Adaptive Channel Bonding in Wireless LANs Under Demand Uncertainty

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Abstract—Channel bonding is one promising approach to cope with rising WLAN data demand, given scarce spectrum resources. An access point (AP) can aggregate multiple contiguous channels to satisfy demand. Optimizing the network performance under deterministic demands has been well-studied. It is still an open question of how to optimally utilize available frequency bands under uncertainty in AP demands. We propose two approaches to tackle this problem using a stochastic optimization framework. First, we develop a static joint stochastic center frequency and bandwidth allocation scheme. The goal of this scheme is to minimize the total occupied bandwidth while satisfying the demand of each AP with probability at least $\beta$. Second, we develop a two-stage joint stochastic center frequency and adaptive bandwidth allocation scheme. In contrast to the static scheme, the two-stage allocation scheme allows adaptability of the bandwidth allocated to each AP in response to its demand variation, while keeping the total occupied bandwidth fixed. Our numerical results demonstrate the advantages of (i) stochastic compared to naive allocation and (ii) adaptive compared to static allocation. They also explain the bandwidth–user satisfaction trade-off provided by the adaptive allocation approach.

I. INTRODUCTION

Data transfer over wireless networks is increasing [1]. According to [2], by 2020 more than 55% of wireless cellular data will be offloaded to WLANs and small cells. In the context of WLANs, a mobile user is connected to the Internet through an access point (AP). An AP uses a single channel to communicate with its associated users. It may happen that multiple users with high data rate demands are associated with a single AP while another nearby AP is totally unused. This imbalance in the load of APs along with single channel allocation result in inefficient spectrum distribution. To avoid this problem, channels need to be allocated to APs adaptively in order to better utilize the available spectrum.

IEEE standards have facilitated this approach by introducing channel bonding in IEEE 802.11n and IEEE 802.11ac [3]. An AP can expand its bandwidth by seizing consecutive 20 MHz channels within the 5 GHz ISM band, up to an aggregated bandwidth of 160 MHz. However, this requires these consecutive channels to be interference-free. Due to the nature of WLAN deployment, interference cannot be avoided between APs operating near each other on the same channel. Therefore, when allocating multiple consecutive channels to an AP, these channels cannot be utilized at the same time by a nearby AP. We use the term “interfering APs” to represent a subset of APs that can interfere with a chosen AP if operated on the same frequency channel.

In [4], Moscibroda et al. suggested using channel bonding to better utilize the spectrum and satisfy the deterministic demands of a set of APs. In this paper, we consider uncertainty in the AP demands. That is, the nature of user association with APs and their data demands are stochastic. Therefore, the aggregated demand at each AP changes over time. This raises a question of how to dynamically allocate channels to each AP as needed to satisfy its demand while avoiding interference from interfering APs. This can be achieved by careful allocation of the center frequency and bandwidth for each AP. Our objective is to minimize the total bandwidth utilized by the network. The smaller the bandwidth seized by our WLAN, the more space is facilitated for other networks to operate without interference. That is, our solution builds toward smoother coexistence between WLANs and other access technologies (e.g. LTE).

A naive approach for channel allocation under AP demand uncertainty is to optimize network performance for either average or peak value. Considering the average demand value does not provide guarantees on satisfying a significant percentage of AP demands, as the ability to meet actual demands will depend on the probability that the actual demand will exceed its average value. On the other hand, considering the maximum demand value, although potentially meeting all AP demands, usually results in huge and unnecessary consumption of scarce channel resources. In this work, we propose a new channel allocation framework in which the level of AP demand satisfaction is controllable. Furthermore, our framework goes a step further by adapting the number of allocated channels according to the AP's demand.

To realize the channel allocation framework proposed above, we adopt a stochastic optimization approach. Stochastic optimization provides a powerful mathematical tool to handle optimization under uncertainty [5]. It has been recently exploited to optimize resource allocation in various types of wireless networks operating under uncertainties (e.g. [6], [7], [8], [9]). We discuss the superiority of stochastic channel bonding over naive schemes, which consider the mean or maximum AP’s demand values, through the following example.

Motivational example—Consider a network with two interfering APs as shown in Fig. 1(a). The demands of APs 1 and 2 are stochastic, as shown in Fig. 1(b).
changes. In our scheme, we eliminate this overhead by its associated users whenever the allocated bandwidth to form one wide channel. This mandates the AP to "primary" channel by seizing more contiguous channels utilization and demand satisfaction when \( \beta \) with probability \( \geq \) determined once such that the given AP demands are met approach, the center frequency and bandwidth are de-

Fig. 1: An illustration of static and adaptive channel bonding.

