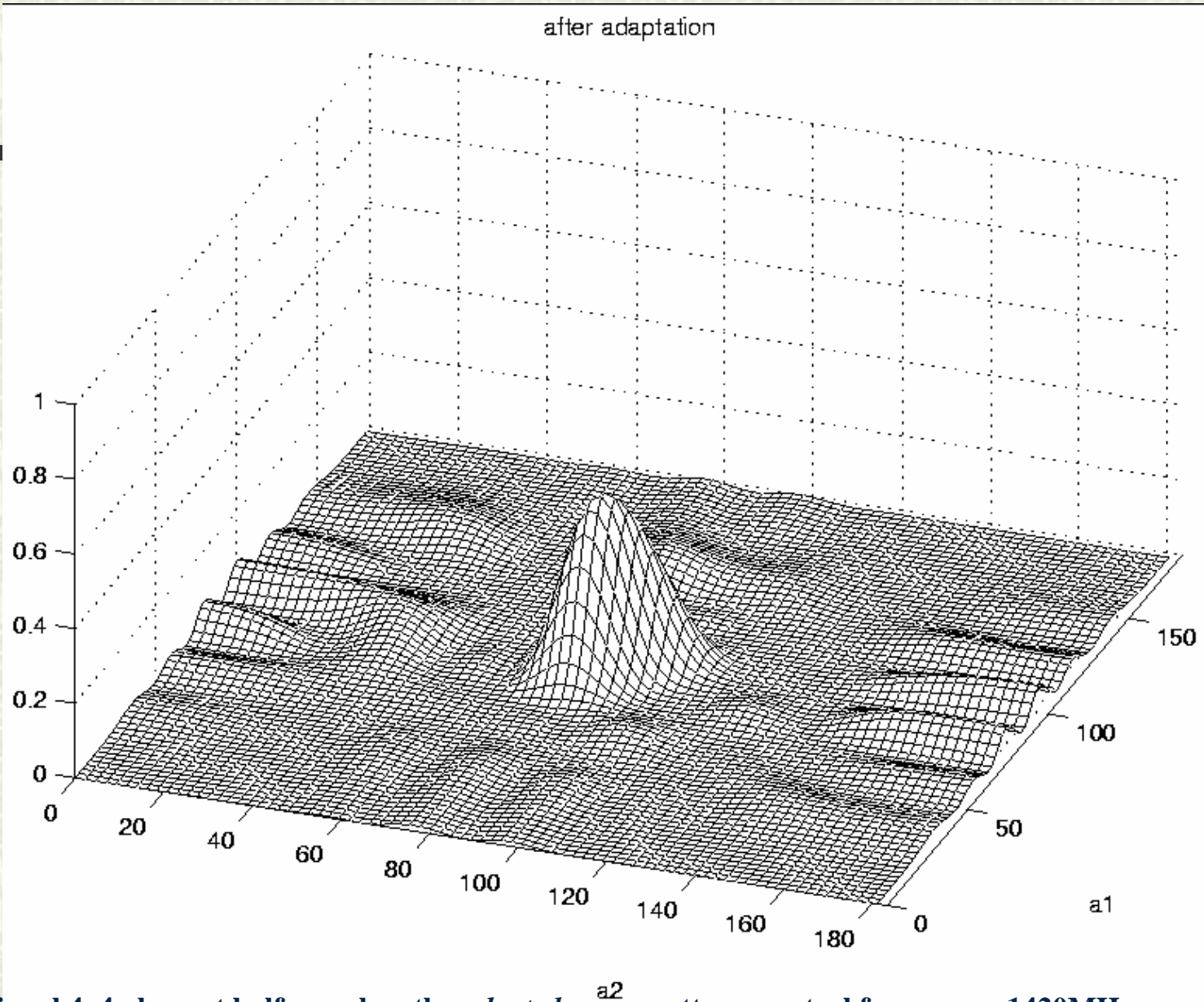




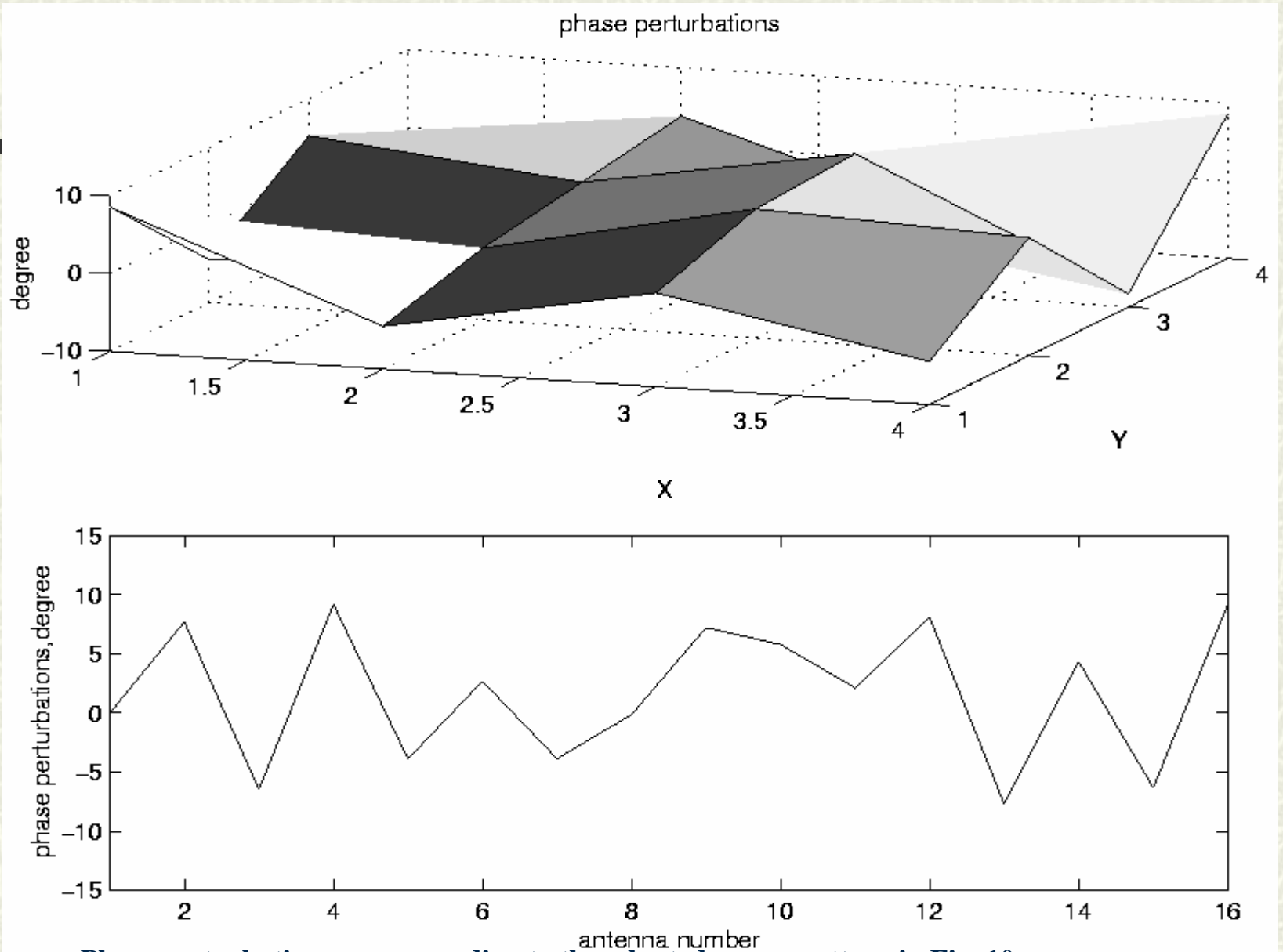
**Phase perturbations corresponding