RFI mitigation with the phase-only adaptive beamforming

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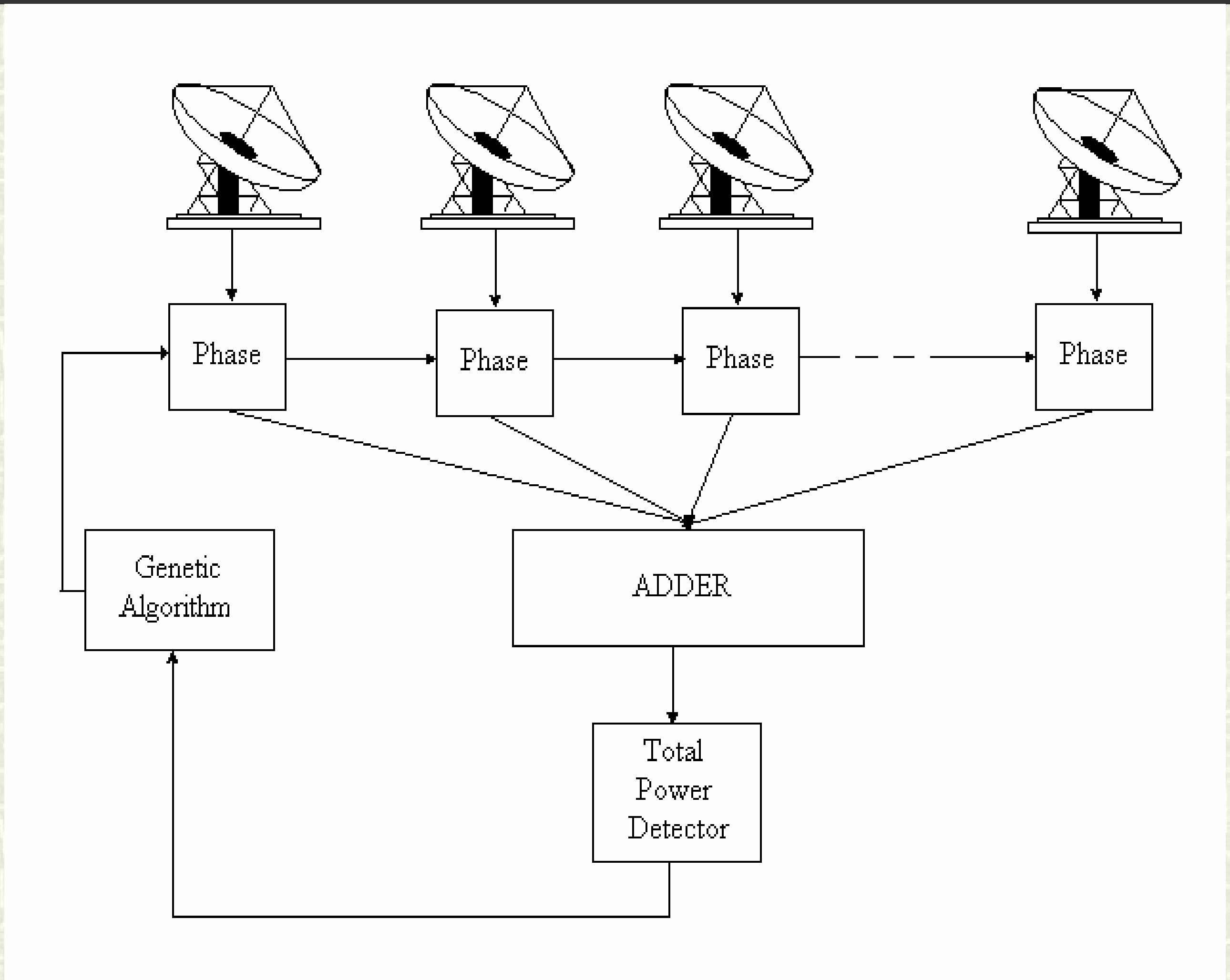
ASTRON



Adaptive phase control

Stochastic gradient search

Fitness function (calibration) - total power detector output:

$$C(\alpha) = -\int_0^T \left\{ \sum_{m=1}^{2} \cos[2\pi f_0 t + 2\pi(m-1)d\sin(\theta_{sig})/\lambda + \alpha_m + \beta_m] \right\}^2 dt$$

Algorithm :

$$\alpha_n = \alpha_{n-1} + \mu \bullet \tilde{\nabla}_{\alpha^\mp} C(\alpha)$$

$$C^+(\alpha, d\alpha) = C^+(\alpha + d\alpha),$$

$$C^-(\alpha, d\alpha) = C^+(\alpha - d\alpha),$$

$$\tilde{\nabla}_{\alpha^\mp} = \frac{C^-(\alpha, d\alpha) - C^+(\alpha, d\alpha)}{2d\alpha}$$

Genetic algorithm search

Fitness function (calibration) - total power detector output:

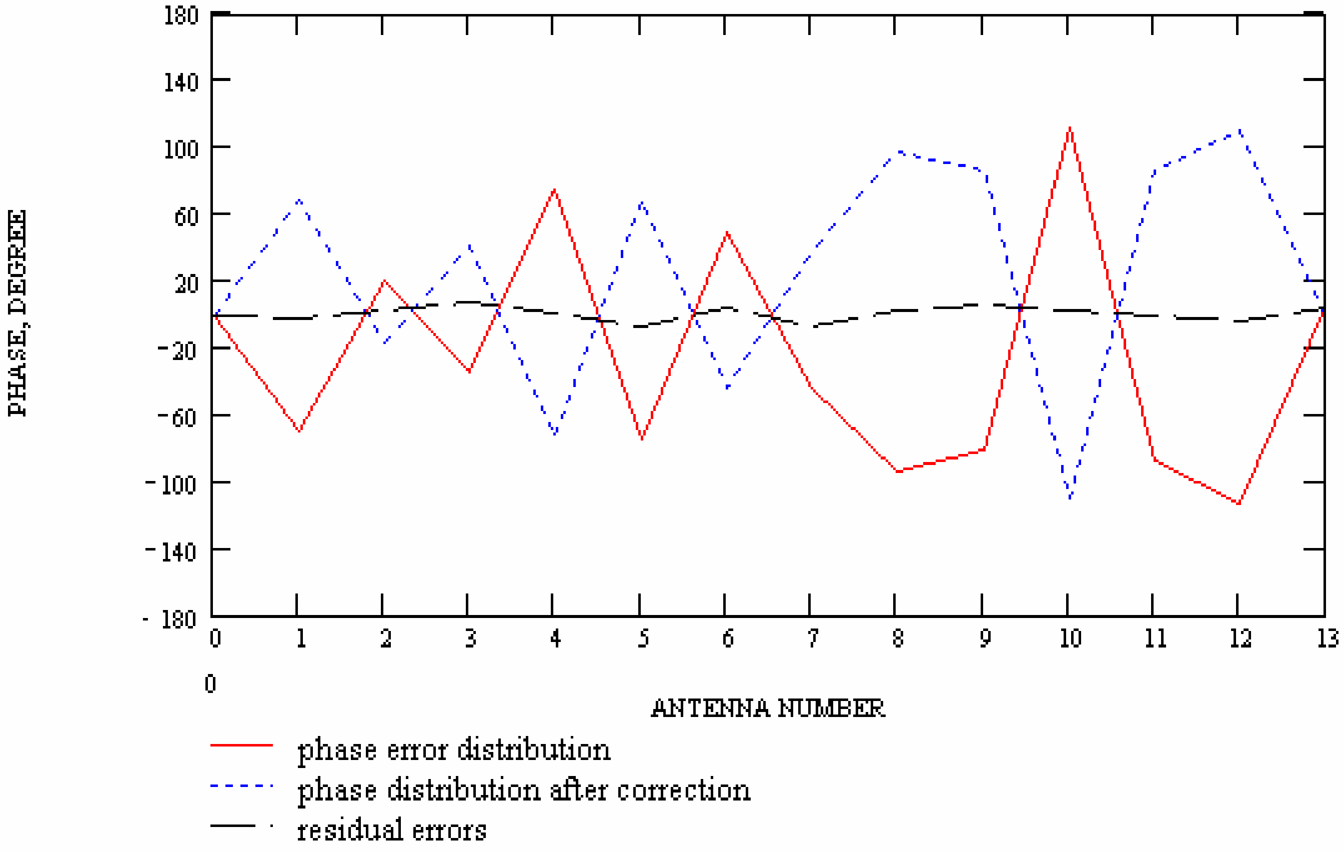
$$C(\alpha) = \int_0^T \left\{ \sum_{m=1}^{M} \cos\left[2\pi f_0 t + 2\pi(m-1)d\sin(\theta_{sig})/\lambda + \alpha_m + \beta_m\right] \right\}^2 dt$$

Fitness function (RFI mitigation) - total power detector output:

$$TPD_{RFI_i}(\alpha) = \int_0^T \left\{ \sum_{m=1}^{M} \cos\left[2\pi f_0 t + 2\pi(m-1)d\sin(\theta_{RFI_i})/\lambda + \alpha_m\right] \right\}^2 dt, \quad \text{RFI response}$$

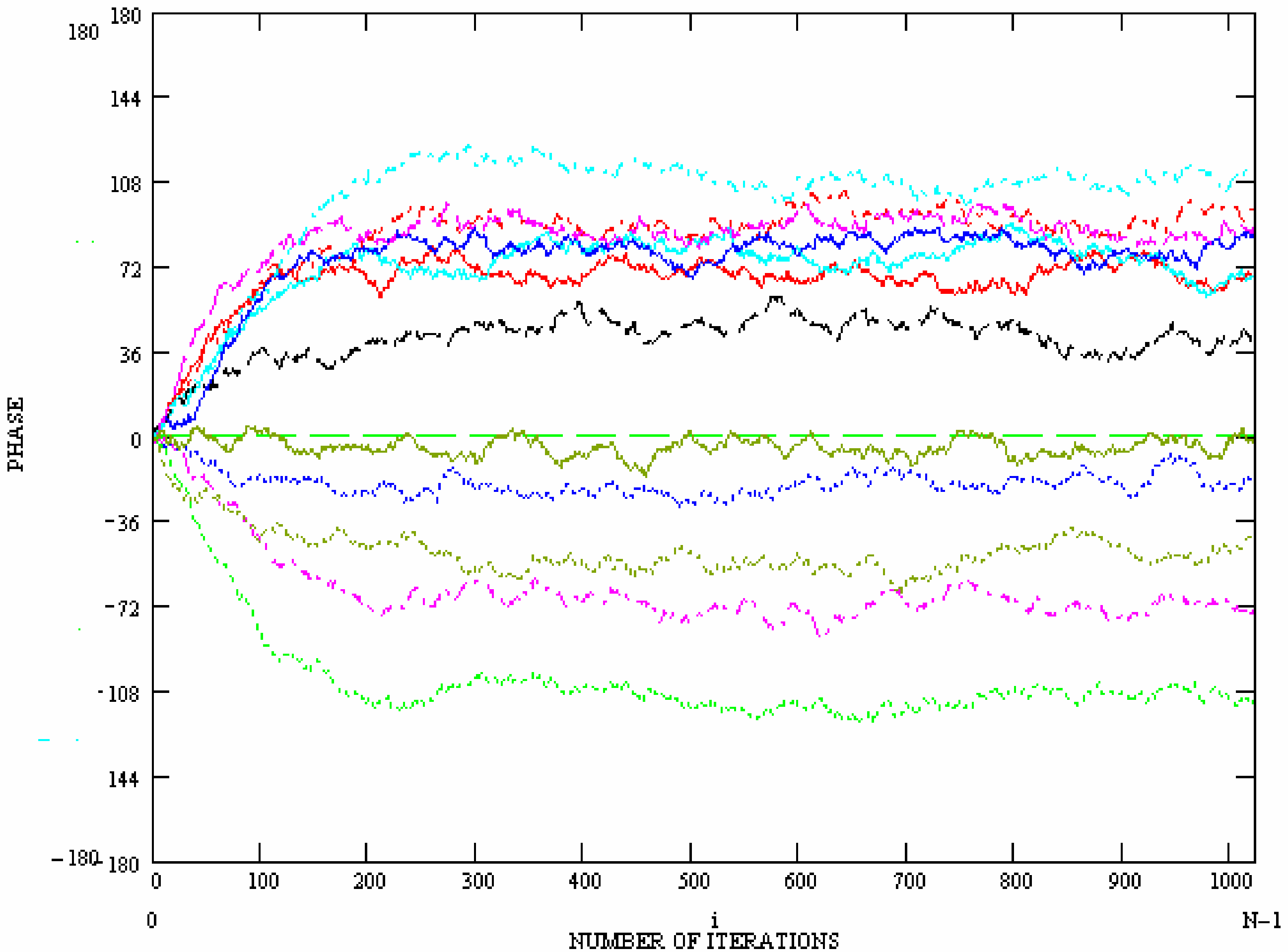
$$TPD_{sig}(\alpha) = \int_0^T \left\{ \sum_{m=1}^{M} \cos\left[2\pi f_0 t + 2\pi(m-1)d\sin(\theta_{sig})/\lambda + \alpha_m\right] \right\}^2 dt, \quad \text{signal response}$$

$$C(\alpha) = \frac{TPD_{sig}(\alpha)}{\sum_{i=1}^{N} TPD_{RFI_i}(\alpha)}$$



180
140
100
60
20
-20
-60
-100
-140
-180
0 1 2 3 4 5 6 7 8 9 10 11 12 13
0
PHASE, DEGREE
ANTENNA NUMBER
phase error distribution
phase distribution after correction
residual errors

RFI mitigation workshop, Penticton, July 2004



Process of the phases correction: each of 14 phases is adjusted to compensate the initial phase errors.