For illustration purposes, we assume that one channel is sufficient to satisfy one unit of AP demand. From here on, we assume that the deterministic naive approach considers the average value of each AP demand. As shown in Fig. 1(c), this approach results in allocating three channels for each AP all the time. Consequently, the AP demands will not be satisfied when they take the value of five. Besides, when an AP demand is one, the AP needs only one channel but this approach unnecessarily allocates three channels to it.

In contrast to the naive approach, the stochastic static approach provides the ability to balance the level of AP demand satisfaction and the total number of utilized channels through a predefined threshold \( \beta \). Under this approach, the center frequency and bandwidth are determined once such that the given AP demands are met with probability \( \geq \beta \). Fig. 1(d) and (e) show the channel utilization and demand satisfaction when \( \beta \) is set to 0.8 and 1, respectively. It is easy to see that fewer channels will be utilized if \( \beta \) is set to a lower value but at the cost of satisfying the demand with a lower probability. Although this approach has an advantage over the naive approach, the channel allocation here is static and cannot be changed when the demand value varies.

The stochastic adaptive approach goes a step further by adapting the allocated bandwidth according to the demand. Fig. 1(f) shows that the two AP demands can be fully satisfied under all scenarios while the total number of utilized channels is eight (as opposed to ten in the static case when \( \beta = 1 \)).

In the IEEE standard procedure, an AP can extend its "primary" channel by seizing more contiguous channels to form one wide channel. This mandates the AP to move to a new center frequency and re-synchronize with its associated users whenever the allocated bandwidth changes. In our scheme, we eliminate this overhead by allowing the AP to extend/shrink its bandwidth symmetrically around the same center frequency.

**Main contributions.**

- Given the AP demand variability, we provide two mathematical formulations for our problem under the stochastic optimization framework.
- We introduce an equivalent deterministic formulation to each stochastic optimization problem in order to solve it using CPLEX.
- We show the advantages of the proposed stochastic allocation approaches over the naive approach.
- We discuss insights on how to set different parameters under each stochastic allocation approach to best utilize the available spectrum.

**Paper organization—**The rest of the paper is organized as follows. In Section II, we introduce the system model and state our problem. In Section III, we formulate our problem when the AP demands are deterministic. In Sections IV and V, we provide two stochastic optimization formulations to tackle our problem under AP demands uncertainty. Equivalent deterministic formulations are then derived. Section VI demonstrates the superiority of the stochastic optimization framework and gives insights on the performance of the proposed approaches. We provide a literature review in Section VII. In Section VIII, we conclude our work and indicate directions for future research.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. System Model

Consider a WLAN, where a set \( P = \{1, 2, \ldots, P\} \) of IEEE 802.11ac APs is deployed. The transmission range of each AP is adjusted through controlling its power level so that a minimum signal strength is achieved at all associated users. Due to the stochasticity of user distribution, the transmission range of an AP might be overheard by a subset of other APs or their associated users. Let \( T_p \) be the set of APs interfering with AP \( p \). Let \( C = \{1, 2, \ldots, C\} \) be the set of all available consecutive channels. Each AP is capable of extending its channel bandwidth by seizing a subset of these channels when needed. We assume a fixed data rate per channel for all APs. To avoid interference, interfering APs cannot operate simultaneously on the same frequency channel. Each AP has a demand that collectively represents the associated users’ data rate requirements\(^1\). We assume a centralized controller in the back-end linked to all APs with a high speed connection. The controller module can also be implemented inside one of the deployed APs. All control signals are collected at the central controller via the APs. The central controller makes the needed decisions regarding channel assignment and bandwidth allocation then conveys these decisions to the APs. This network architecture is suitable for enterprise and campus environments where all APs are managed through one entity [10], [4].

\(^1\)How each AP schedules its users to meet their data rate require-

ments is not within the scope of this work.
B. Distribution of the AP Demands

To study our problem, we need to model the AP demand distribution in order to accurately represent the demand variation in practical networks. In [10], Chen, Kurose, and Towsley developed a mixed queueing network model to capture user mobility in a campus network. In their model, two classes of users were considered: open and closed.