to the adapted array pattern.**



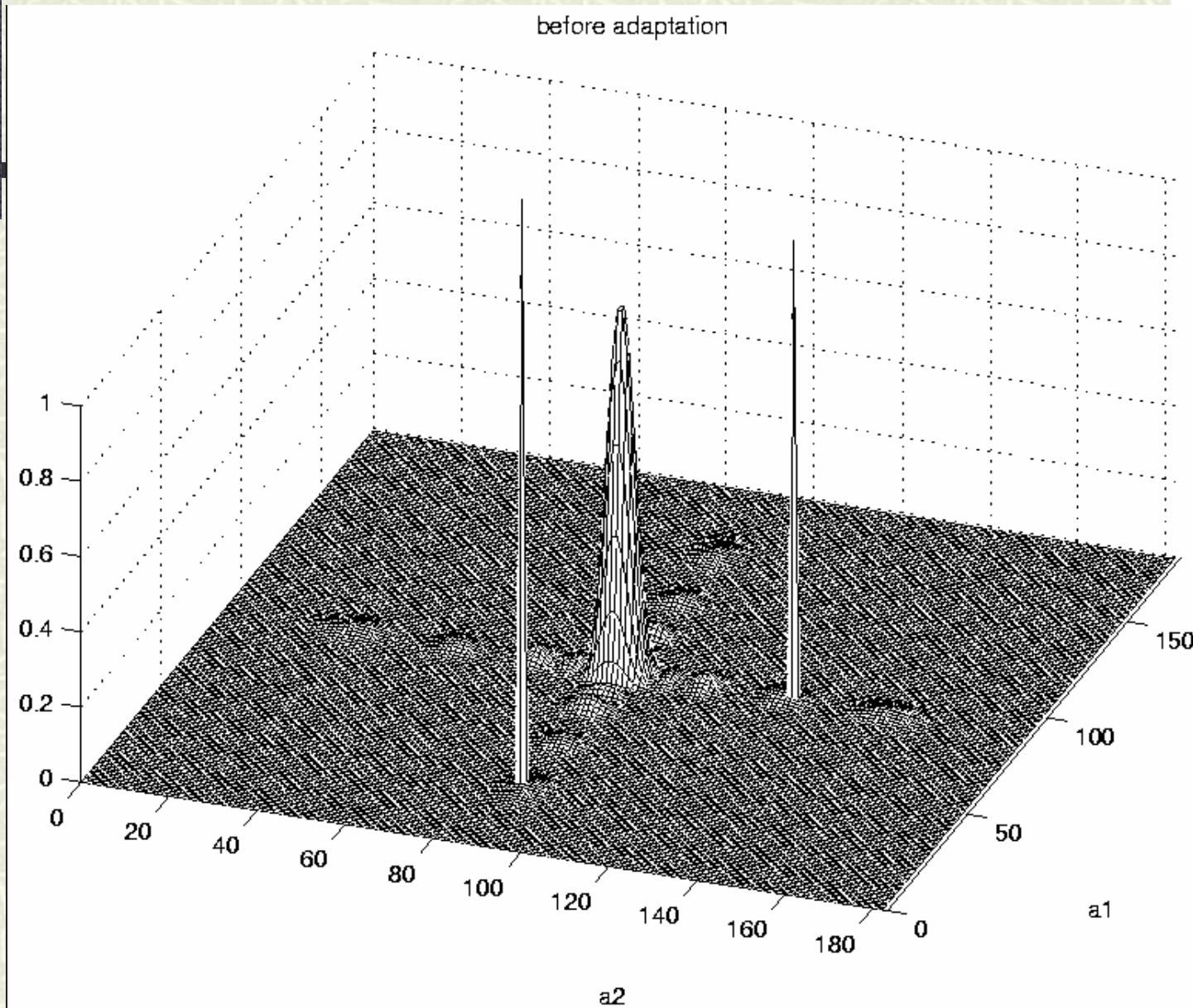
Two-dimensional 4x4-element half-wavelength, *quiescent* array pattern, central frequency=1420MHz.  
RFI-1 at (45, 90)dg, RFI-2 at (90, 135) dg, linear scale.