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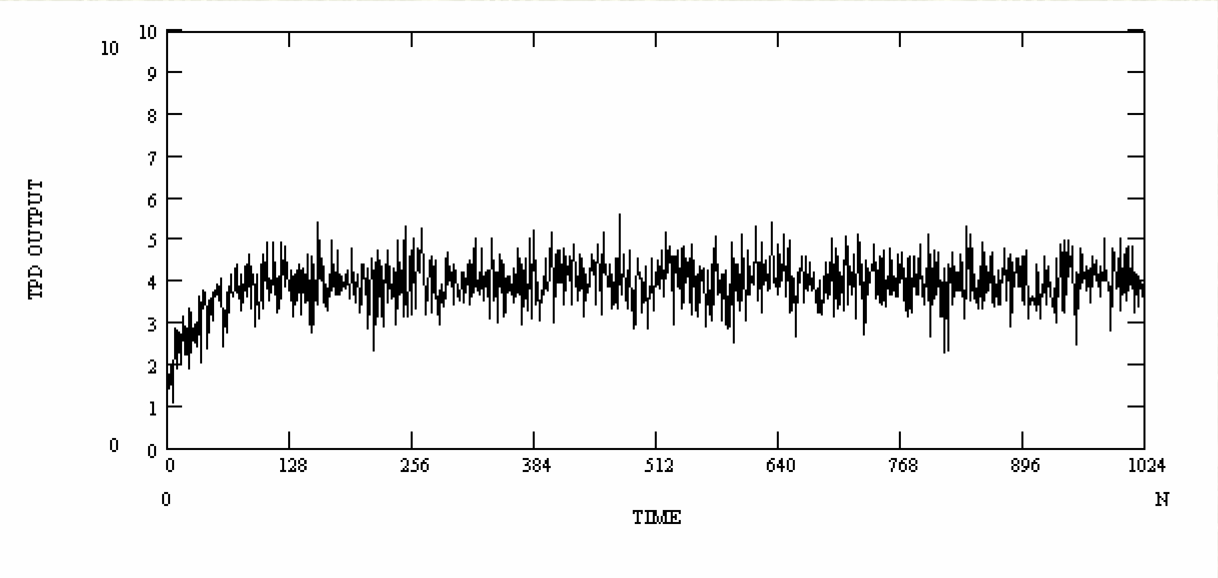
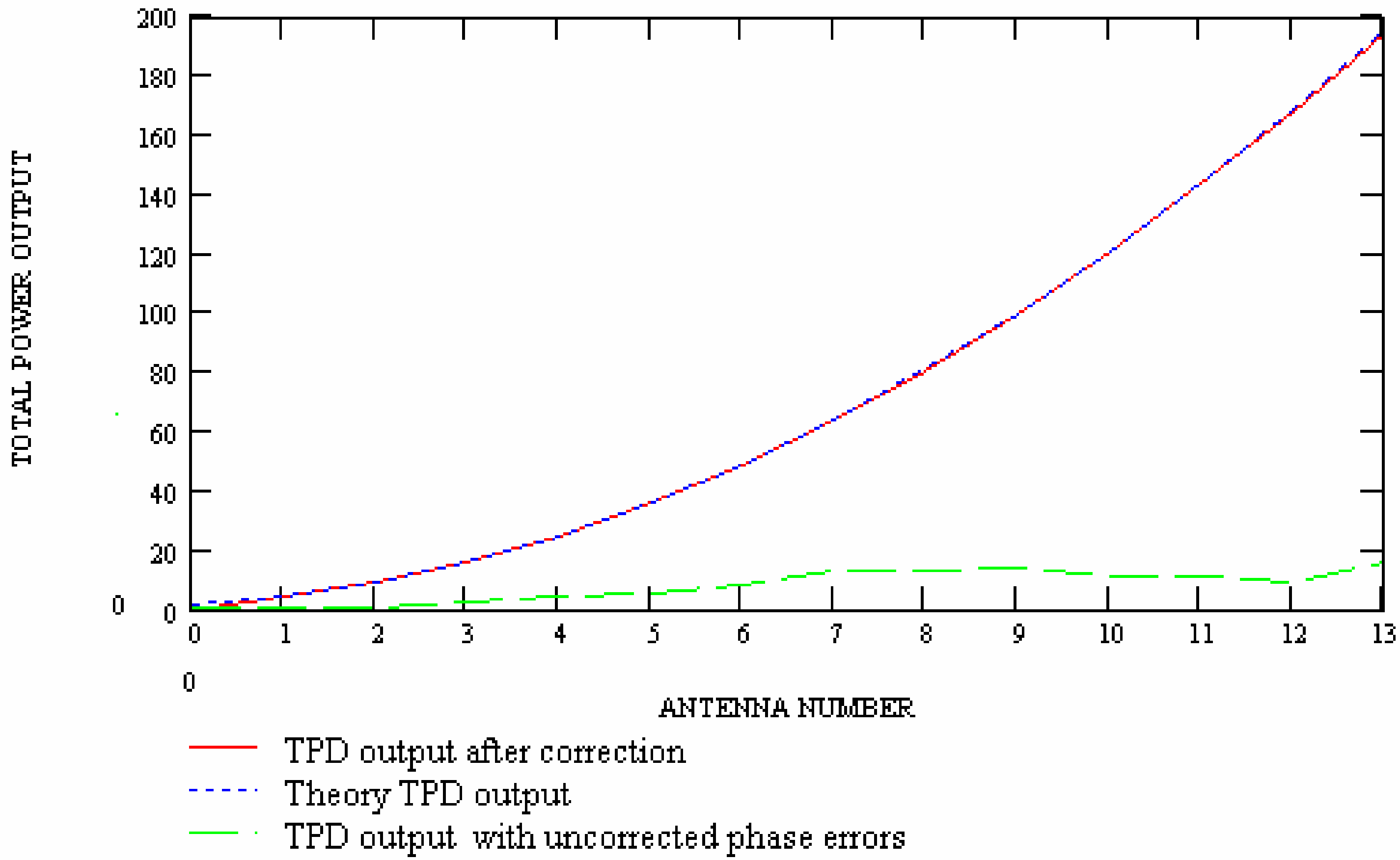


Total power detector output during the process of the phases correction: each of 14 phases is adjusted to compensate the initial phase errors.


200
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TOTAL POWER OUTPUT
ANTENNA NUMBER
TPD output after correction
Theory TPD output
TPD output with uncorrected phase errors

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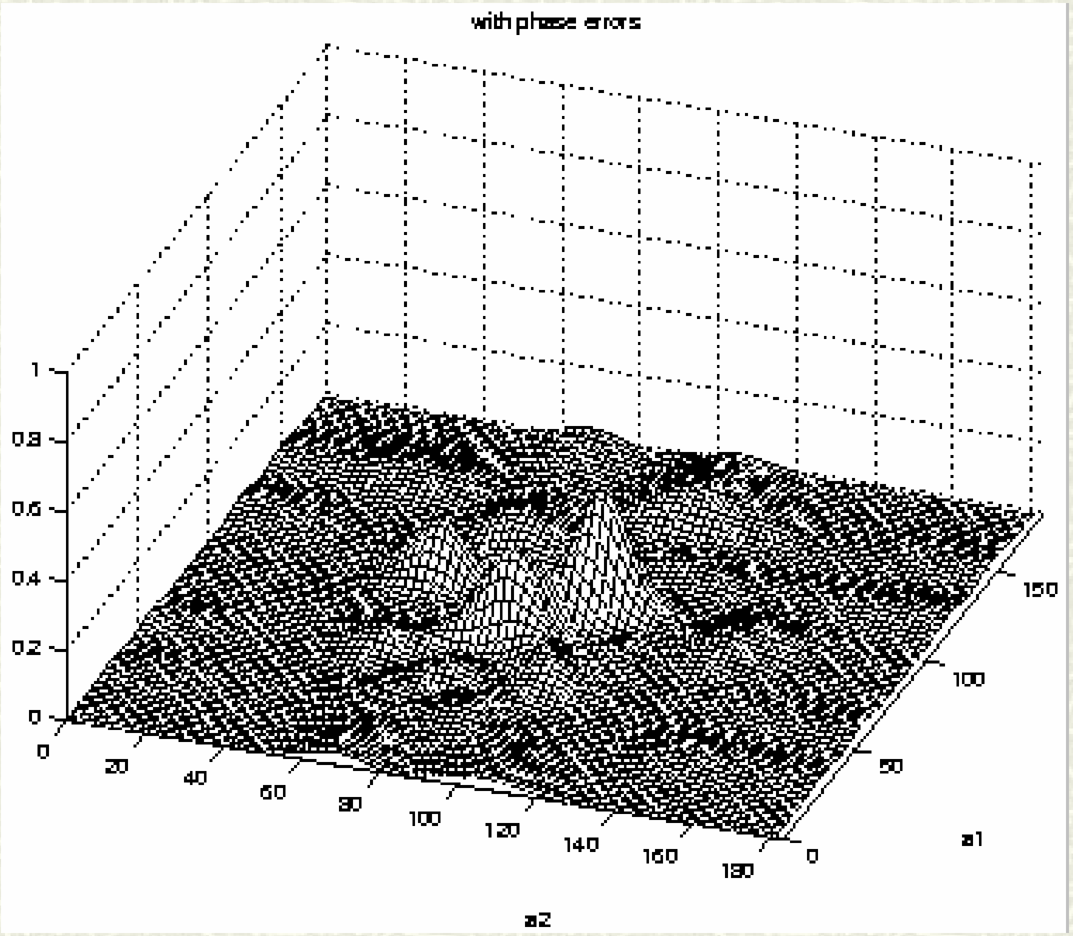

Array pattern with phase errors

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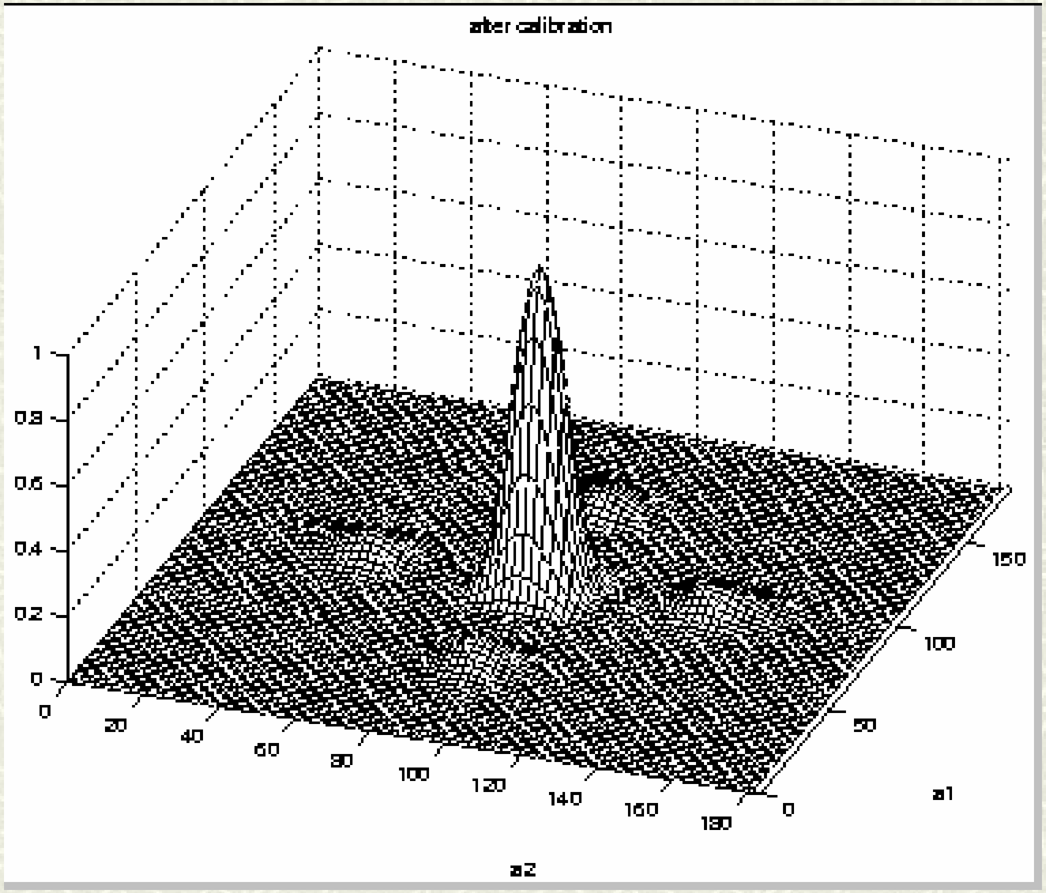