The open class considers users who visit the network for a short period of time then leave. The exogenous arrival of users to an AP $i$ in this class has been modeled as a Poisson process with rate $\gamma_i$. Then, the aggregate arrival rate at AP $i$, denoted by $\lambda_i$, can be expressed as

$$\lambda_i = \gamma_i + \sum_{j \neq i} \lambda_j \rho_{ji}, \quad \forall i \in \{1, 2, \ldots, M\},$$

where $\lambda_j$ denotes the load of AP $j$, and $\rho_{ji}$ denotes the probability that a user switches from AP $j$ to AP $i$. The marginal occupancy distribution at AP $i$ of the open-class users can be expressed as:

$$\Pr\{U_i = u_i\} = e^{-\rho_i} \frac{\rho_i^{u_i}}{u_i!},$$

where $\rho_i$ is the load of AP $i$ generated from the open-class users and can be calculated by multiplying the arrival rate $\lambda_i$ by the expected time the user stays in the network, given by $1/\mu_i$ (i.e. $\rho_i = \frac{\lambda_i}{\mu_i}$).

The closed class consists of users who stay in the campus for a long period and switch between multiple APs. The marginal occupancy distribution at AP $i$ of the closed-class users has been modeled as a binomial distribution and can be expressed as follows:

$$\Pr\{U_i = u_{ci}\} = \binom{N}{\nu_i} \nu_i^{u_{ci}} (1 - \nu_i)^{N-u_{ci}},$$

where $N$ is the total number of users in the closed class, and $\nu_i$ is the fraction of time during which a closed-class user is associated with AP $i$. For large networks, the probability that a user associates with a certain AP is relatively small. Therefore, the binomial distribution of closed-class users at AP $i$ has been approximated by a Poisson distribution with parameter $\rho_{ci} = N \nu_i$.

As the distributions of the two classes are independent, the overall marginal occupancy distribution of AP $i$ is the convolution of two Poisson distributions, which is also Poisson, and can be expressed as:

$$\Pr\{U_i = u_{ci}\} = e^{-\rho_i} \frac{\rho_i^{u_{ci}}}{u_{ci}!},$$

where $\rho_i = \rho_{ci} + \rho_{oi}$.

C. Problem Statement

Consider a network of APs where different subsets of APs interfere with each other, and each AP has a stochastic demand. Each AP is to be assigned a center frequency index and allocated contiguous frequency channels to meet all user data rate requirements while avoiding interference between interfering APs. The objective is to minimize the total bandwidth utilized by the network. In the next section we will first explain the simple case where all AP demands are deterministic. Then, in Sections IV and V, we will discuss the details of our problem, in which AP demands are stochastic.

III. Spectrum Distribution under Deterministic AP Demands

In this section, we assume the user demand of each AP is fixed. The available spectrum is divided into $b_{\text{tot}}$ small channels. Each AP will be allocated a contiguous band from these channels. Let $f_p$ and $b_p$ denote the center frequency index and number of channels allocated to AP $p$, respectively. Each AP has a demand denoted by $D_p, p \in \mathcal{P}$. Let $R_p$ be the achievable rate on a single channel between AP $p$ and the associated users. We assume that $R_p, p \in \mathcal{P}$, is fixed. The maximum center frequency index that can be assigned to an AP, denoted by $f_{\text{max}}$, is given by $b_{\text{tot}} - \frac{1}{2}$. We assume that each AP cannot be allocated more than $b_{\text{max}}$ channels.

A. Optimization Constraints

The achievable rate across the allocated band for any AP needs to meet its demand. This can be formulated as follows:

$$R_p b_p \geq D_p, \quad \forall p \in \mathcal{P}. \quad (4)$$

To ensure that two interfering APs will not be allocated the same channel, the following two constraints are needed [4]:

$$f_p + \frac{b_p}{2} < f_k - \frac{b_k}{2}, \quad \forall p, k \in \mathcal{T}_p, \quad (5)$$

$$y_{pk} + y_{kp} \leq 1, \quad \forall p, k \in \mathcal{T}_p, \quad (6)$$

where $y_{pk}$ is a binary variable to indicate whether or not the upper-frequency index of AP $p$, given by $\left(f_p + \frac{b_p}{2}\right)$, is greater than the lower-frequency index of AP $k$, given by $\left(f_k - \frac{b_k}{2}\right)$. If both $y_{pk}$ and $y_{kp}$ are set to one, the bands allocated to APs $p$ and $k$ are overlapping. The second constraint prevents such overlap in the allocated bands if the AP transmission ranges are overlapping.