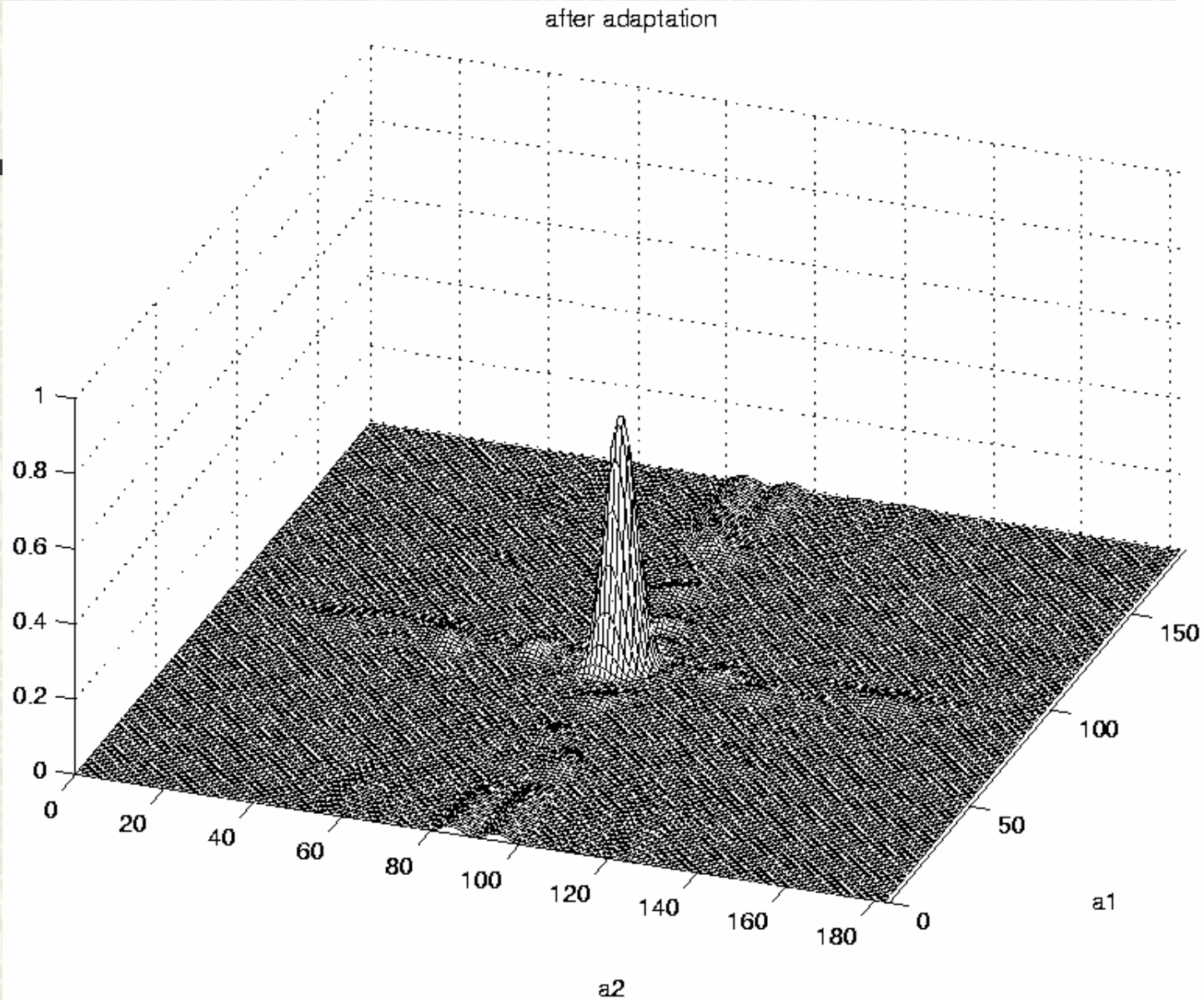
Two-dimensional 4x4-element half-wavelength, *adapted* array pattern, central frequency=1420MHz, RFI-1 at (45, 90) dg, RFI-2 at (90, 135) dg, linear scale. RFI-1 suppression=76.6dB, RFI-2 suppression=74dB.