Array pattern after calibration

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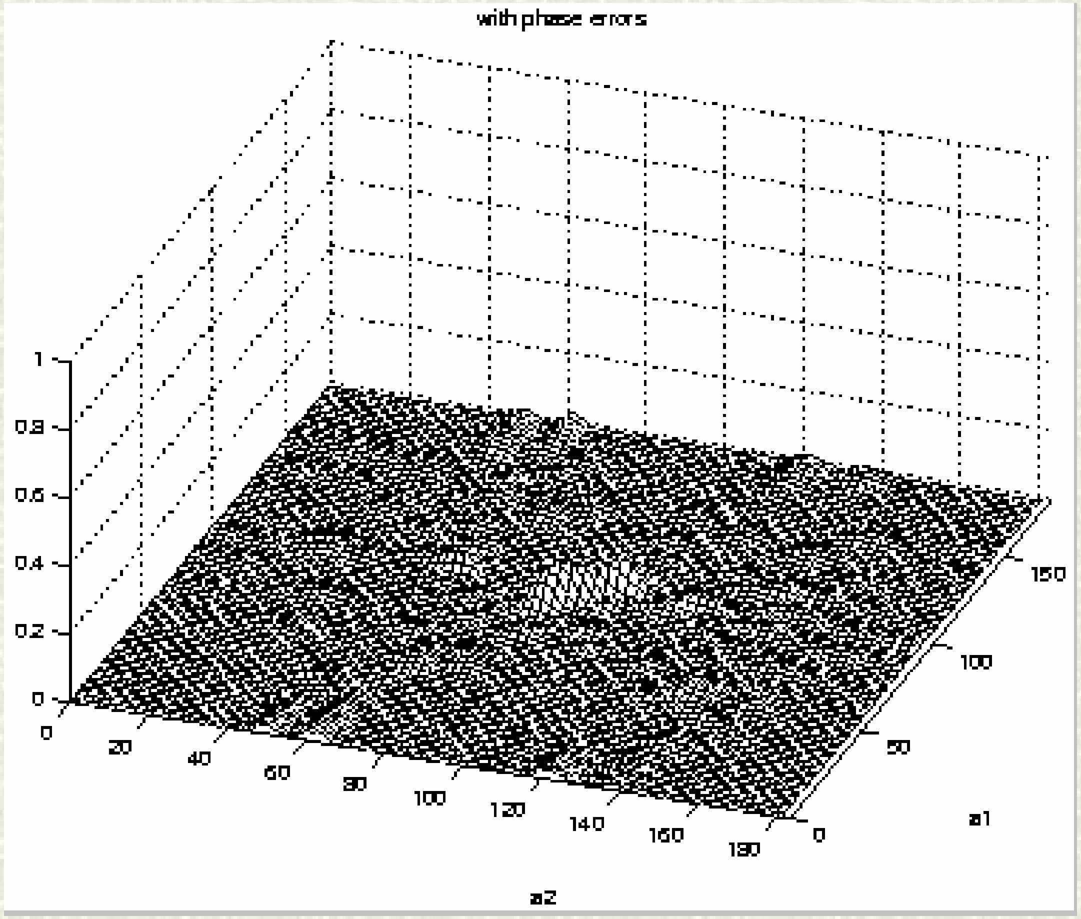
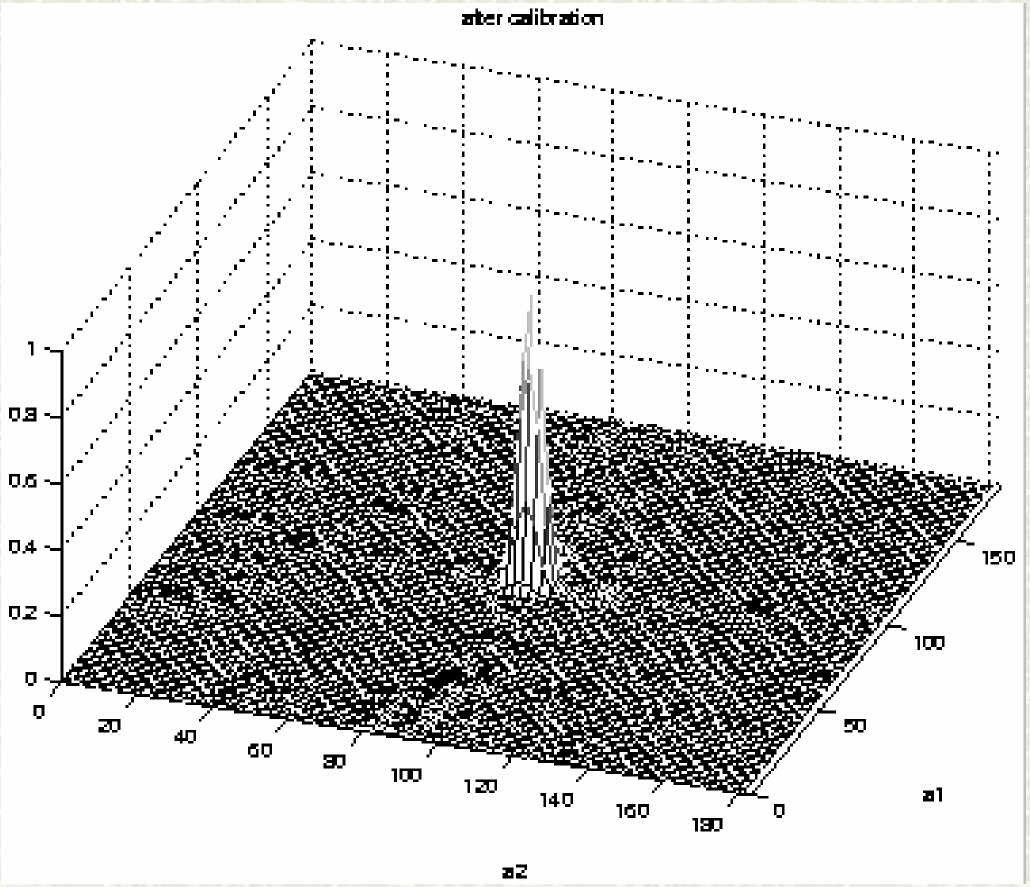
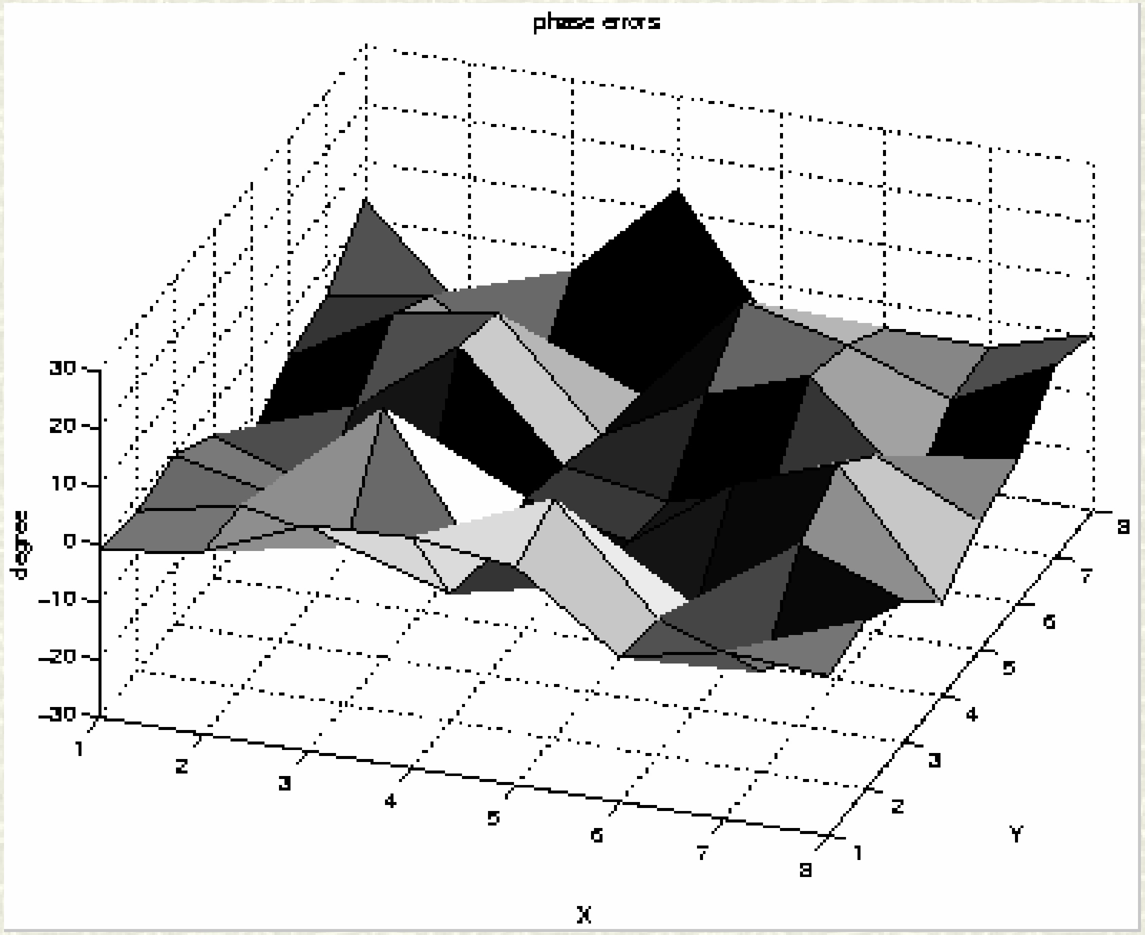


Array pattern with phase errors



Array pattern after calibration



Phase errors

The complex amplitude of a signal $x(t) = X(\theta)e^{j2\pi f_0 t}$, consists of the following components :

$$X(\theta) = S(\theta_0) + \sum_{n=1}^{N} RFI_n(\theta_n) + N_{sys}, \qquad (1)$$

$$S(\theta_0) = [1, e^{-i\varphi_0}, \dots e^{-i(M-1)\varphi_0}]^T, \qquad (2)$$

where the phase shift $\varphi_0 = (2\pi d/\lambda)\sin(\theta_0)$, d is the spacing between array's elements , λ is the wavelength . The phase of the first antenna is chosen equal to 0.

The beamformer , in general , consists of the number of complex weights $w_m e^{i\phi_m}, m = 1 \dots M$, which form the beamformer vector W :

$$W = [1, w_2 e^{i\phi_2}, \dots w_M e^{i\phi_M}]^T. \qquad (3)$$

The output of the phased array is

$$Y = W^H X. \qquad (4)$$

The beamformer must satisfy the two requirements :

a) *steering capability*, the SOI is protected ($W^H S = g$), that is for a prescribed direction θ_0, the response of the array is maintained constant, no matter what values are assigned to the weights W. This is a *linearly constrained minimum variance* (LCMV) beamforming (Frost 1972) ;

b) effects of RFI should be minimized .

The *minimum-variance distortionless response* (MVDR) beamforming algorithm , subject to this constraint when g = 1, was proposed to *minimize the variance* of the beamformer output (Capon 1969). The solution for W in this case is

$$W_{MDVR} = R^{-1}S(\theta_0)[S(\theta_0)^H R^{-1} S(\theta_0)]^{-1}, \qquad (5)$$

where R is the correlation matrix of X.

WSRT and other large radio astronomy arrays cannot use this algorithm in real time with the **existing** equipment because there is no amplitude control facilities . That is the "RFI nulling " can be performed with the **phase-only** control .