B. Optimization Objective

As mentioned earlier, the goal of our problem is to minimize the total bandwidth utilized by the network while meeting all AP demands. Minimizing the total bandwidth can be achieved by minimizing the maximum upper-frequency index allocated to any AP. That is,

$$\min_{p \in \mathcal{P}} \left\{ f_p + \frac{b_p}{2} \right\}. \quad (7)$$

This objective function can be linearized by introducing a new variable $W$ which represents the total number of utilized channels, and adding the following set of constraints:

$$f_p + \frac{b_p}{2} \leq W, \quad \forall p \in \mathcal{P}. \quad (8)$$

This set of constraints ensures that the upper-frequency index allocated to any AP cannot exceed the total number of channels allocated to the network.
C. Optimization Formulation

The joint center frequency index and bandwidth allocation problem under deterministic AP demands can be formulated as an integer linear program (ILP) as follows:

<table>
<thead>
<tr>
<th>Channel Allocation Under Deterministic AP Demands</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \min W )</td>
</tr>
<tr>
<td>s.t. ( f_p + \frac{b_p}{2} - (f_k - \frac{b_k}{2}) &lt; b_{\text{tot}} y_{pk} ), \quad \forall , p \in \mathcal{P}, k \in \mathcal{T}_p; )</td>
</tr>
<tr>
<td>( y_{pk} + y_{kp} \leq 1 ), \quad \forall , p \in \mathcal{P}, k \in \mathcal{T}_p; )</td>
</tr>
<tr>
<td>( f_p + \frac{b_p}{2} \leq W, \quad \forall , p \in \mathcal{P}; )</td>
</tr>
<tr>
<td>( R_p b_p \geq D_p, \quad \forall , p \in \mathcal{P}; )</td>
</tr>
<tr>
<td>( y_{pk} \in {0, 1}, \quad \forall , p \in \mathcal{P}, k \in \mathcal{T}_p; )</td>
</tr>
<tr>
<td>( f_p \in {0.5, 1, 1.5, \ldots, f_{\text{max}}}, \quad \forall , p \in \mathcal{P}; )</td>
</tr>
<tr>
<td>( b_p \in {0, 1, \ldots, b_{\text{max}}}, \quad \forall , p \in \mathcal{P}; )</td>
</tr>
<tr>
<td>( W \in {0, 1, \ldots, b_{\text{tot}}} ).</td>
</tr>
</tbody>
</table>

In the following two sections, we consider the uncertainty in the AP demands. The demand of AP \( p \in \mathcal{P} \) is represented as a stochastic variable \( D_p \). First, we propose a static joint stochastic center frequency index and bandwidth allocation scheme. Then, we develop a two-stage joint stochastic center frequency index and adaptive bandwidth allocation scheme.

IV. SPECTRUM DISTRIBUTION UNDER STOCHASTIC AP DEMANDS: A STATIC APPROACH

In this section, we develop the static single-stage joint stochastic center frequency index and bandwidth allocation problem formulation and provide its deterministic equivalent problem (DEP).

A. Problem Formulation

Using chance-constrained stochastic programming [5], we formulate our static probabilistic allocation problem. The objective is to minimize the amount of spectrum that is collectively utilized by all APs. At the same time, we want to guarantee that each AP demand is satisfied with probability \( \geq \beta \), where \( \beta \in [0, 1] \).

The probabilistic satisfaction of AP demands can be formulated using the following “chance constraint”:

\[
\Pr \left\{ R_p b_p \geq \bar{D}_p \right\} \geq \beta, \quad \forall \, p \in \mathcal{P}. \tag{9}
\]

B. Problem Reformulation and Solution Procedure

To convert our chance-constrained program to a deterministic program, we need to reformulate the chance constraint as follows:

\[
R_p b_p \geq F^{-1}_{\bar{D}_p}(\beta), \quad \forall \, p \in \mathcal{P}, \tag{10}
\]

where \( F^{-1}_{\bar{D}_p}(\beta) \) is the \( \beta \)-quantile function of \( \bar{D}_p \) (equivalently, the inverse CDF of \( \bar{D}_p \) evaluated at \( \beta \)) and can be obtained numerically using the distribution of \( \bar{D}_p \).

V. SPECTRUM DISTRIBUTION UNDER STOCHASTIC DEMANDS: AN ADAPTIVE APPROACH

In this section, we present the two-stage joint stochastic center frequency index and adaptive bandwidth allocation problem formulation and provide its DEP.