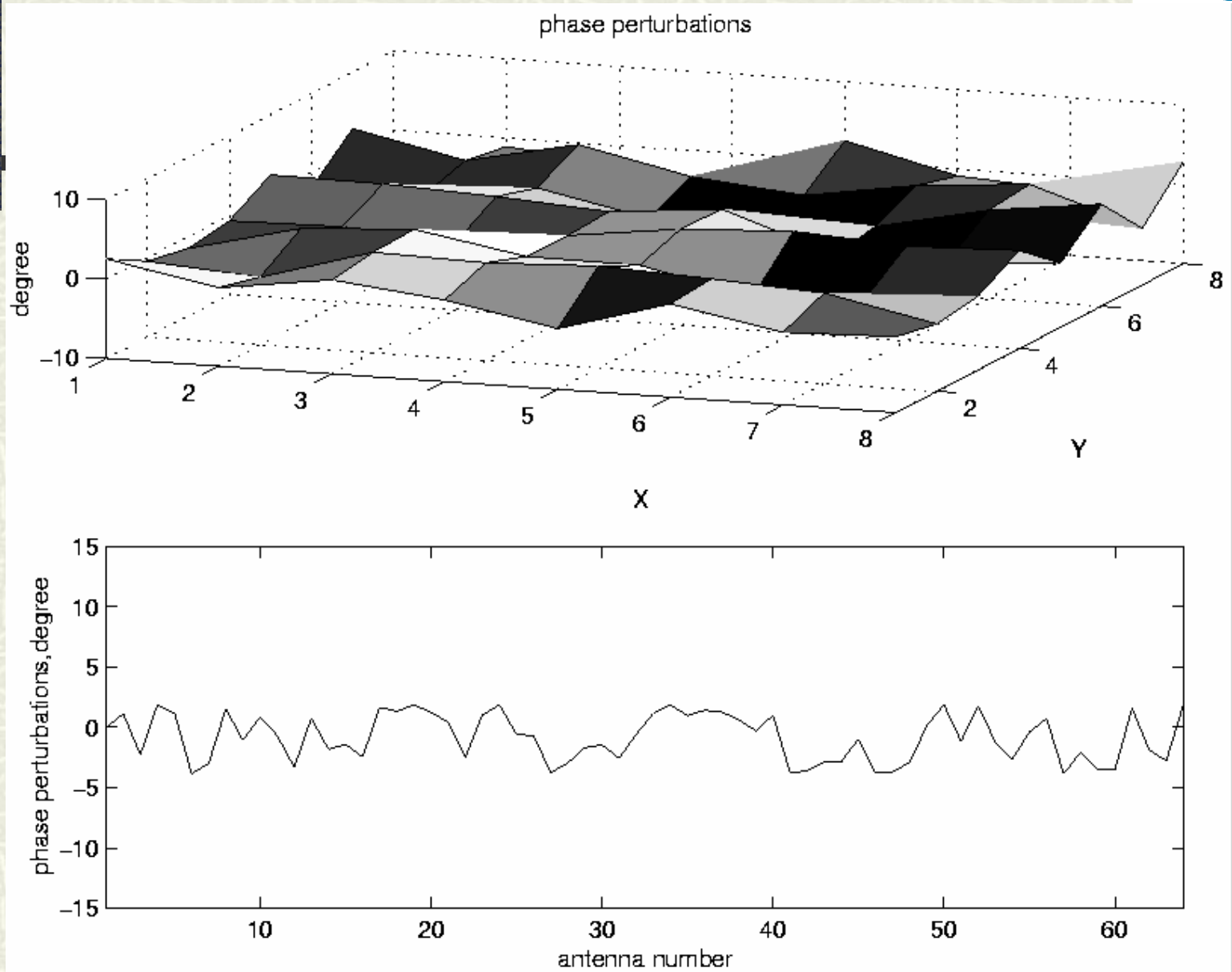
**Phase perturbations corresponding to the adapted array pattern in Fig. 10:  
a) 3D-presentation of the phase surface; b) phase distribution in linear order.**



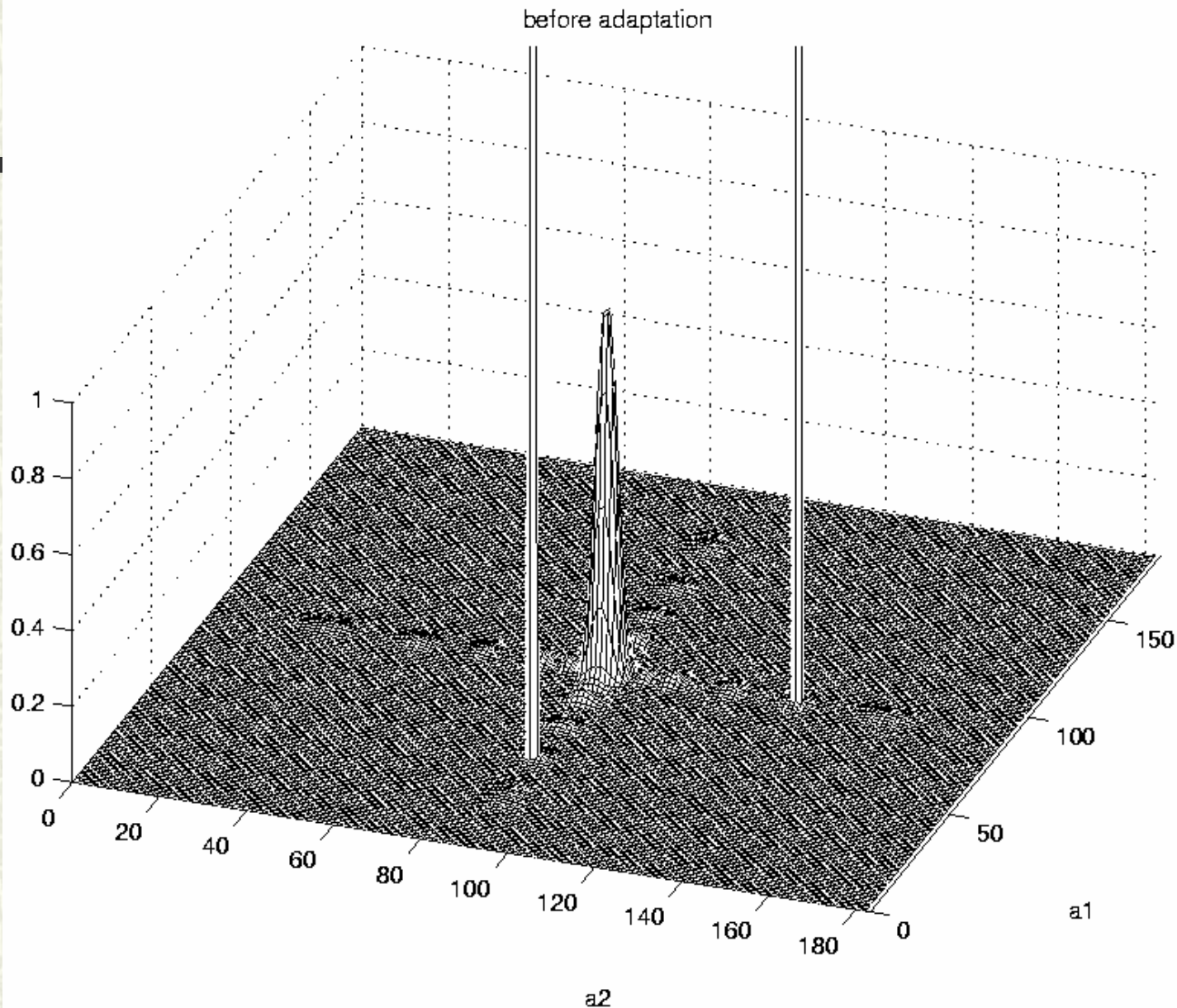
Two-dimensional 8x8-element half-wavelength, *quiescent* array pattern, central frequency=1420MHz.  
RFI-1 at (30, 90) dg, RFI-2 at (90, 130) dg, linear scale.



Two-dimensional 8x8-element half-wavelength, *adapted* array pattern, central frequency=1420MHz  
RFI-1 at (30, 90) dg, RFI-2 at (90, 130) dg, linear scale; RFI-1 suppression=98.5dB, RFI-2 suppression=96.6dB.