Phase -only weights can be found as a solution of the following system of nonlinea equaitons :

$$\mathrm{Re}\left\{\sum_{m=1}^{M} e^{i\phi_m} S_m\right. = M, \tag{6}$$

$$\mathrm{Im}\left\{\sum_{m=1}^{M} e^{i\phi_m} S_m\right. = 0,$$

$$\mathrm{Re}\left\{\sum_{m=1}^{M} e^{i\phi_m} RFI_m\right. = 0,$$

$$\mathrm{Im}\left\{\sum_{m=1}^{M} e^{i\phi_m} RFI_m\right. = 0.$$

The array total power detector (TPD) output is

$$TPD_{TA} = < \int_0^T [x(t)]^2 dt > = TPD_{sig} + TPD_{RFI} + TPD_N, \tag{7}$$

The mean value of the signal component is

$$TPD_{sig}(\phi_m) = < \int_0^T \left\{ \sum_{m=1}^{M} \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_0)/\lambda + \phi_m]^2 dt \right\} >, \tag{8}$$

the mean value of the n-th RFI component is

$$TPD_{RFIn}(\phi_m) = < \int_0^T \left\{ \sum_{m=1}^{M} A_{RFIn} \cos[2\pi f_0 t + 2\pi(m-1)d \sin(\theta_{RFIn})/\lambda + \phi_m]^2 dt \right\} >, \tag{9}$$

Considering the TPD output as a function of M variables ϕ_m, the following criterium for a "good" vector $\phi = [\phi_1 \ldots \phi_M]^T$ can be proposed:

$$C(\phi) = \frac{TPD_{sig}(\phi)}{\sum_{n=1}^{N} TPD_{RFIn}(\phi) + TPD_N} \to \max . \tag{10}$$

Genetic algorithm search

Fitness function (calibration) - total power detector output:

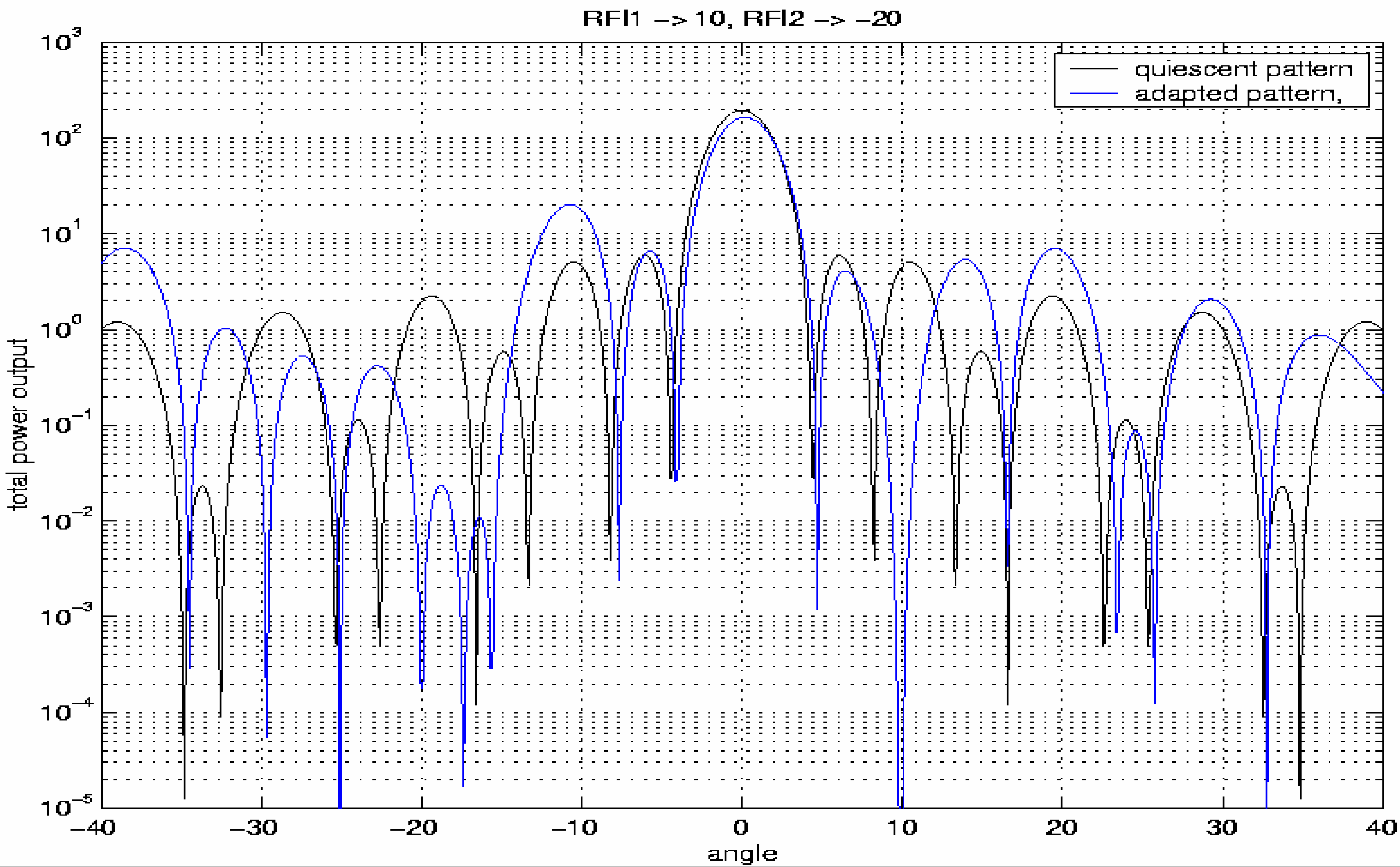
$$C(\alpha) = \int_0^T \left\{ \sum_{m=1}^{M} \cos[2\pi f_0 t + 2\pi(m-1)d\sin(\theta_{sig})/\lambda + \alpha_m + \beta_m] \right\}^2 dt$$

Fitness function (RFI mitigation) - total power detector output:

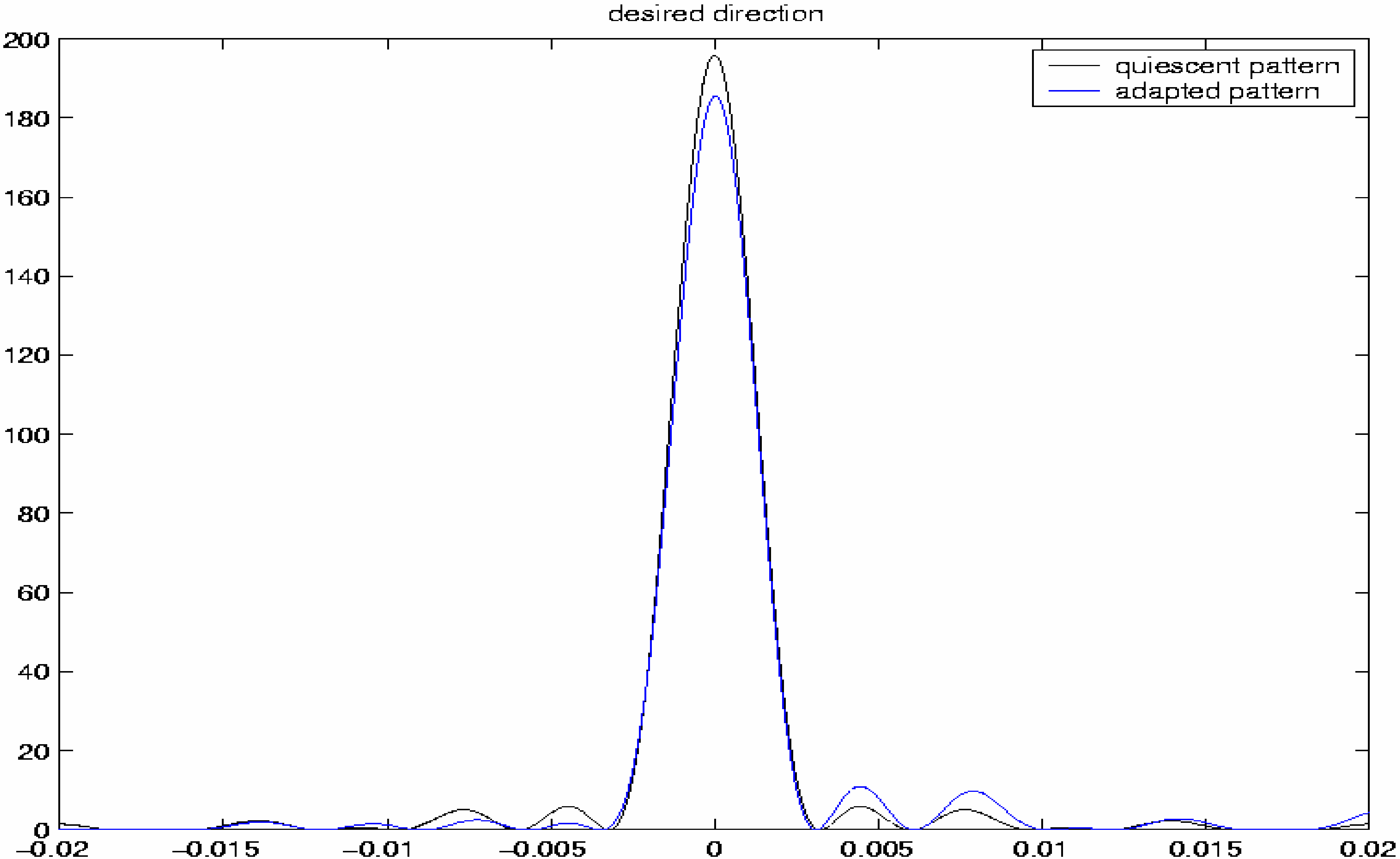
$$TPD_{RFI_i}(\alpha) = \int_0^T \left\{ \sum_{m=1}^{M} \cos[2\pi f_0 t + 2\pi(m-1)d\sin(\theta_{RFI_i})/\lambda + \alpha_m] \right\}^2 dt, \text{ RFI response}$$

$$TPD_{sig}(\alpha) = \int_0^T \left\{ \sum_{m=1}^{M} \cos[2\pi f_0 t + 2\pi(m-1)d\sin(\theta_{sig})/\lambda + \alpha_m] \right\}^2 dt, \text{ signal response}$$

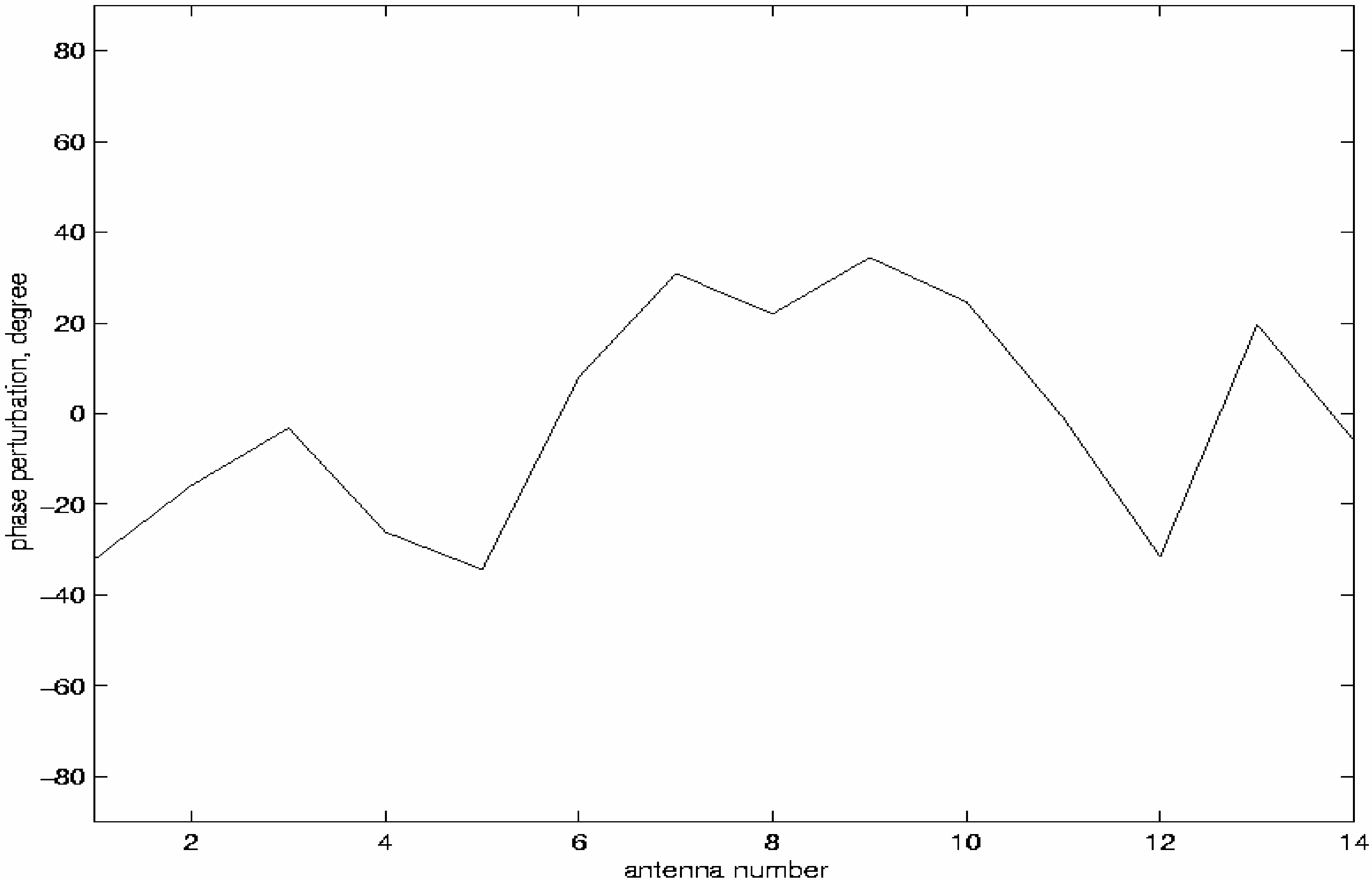
$$C(\alpha) = \frac{TPD_{sig}(\alpha)}{\sum_{i=1}^{N} TPD_{RFI_i}(\alpha)}$$