A. Problem Formulation

Using two-stage stochastic programming, we formulate our spectrum distribution problem. In contrast to the static approach, here the bandwidth assignment adapts to the variations in the AP demands. The goal of the first-stage problem is to optimally determine the center frequency index for each AP, knowing the distribution of each AP demand. The first-stage problem decision is static and is taken before knowing the realization of each AP demand. In the second-stage problem, the bandwidth is optimized for each AP under each realization of AP demands. The target is to minimize the maximum difference between an AP demand and its achieved rate. Let \( y = \{y_{pk} : p \in \mathcal{P}, k \in \mathcal{T}_p\}, \) \( b = \{b_p : p \in \mathcal{P}\}, \) and \( D = \{D_p : p \in \mathcal{P}\}. \) Then, the two-stage stochastic optimization problem can be formulated as follows:

<table>
<thead>
<tr>
<th>Adaptive Channel Allocation Under Stochastic AP Demands</th>
</tr>
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<tbody>
<tr>
<td>( \min \left{ W + \alpha \mathbb{E} \left[ \psi \left( y, b, D \right) \right] \right} )</td>
</tr>
<tr>
<td>s.t. ( f_p \in {0.5, 1, 1.5, \ldots, f_{\text{max}}}, \quad \forall , p \in \mathcal{P}; )</td>
</tr>
<tr>
<td>( W \in {0, 1, \ldots, b_{\text{tot}}} ),</td>
</tr>
<tr>
<td>where ( \psi \left( y, b, D \right) ) is the optimal value of the second-stage problem, which is given by:</td>
</tr>
<tr>
<td>( \min_{p \in \mathcal{P}} \max_{b_p \in \mathcal{P}} \left{ \bar{D}_p - R_p b_p \right} )</td>
</tr>
<tr>
<td>s.t. ( f_p + \frac{b_p}{2} - (f_k - \frac{b_k}{2}) &lt; b_{\text{tot}} y_{pk}, \quad \forall , p \in \mathcal{P}, k \in \mathcal{T}_p; )</td>
</tr>
<tr>
<td>( y_{pk} + y_{kp} \leq 1, \quad \forall , p \in \mathcal{P}, k \in \mathcal{T}_p; )</td>
</tr>
<tr>
<td>( f_p + \frac{b_p}{2} \leq W, \quad \forall , p \in \mathcal{P}; )</td>
</tr>
<tr>
<td>( f_p - \frac{b_p}{2} \geq 0, \quad \forall , p \in \mathcal{P}; )</td>
</tr>
<tr>
<td>( y_{pk} \in {0, 1}, \quad \forall , p \in \mathcal{P}, k \in \mathcal{T}_p; )</td>
</tr>
<tr>
<td>( b_p \in {0, 1, \ldots, b_{\text{max}}}, \quad \forall , p \in \mathcal{P}, )</td>
</tr>
</tbody>
</table>

where \( \alpha \) is a design coefficient to control the trade-off between the total number of utilized channels and the APs demand satisfaction.

B. Problem Reformulation and Solution Procedure

The objective function of the second-stage problem is not linear. To have an equivalent linear formulation, we define a new objective term \( S \) and add the following constraint:

\[
\bar{D}_p - R_p b_p \leq S, \quad \forall \, p \in \mathcal{P}. \tag{11}
\]

Here, we need to represent each scenario which corresponds to a specific realization of AP demands in the
new formulation. Denote $\Omega$ as the set of “scenarios”, or all possible demand realizations, and $\omega \in \Omega$ is a specific realization. Let $p^{(\omega)}$ be the probability of scenario $\omega \in \Omega$. The DEP can be expressed as follows:

$$
\min \left\{ W + \alpha \sum_{\omega \in \Omega} p^{(\omega)} S^{(\omega)} \right\}
$$

s.t.