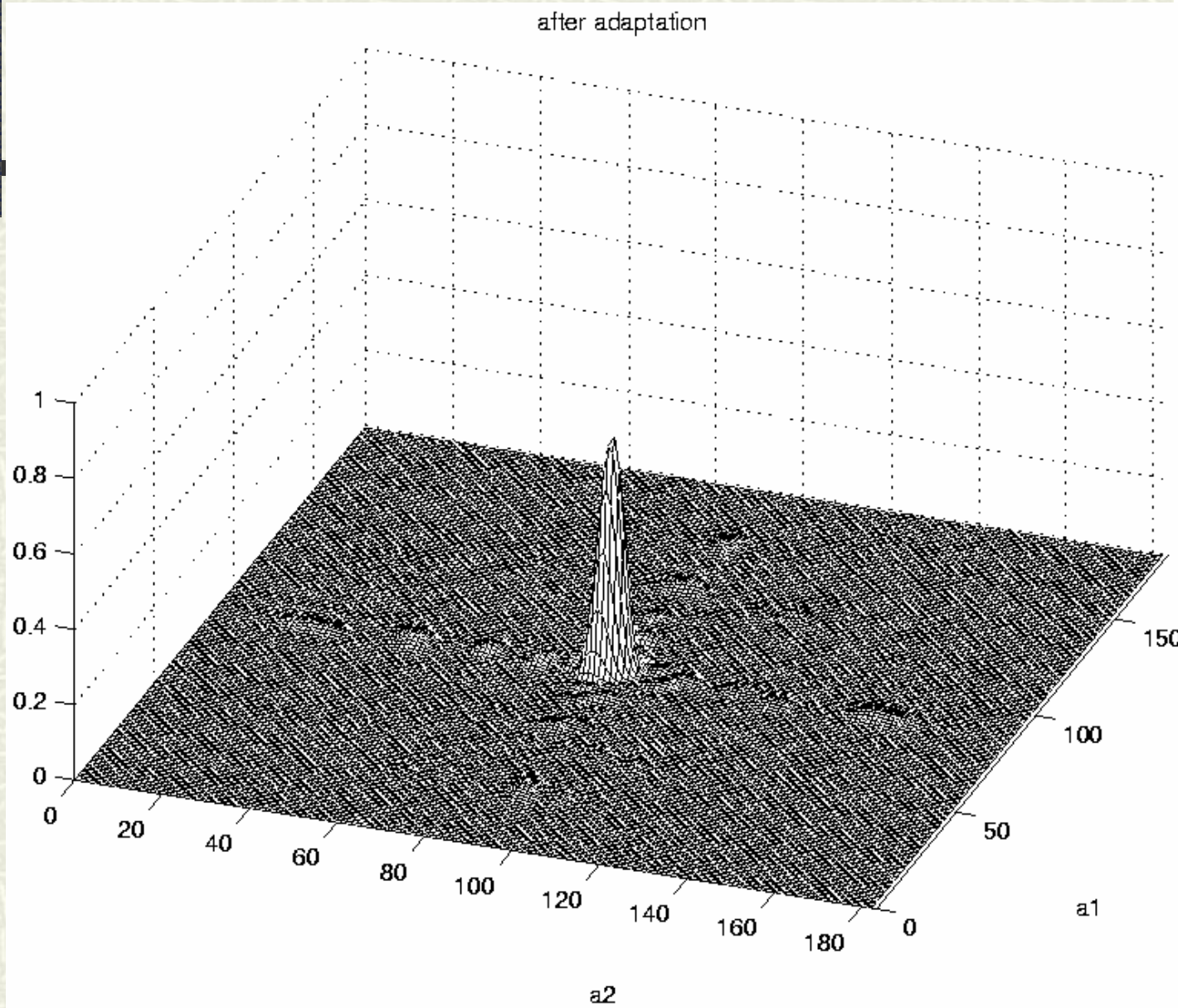


**Phase perturbations corresponding to the adapted array pattern :**  
a) 3D-presentation of the phase surface; b) phase distribution in linear order.