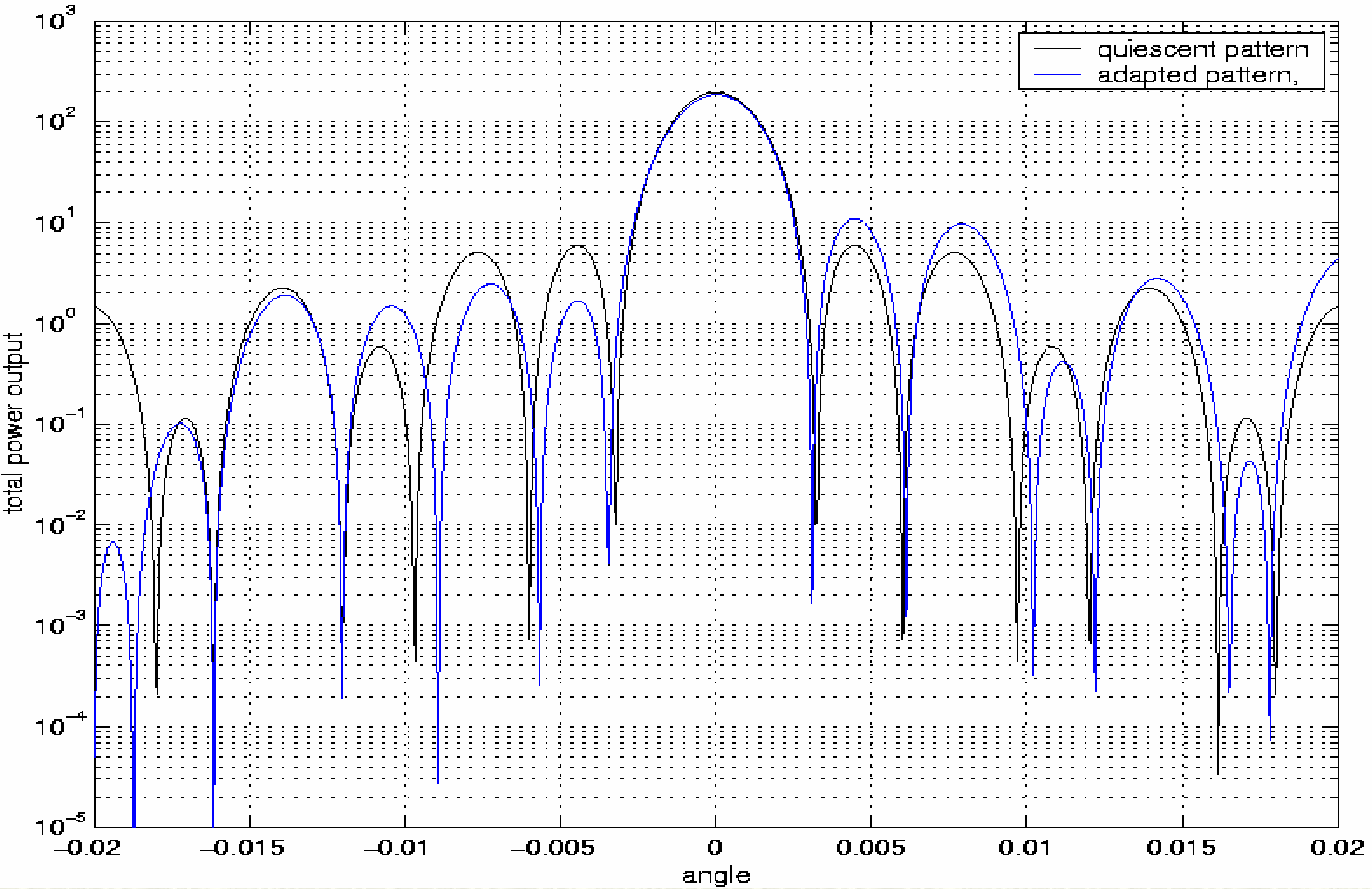
Quiescent (red line) and adapted (blue line) 14-element half-wavelength array pattern, logarithmic scale; two nulls in the adapted pattern at –20dg and +10dg.



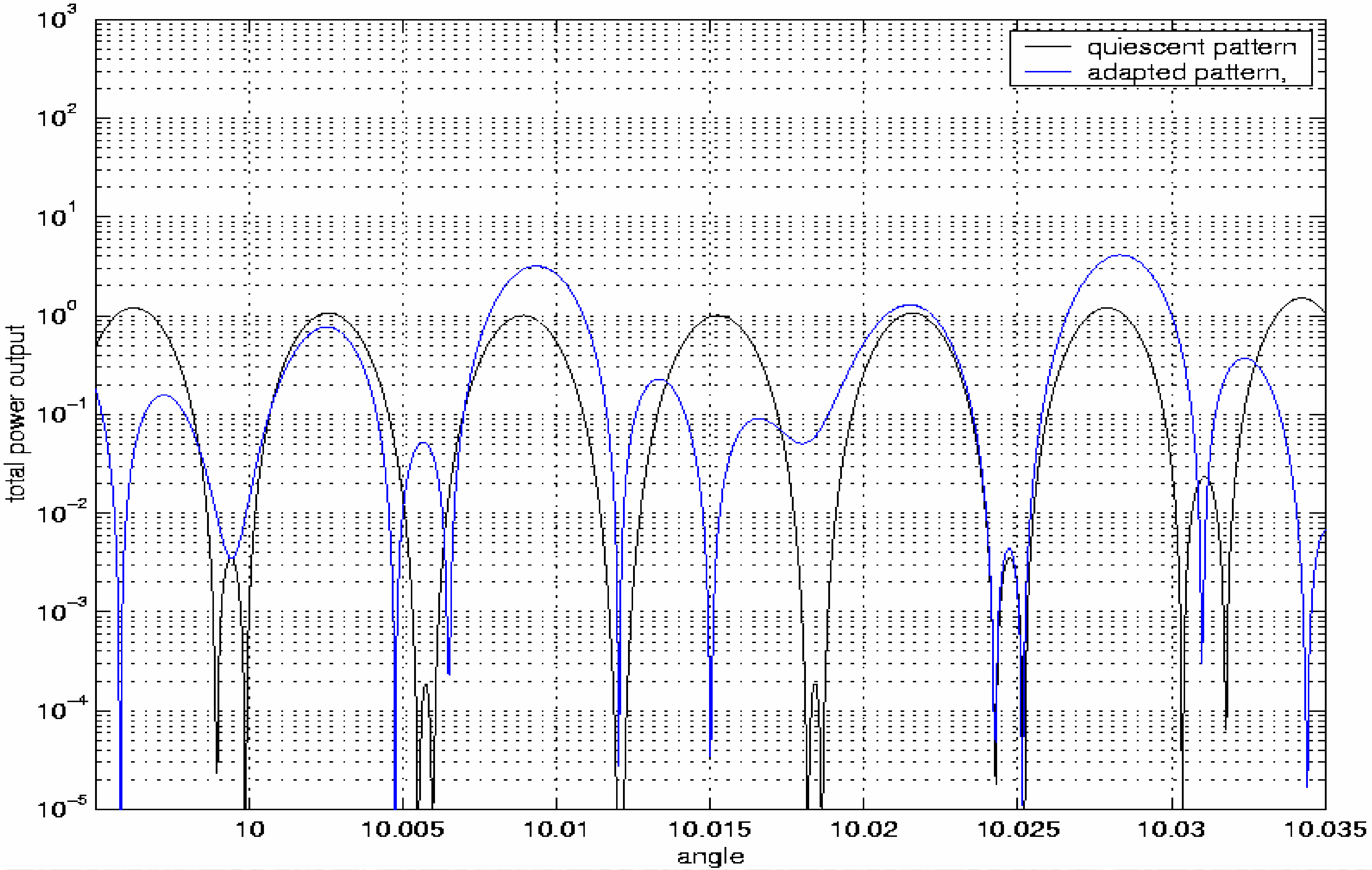
Quiescent (red line) and adapted (blue line) 14-element half-wavelength array pattern, linear scale



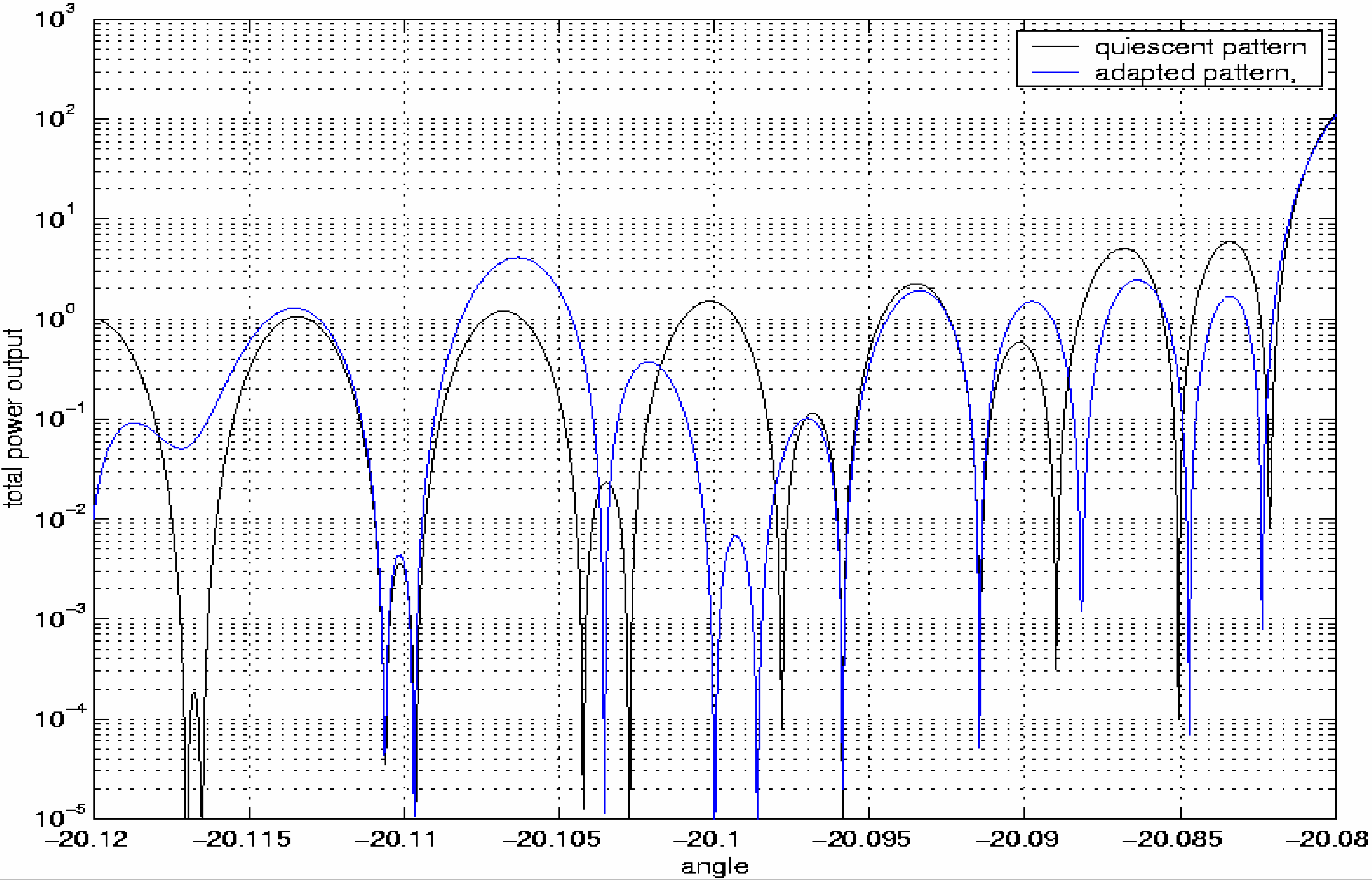
Phase perturbations corresponding to the adapted array pattern.