$$
\begin{align*}
D_p^{(\omega)} - R_p b_p^{(\omega)} & \leq S^{(\omega)}, \quad \forall p \in \mathcal{P}, \omega \in \Omega; \\
\left( f_p + \frac{b_p^{(\omega)}}{2} \right) - \left( f_k - \frac{b_k^{(\omega)}}{2} \right) & < b_{\text{hot}} y_{pk}^{(\omega)}, \quad \forall p \in \mathcal{P}, k \in \mathcal{T}_p, \omega \in \Omega; \\
y_{pk}^{(\omega)} & \leq 1, \quad \forall p \in \mathcal{P}, k \in \mathcal{T}_p, \omega \in \Omega; \\
f_p + \frac{b_p^{(\omega)}}{2} & \leq W, \quad \forall p \in \mathcal{P}, \omega \in \Omega; \\
f_p - \frac{b_p^{(\omega)}}{2} & \geq 0, \quad \forall p \in \mathcal{P}, \omega \in \Omega; \\
f_p & \in \{0, 1, 1.5, \ldots, f_{\text{max}}\}, \quad \forall p \in \mathcal{P}; \\
y_{pk}^{(\omega)} & \in \{0, 1\}, \quad \forall p \in \mathcal{P}, k \in \mathcal{T}_p, \omega \in \Omega; \\
b_p^{(\omega)} & \in \{0, 1, \ldots, b_{\text{max}}\}, \quad \forall p \in \mathcal{P}, \omega \in \Omega; \\
W & \in \{0, 1, \ldots, b_{\text{hot}}\}.
\end{align*}
$$

### VI. Performance Evaluation

#### A. Evaluation Setup

In our experiments, we use two network setups. The first network setup is to demonstrate the benefits of the static stochastic allocation approach. The network consists of ten APs, each of which can have up to 15 users. The interference relationships between APs are shown in Fig. 2, where a line between two APs exists if they are interfering. The exogenous arrival rate of users at each AP is uniformly and independently generated between one and five users per minute. The expected stay time of users at each AP is randomly selected between one and five minutes. $p_{ij}$ is set to $1/(P + 1)$ for all APs which means that the probability that a user leaves the network is the same as being associated to any other AP.

For the closed-class users, we used the same values for $N$ and $v_i$ as in [10]. The data rate per channel is assumed to be the same for all APs, i.e. $R_p \triangleq R, \forall p \in \mathcal{P}$. The demand per user is also assumed to be the same for all APs and users (we denote it by $d$). The ratio $R/d$ is used as a parameter in our simulations. The second network setup is to show the advantage of the two-stage adaptive stochastic allocation approach. The network consists of three APs, each of which can have up to seven users. We consider a general case where AP 1 interferes with both AP 2 and 3 while AP 2 and 3 are not interfering with each other. We ran our experiments on a general-purpose desktop computer, which is Dell Precision T7600 with 16 processor cores (Intel Xeon CPU E5-2687W 0 @ 3.1 GHz) and 64 GB RAM. CPLEX was used to solve our optimization problems. For the naive and the static (single-stage) stochastic schemes, whenever CPLEX reports multiple optimal solutions (with the same value of $W$), we pick the solution that gives the maximum average probability of demand satisfaction across all APs.

We use the following metrics to evaluate our proposed stochastic allocation approaches: (i) the total number of utilized channels ($W$), (ii) the average probability of demand satisfaction across all APs, denoted as $\xi$, and (iii) the average AP demand dissatisfaction, denoted as $\chi$. This metric considers only the cases where the AP demands are not satisfied. The relative deficit in each AP demand satisfaction is obtained then we calculate the average over all AP deficits. For the static (single-stage) and naive schemes, the relative deficit for each AP $p \in \mathcal{P}$ can be computed as follows:

$$
\sum_{d_p \in \mathcal{D}_p} \left\{ \Pr\{\bar{D}_p = d_p\} \frac{\max(D_p - R b_p, 0)}{d_p} \right\} \times 100%.
$$

For the two-stage problem, the relative deficit for each AP $p \in \mathcal{P}$ is calculated as follows:

$$
\sum_{\omega \in \Omega \cdot D_p^{(\omega)} > R b_p^{(\omega)}} \left\{ p^{(\omega)} \frac{\max(D_p^{(\omega)} - R b_p^{(\omega)}, 0)}{D_p^{(\omega)}} \right\} \times 100%.
$$