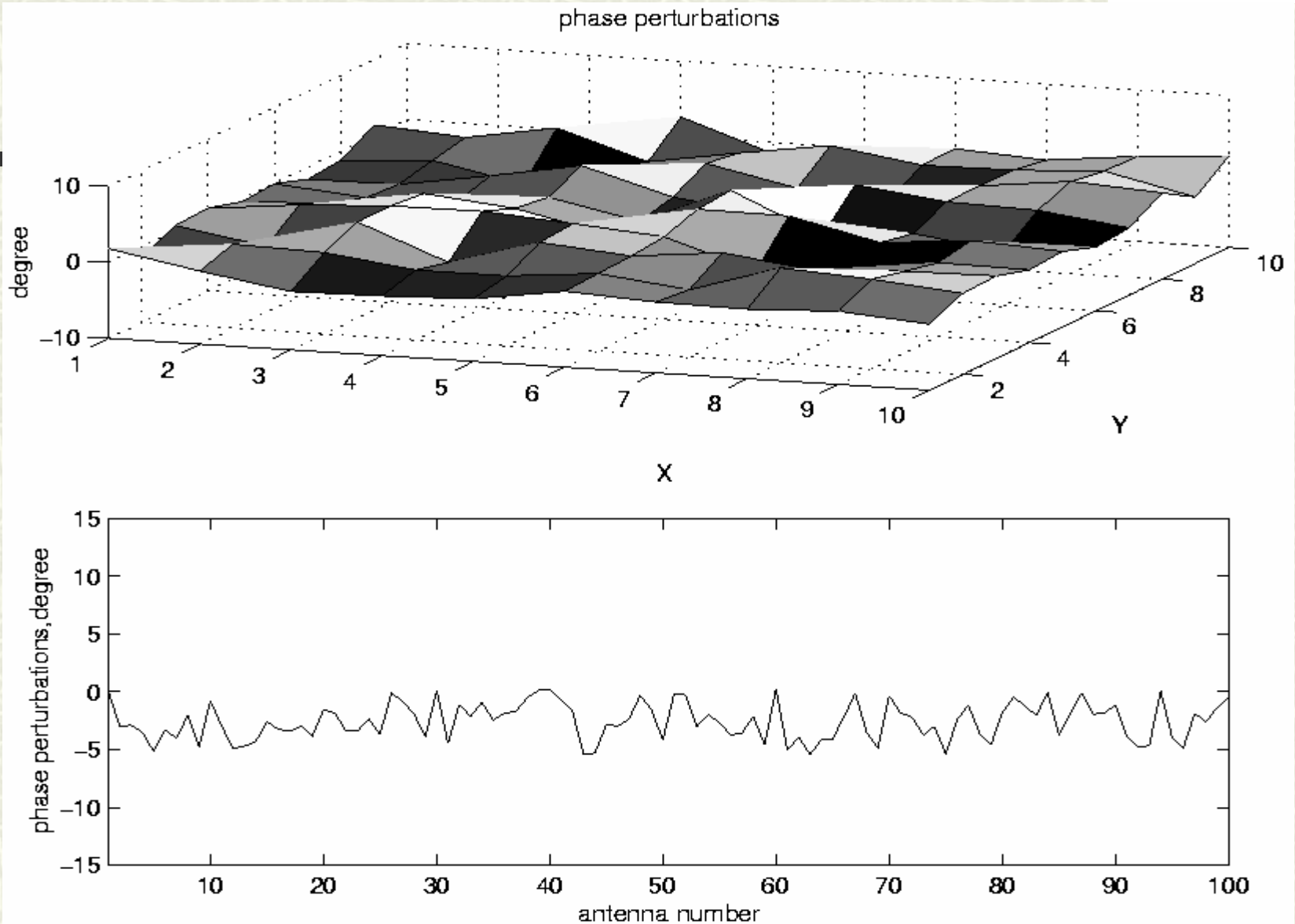


**Two-dimensional 10x10-element half-wavelength, quiescent array pattern, central frequency=1420MHz, RFI-1 at (45, 90) dg, RFI-2 at (90, 135) dg, linear scale.**





Two-dimensional 10x10-element half-wavelength, *adapted* array pattern, central frequency=1420MHz, RFI-1 at (45, 90) dg., RFI-2 at (90, 135) dg, linear scale; RFI-1 suppression=106.2dB, RFI-2 suppression=103.1dB.



Phase perturbations corresponding to the adapted array pattern in Fig. 16:  
a) 3D-presentation of the phase surface; b) phase distribution in linear order.