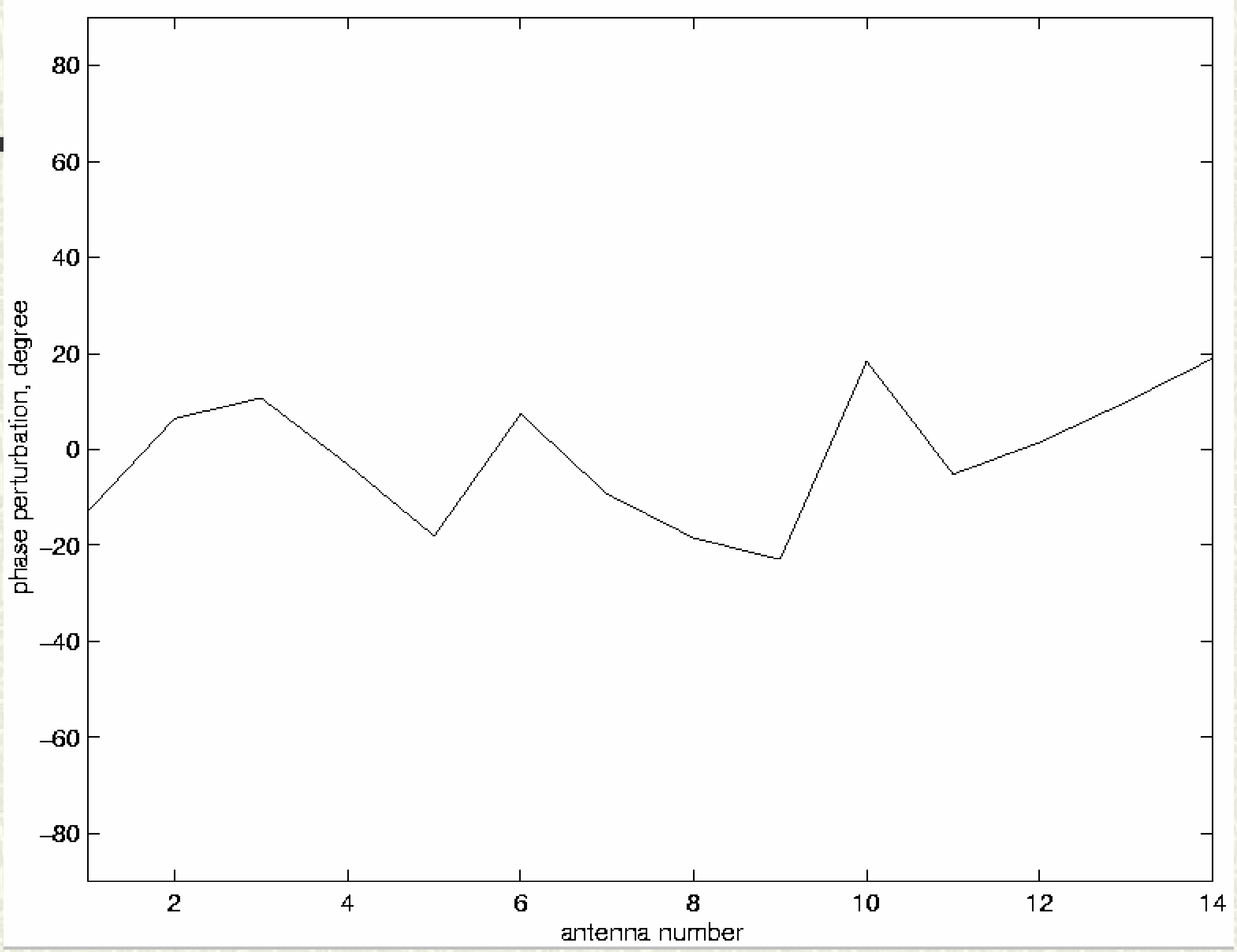
Quiescent (red line) and adapted (blue line) 14-element array pattern, spacing=144m, central frequency=1420MHz, main beam, logarithmic scale.



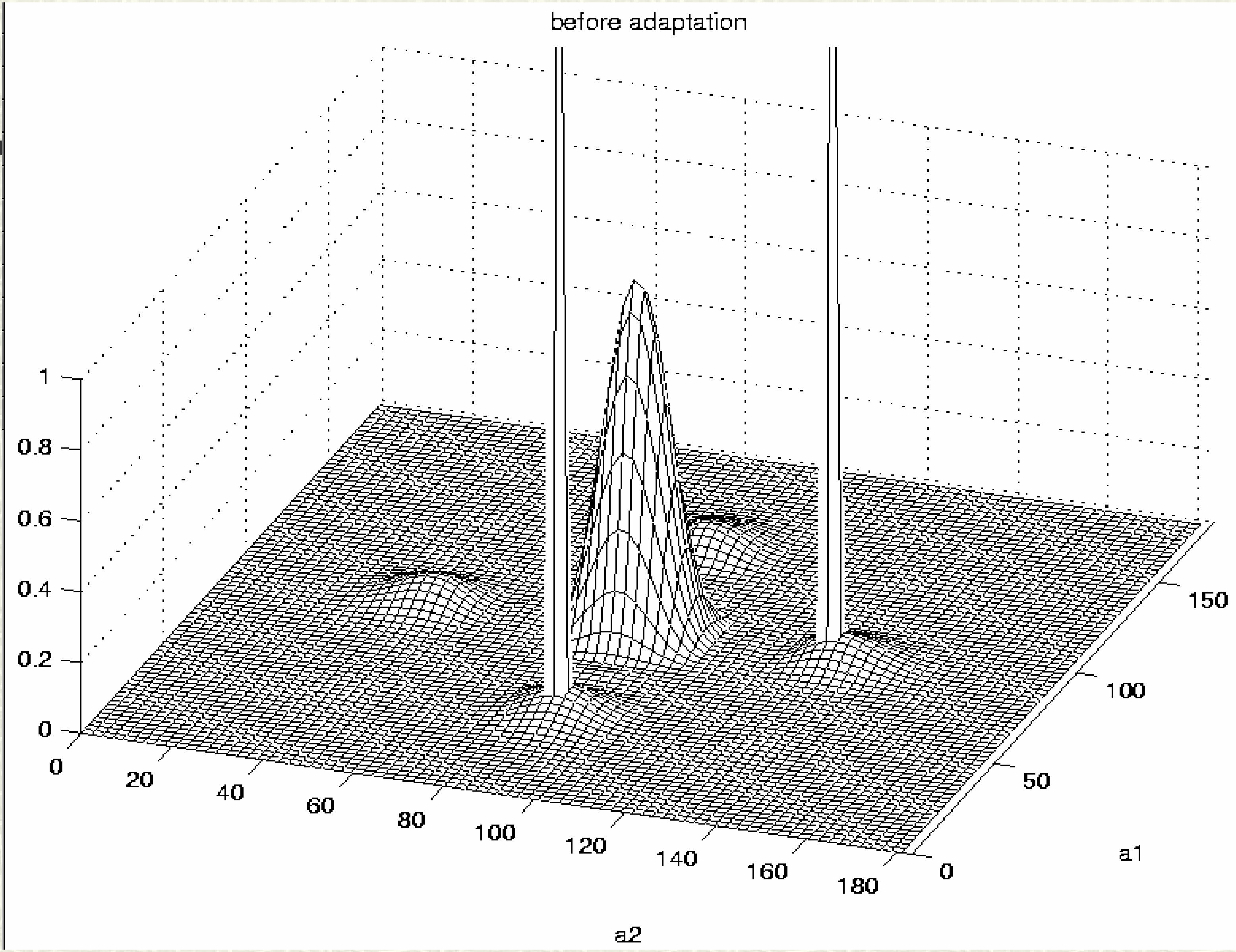
Quiescent (red line) and adapted (blue line) 14-element array pattern, spacing=144m, central frequency=1420MHz, direction +10.015dg, logarithmic scale.



Quiescent (red line) and adapted (blue line) 14-element array pattern, spacing=144m, central frequency=1420MHz, direction -20.1dg, logarithmic scale.



Phase perturbations corresponding to the adapted array pattern .



Two-dimensional 4x4-element half-wavelength, *quiescent* array pattern, central frequency=1420MHz, RFI-1 at (45, 90)dg, RFI-2 at (90, 135) dg, linear scale.

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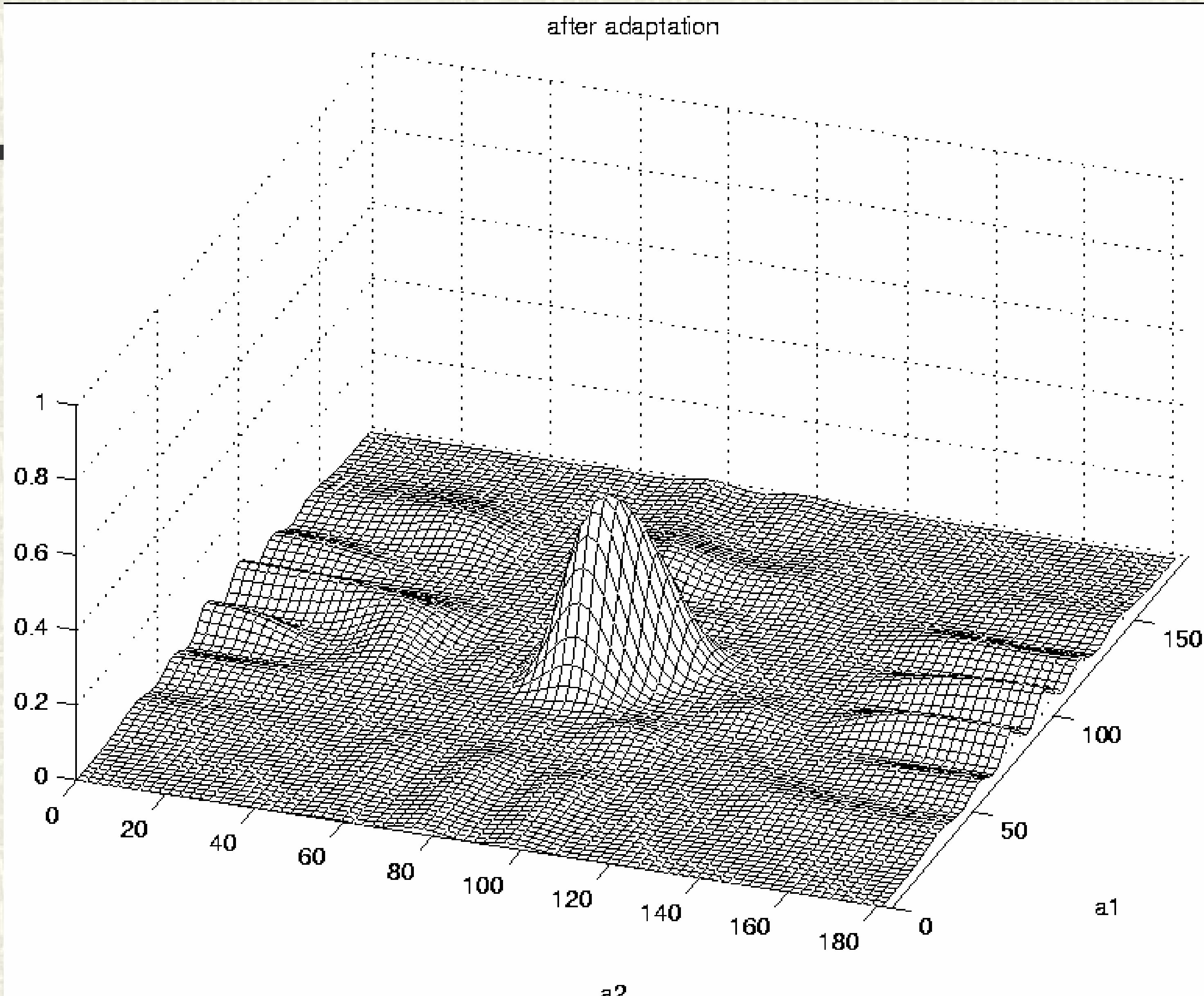
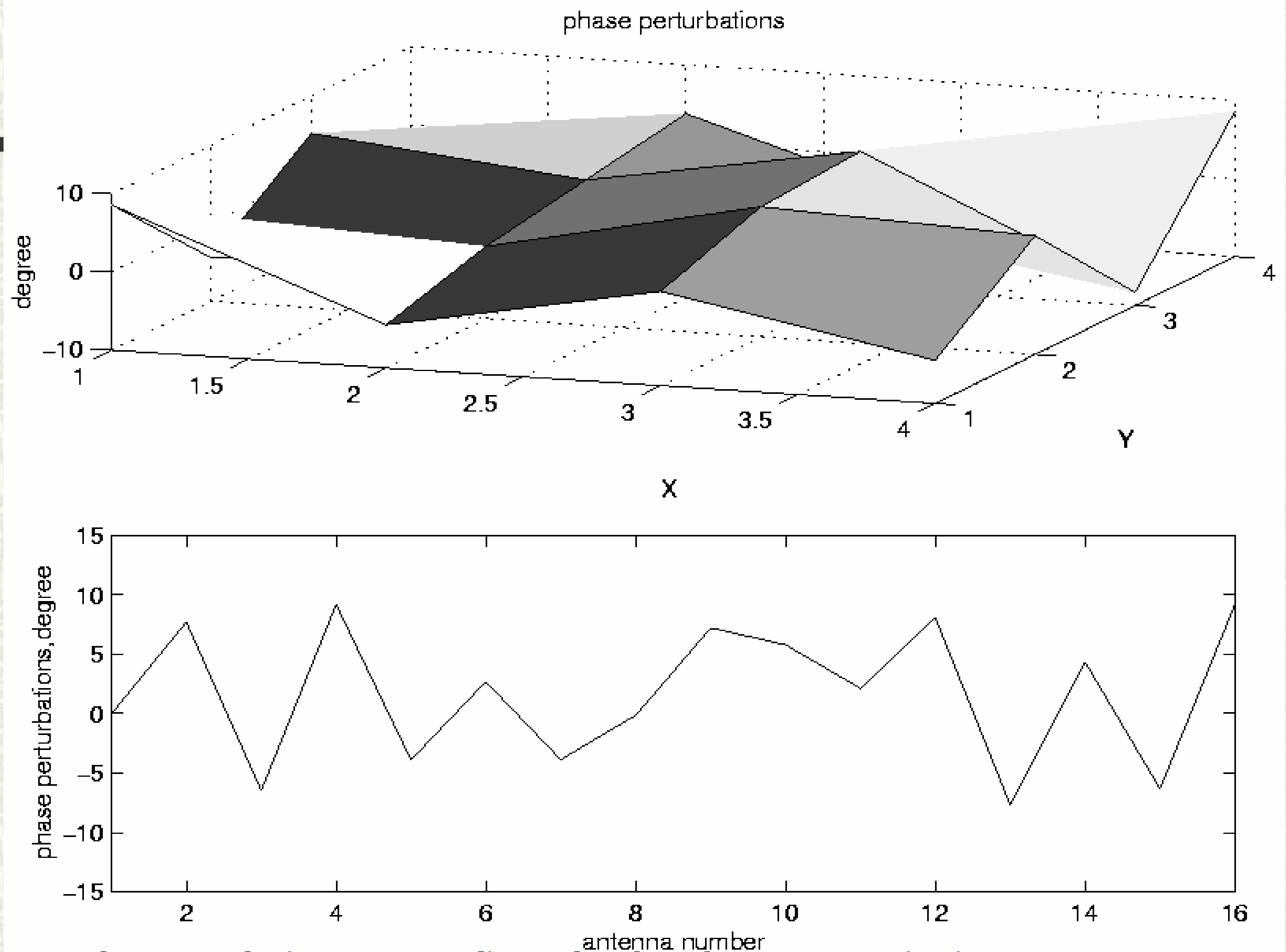
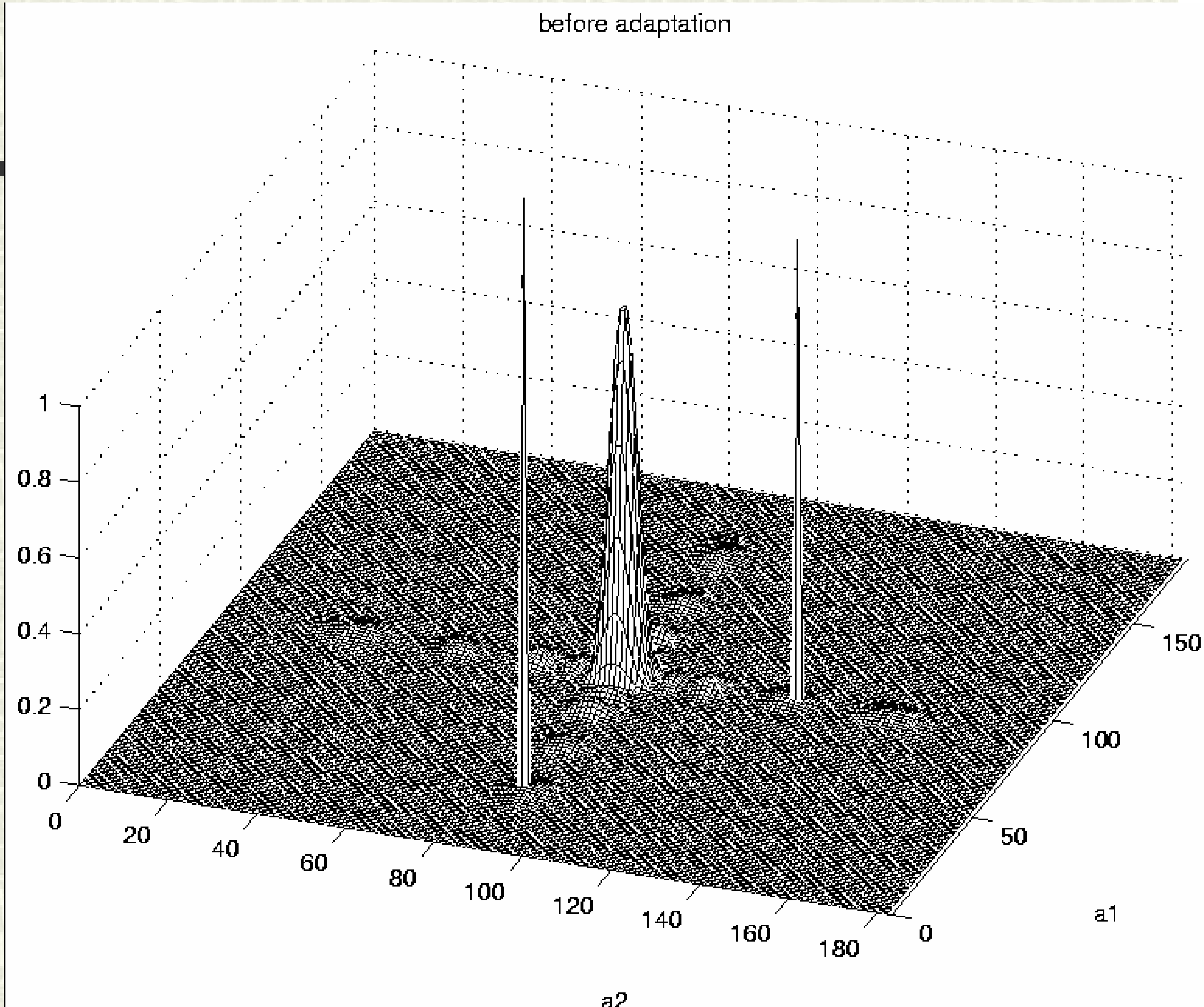