#### B. Static vs. Naive

In this subsection, we illustrate the gains of static approach. More specifically, we compare the performance of the deterministic naive scheme (which considers the mean value of the demand) with the static approach under two values of $\beta$. In our experiments, $\Pr\{\bar{D}_p \leq \mathbb{E}\{D_p\}\}$ is between 0.41 and 0.55. As shown in Fig. 3, when $\beta = 0.45$, the static approach consumes slightly larger number of channels but both $\xi$ and $\chi$ are better than the naive approach. When $\beta = 0.85$, both $\xi$ and $\chi$ have much better values at the cost of more allocated channels. Also, when $R/d \leq 1.5$, there is no feasible solution as $\beta$ is too large to be satisfied given the available number of channels. The fluctuations of $\xi$ and $\chi$ in Fig. 3(b) and (c), respectively, can be explained through an example as follows. Let us consider the curves corresponding to $\beta = 0.45$. When the value of $R/d$ increases from 1.5 to 1.6, $W$ decreases from 15 to 14, $\xi$ increases, and $\chi$ decreases because the increment in $R/d$ could make-up for the decrement in $\xi$. However, when $R/d$ is too large, $\xi$ decreases and $\chi$ increases because the increment in $R/d$ cannot make up for the decrement in $\xi$. In this case, $\chi$ increases more than $\xi$.

Fig. 2: Interference relationships in the first network setup.
Total number of utilized channels (W)

Average probability of AP demand satisfaction (ξ)

Average AP demand dissatisfaction (χ) (%)

C. Adaptive Approach

Fig. 4 demonstrates the capability of the adaptive approach to control the trade-off between W and both ξ and χ. When α is sufficiently small (i.e. ≤ 1), the network seizes as few as one channel at the cost of small ξ and big χ. On the other hand, when α is sufficiently large (i.e. ≥ 10³), ξ approaches one, and χ drops to zero at the cost of seizing more channels.

D. Adaptive vs. Static

Here, we compare the adaptive and static approaches. As discussed in Section V, α controls the tradeoff between W and the APs demand satisfaction. When α is set to a sufficiently small values (e.g. α = 1), Fig. 5(a) shows that W of the adaptive approach is significantly less than that of the static case. On the other hand, when α is set to a sufficiently large values (e.g. α = 10³), Fig. 5(b) and (c) show that the adaptive approach provides bigger ξ and much smaller χ than that of the static case.

VII. RELATED WORK

Channel bonding was shown in the literature to be an effective technique for optimizing WLAN performance under deterministic demands [4], [11], [12], [13], [14], [15], [16], [17], [18]. In [4], Moscibroda et al. addressed the problem of band allocation with adaptive width in WLANs. They formulated the problem as an ILP and provided an NP-completeness proof. Then, they developed approximation algorithms where the interfering APs were assigned non-overlapping channels. In [11], Zarinni and Das proposed a dynamic spectrum distribution technique in which interfering APs are allowed to use overlapping channels if the links to their users are not interfering. In [12], Arslan et al. proposed an algorithm to dynamically select the channel center frequency and switch between 20 and 40 MHz channel widths in order to maximize the throughput. In [13], Deek et al. characterized the channel bonding behavior in 802.11n networks and studied its impact on the network performance. In [14], Herzen et al. considered solving the problem of jointly allocating channel center frequencies and bandwidths for home WLANs through a decentralized algorithm. They considered the trade-off between interference mitigation and the extra capacity offered by allocating more bandwidth to each AP. In [15], Hanada et al. adopted a game-theoretic approach to achieve coordination between APs at Nash equilibrium. They considered the scenario of interfering APs where each AP selects multi-bandwidth channels to maximize its own throughput. In [16], a continuous-time Markov network model was utilized to analyze the CSMA/CA behavior, and capture the interactions between a group of neighboring WLANs. In [18], Abdel-Rahman and Krunz jointly optimized channel bonding and guard band allocation to maximize spectrum efficiency assuming deterministic and stochastic channel rates, respectively.

VIII. CONCLUSIONS

In this paper, we proposed a novel stochastic spectrum distribution framework for WLANs, which accounts for the APs’ demand uncertainty. We developed static and adaptive stochastic bandwidth allocation approaches. The objective was to minimize the total number of channels allocated to the network. The static approach also considered guaranteeing a configurable minimum probability of satisfaction for each AP demand. On the other hand, the adaptive approach provided a trade-off between the number of channels and the level of AP demand satisfaction. Through extensive simulations, we showed the superiority of the proposed stochastic allocation framework compared to the naive approach.

This work can be extended in several directions. The problem size grows quickly with the number of APs and possible values of user demands for the adaptive formulation. Approximation methods are then needed in order to facilitate finding good solutions for reasonable network size. On the other hand, we assumed only one
source of uncertainly in our problem: AP demands. Channel conditions and per-user demand are other potential sources of uncertainly which we will consider in future work.

REFERENCES