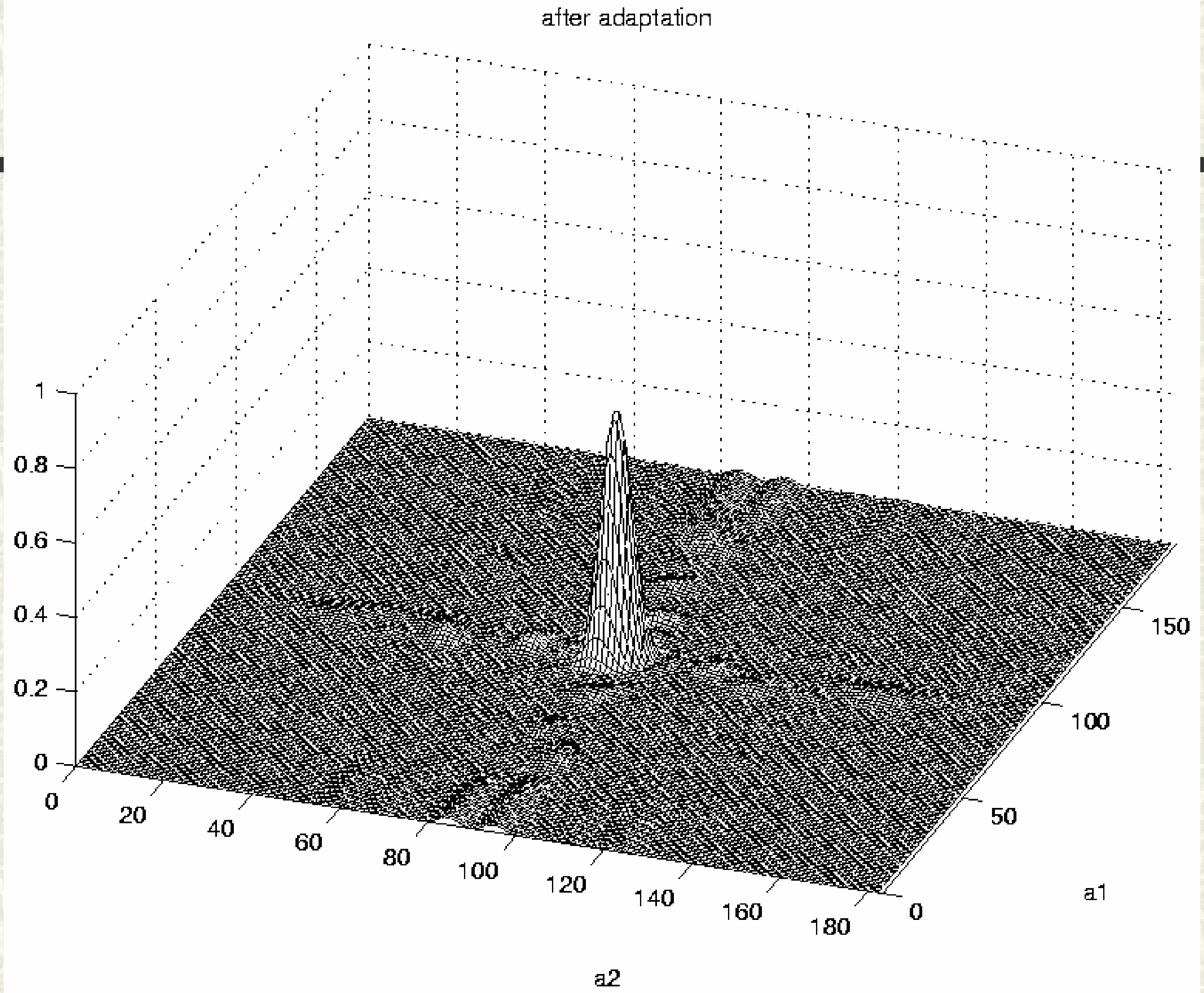
Two-dimensional 4x4-element half-wavelength, *adapted* array pattern, central frequency=1420MHz, RFI-1 at (45, ,90) dg, RFI-2 at (90, 135) dg, linear scale. RFI-1 suppression=76.6dB, RFI-2 suppression=74dB.



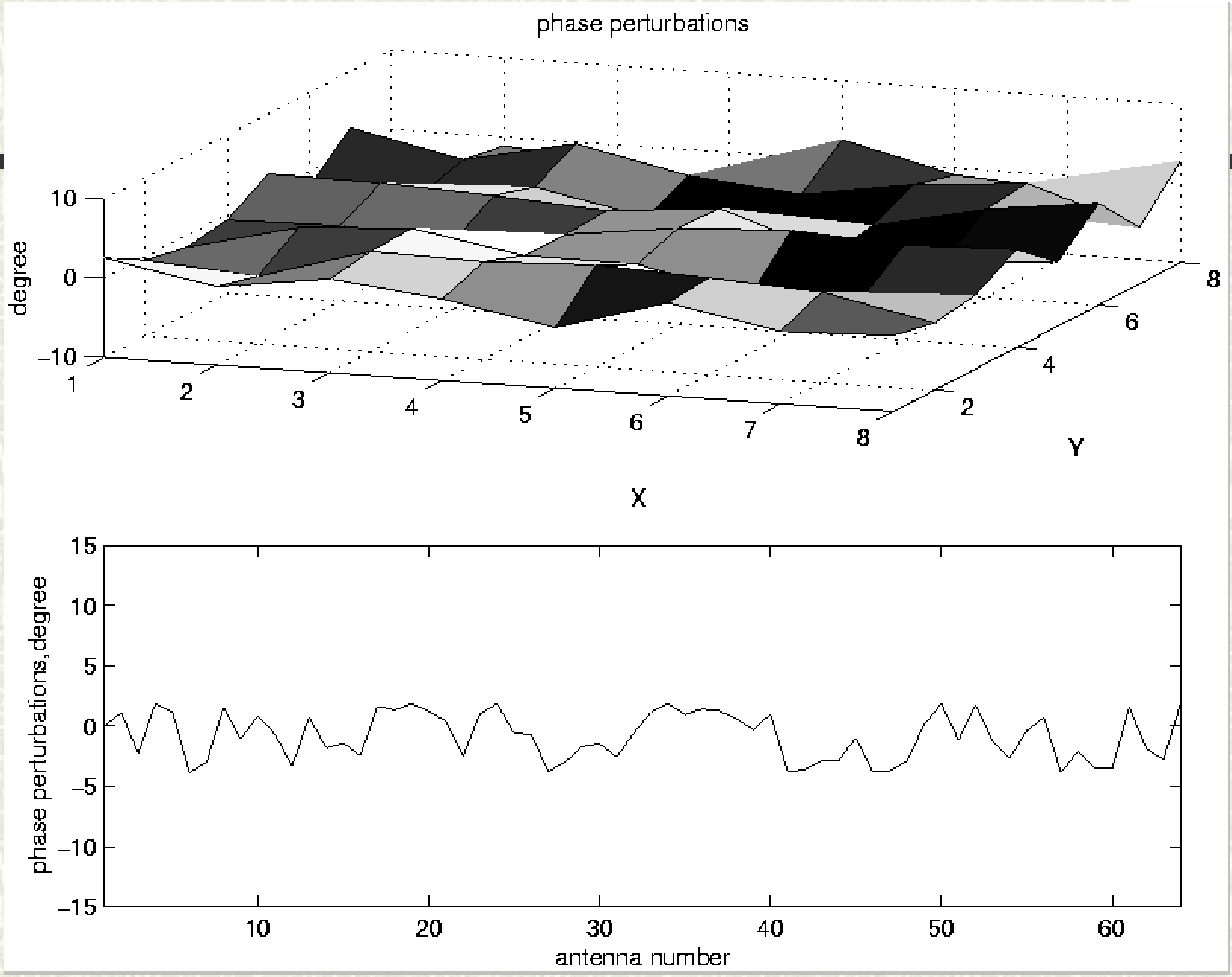
Phase perturbations corresponding to the adapted array pattern in Fig. 10:
a) 3D-presentation of the phase surface; b)phase distribution in linear order.}



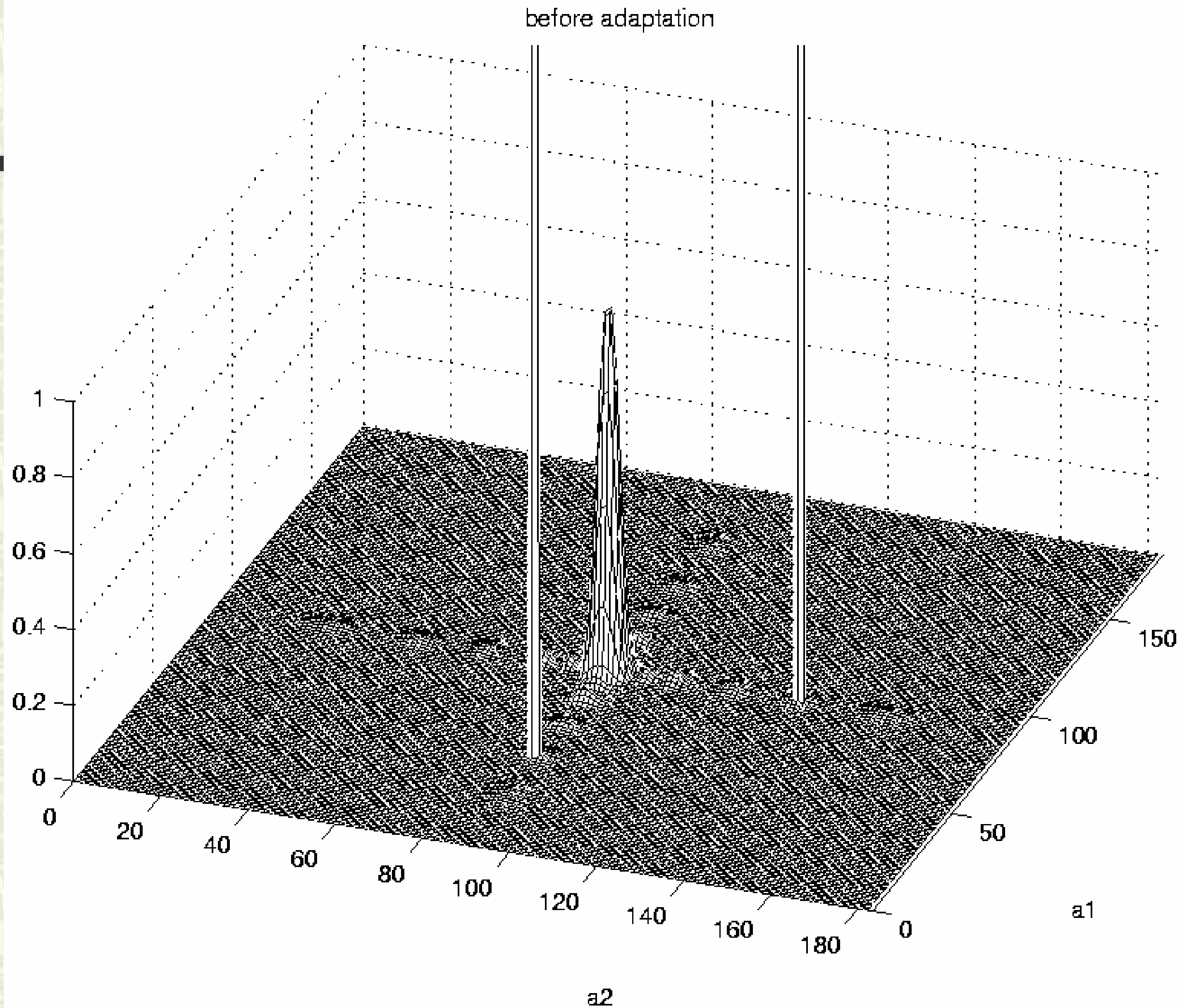
Two-dimensional 8x8-element half-wavelength, *quiescent* array pattern, central frequency=1420MHz, RFI-1 at (30, 90) dg, RFI-2 at (90, 130) dg, linear scale.



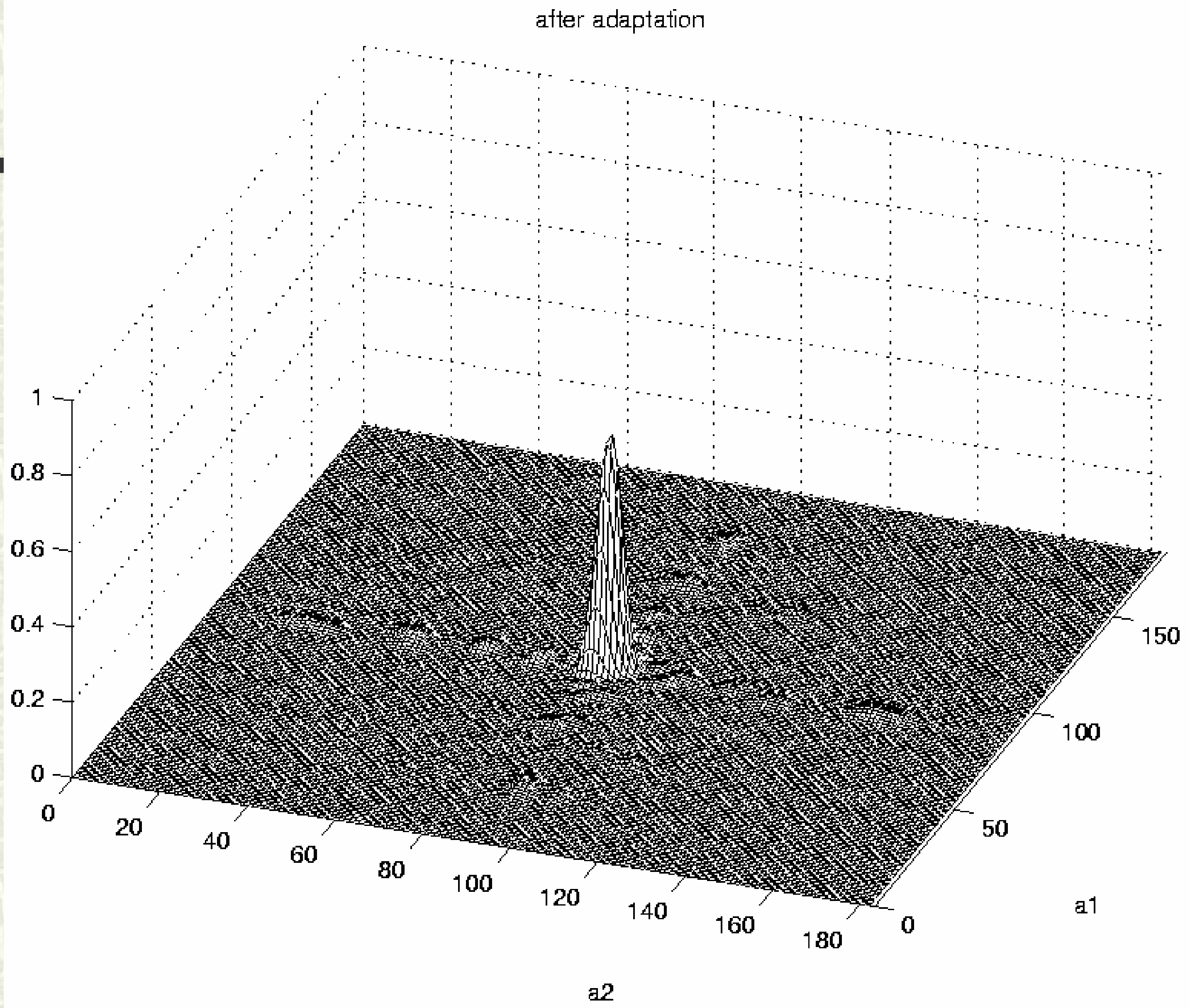
Two-dimensional 8x8-element half-wavelength, *adapted* array pattern, central frequency=1420MHz
RFI-1 at (30, 90) dg, RFI-2 at (90, 130) dg, linear scale; RFI-1 suppression=98.5dB, RFI-2 suppression=96.6dB.



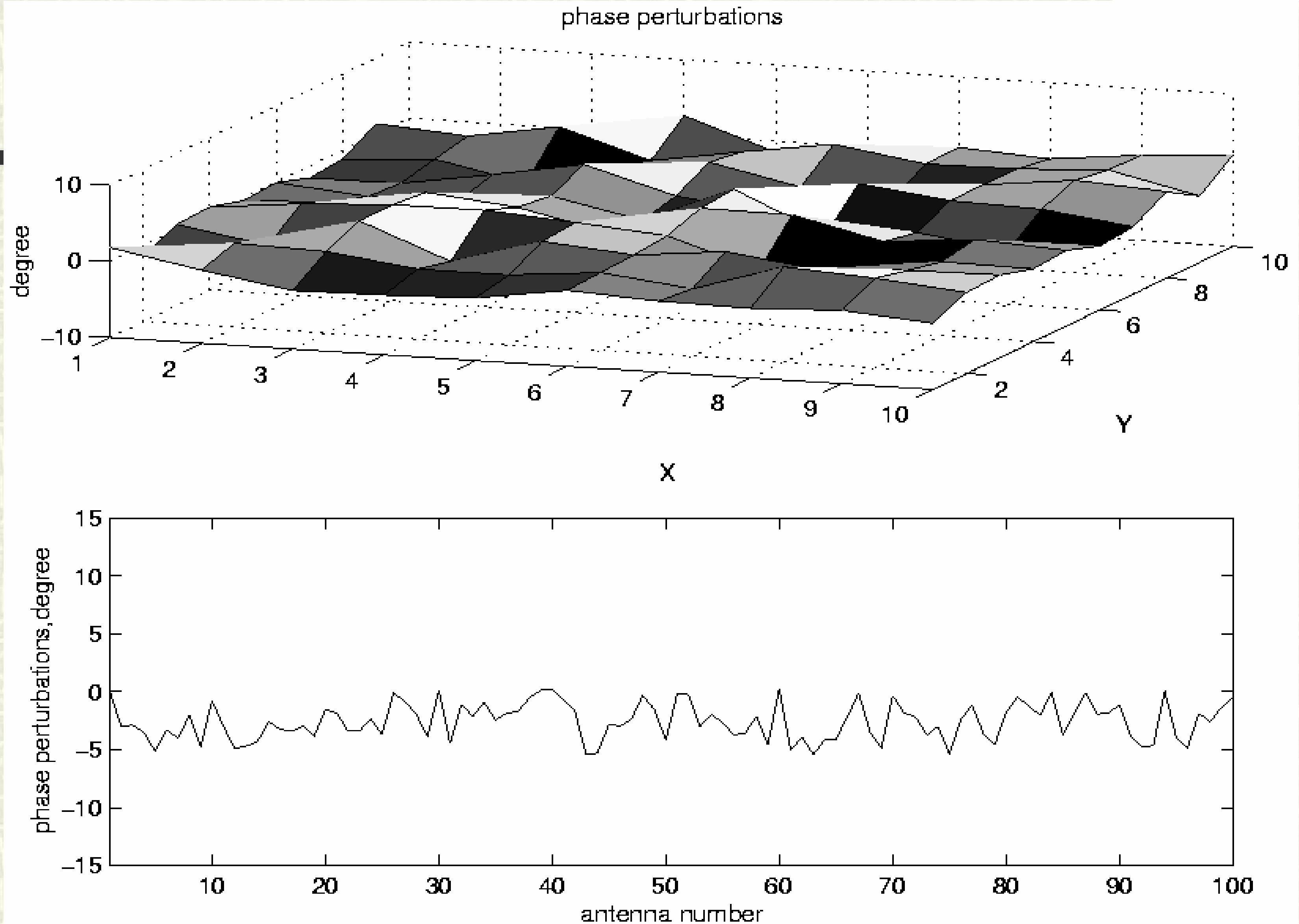
Phase perturbations corresponding to the adapted array pattern :
a) 3D-presentation of the phase surface; b) phase distribution in linear order.



Two-dimensional 10x10-element half-wavelength, quiescent array pattern, central frequency=1420MHz, RFI-1 at (45, 90) dg, RFI-2 at (90, 135) dg, linear scale.



Two-dimensional 10x10-element half-wavelength, *adapted* array pattern, central frequency=1420MHz,
RFI-1 at (45, 90) dg,, RFI-2 at (90, 135) dg,linear scale; RFI-1 suppression=106.2dB, RFI-2 suppression=103.1dB.



Phase perturbations corresponding to the adapted array pattern in Fig. 16:
a) 3D-presentation of the phase surface; b) phase distribution in linear order.