A Mathematical Analysis of Cellular Interference on the Performance of S-band Military Radar Systems

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Abstract—In the United States, the 3500-3650 MHz band is a potential candidate for spectrum sharing between military radars and commercial cellular systems. This paper presents a framework for the analysis of radar performance under cellular interference. The impact on the performance of radar due to cellular interference is studied by deriving bounds on the probability of detection and probability of miss detection. For this purpose, we first derive the distribution of aggregate cellular interference, in a correlated shadow fading environment, at the radar receiver. We prove that the sum of interference signals from a cellular system has a log-normal distribution with probability 1. We then derive a lower bound on the probability of miss target where we consider our target to be a ship and target returns are modeled by a log-normal distribution. Along with the analytical results we also provide the corresponding simulation results showing degradation in radar performance due to interference from cellular systems.

I. INTRODUCTION

Recently, in order to accommodate growing bandwidth demands, regulators and operators have taken initiative to explore secondary access to VHF/UHF bands. This has resulted in many policy level decisions by regulators around the world [1] to accommodate secondary access without harming the incumbents. In order to further facilitate the growth in commercial spectrum utilization, the United States government is exploring ways to share spectrum currently in-use by the federal agencies. This move is motivated by recent studies by the National Telecommunications and Information Administration (NTIA), along with the Federal Communications Commission (FCC), which found under utilization of huge chunks of spectrum reserved for the federal agencies. Spectrum sharing promises huge economic and social prospects but also brings in new challenges for the optimal operation of incumbents and commercial users, from a harmful interference perspective. It is required to first understand the challenges and then propose innovative interference mitigation methods along with the policy level decisions needed to make spectrum sharing a reality. This work is a step towards understanding the challenges posed by sharing radar spectrum with commercial cellular systems.

In order to analyze the problem we first need an accurate model for the distribution of aggregate interference. This also helps in cellular system planning to reduce the harmful effects of interference and ensures performance of both the cellular and radar systems. In determining a model of aggregate interference, from a cellular system, several factors need to be considered which includes their spatial distribution, total number of transmitters, power control, and channel parameters. In wireless communications, interference is usually characterized by sum of log-normal random variables. This is why the distribution of sum of log-normal random variables has been an active topic of research since 1960s [2]. Since then, many people have used approximation techniques to approximate distribution of log-normal sums. Some of the most commonly used approximation methods include Wilkinson's [3], a moment matching approach for the first two moments, and Schwartz and Yeh's exact first two moment expressions for a sum of two log-normal random variables. In the context of cellular communications, aggregate interference has been approximated using various approximations and moment matching techniques, see [4] and references therein. In this work, we prove that the aggregate cellular interference has a log-normal distribution with probability 1.

Dynamic frequency selection (DFS) methods have been successfully used in the past to share the 5 GHz spectrum between radars and wireless LANs [5]. However, the topic of spectrum sharing between radars and cellular systems has received little attention thus far. Recent efforts include spectral, temporal, system level, and spatial approaches for interference mitigation [6], [7], [8]. Others include radar waveform shaping [9], [10], radar waveform design [11], and resource allocation at cellular system [12] for radar spectrum sharing. The authors in [13], propose an exclusion-region and secondary-user-density based model to share spectrum between an aeronautical radar, operating in the 960-1215 MHz band, and indoor femto cell users. The operation of femto cell users is limited by the interference threshold established by the central network in order to protect the radar. However, this paper doesn't address radar performance for macro cell users and for the case when the cellular operation is not constrained by any limit imposed on the level of interference, we seek to explore such a possibility in this paper. This also serves as a motivation to explore the tolerable interference limits for radar under consideration for policy level decisions.

In this paper, we first provide a spectrum sharing model in Section II. In Sections III and III-A we state and prove that

the aggregate interference from a cellular system has a log-normal distribution, respectively. In Section III-B, we compute the interference parameters. In Section IV we explain our radar system and target model. Section V derives bounds on the probability of detection. Section VI presents simulation results along with the discussion and Section VII concludes the paper.

II. SYSTEM MODEL AND NOTATIONS

We consider a model in which a cellular system with N base stations (BS) is sharing radar spectrum to increase its capacity. However, this sharing of spectrum results in cellular interference to radar system. We assume the radar is at a distance of $r_i, i = 1, 2, \cdots, N$, from the ith BS. Our model is general in nature as we do not consider any specific spatial distribution of the cellular system. In addition, all the BS are capable of transmitting at arbitrary power levels of their interest. The only assumption is that the location of all the BSs is known to the radar at which we want to characterize the interference. This is a fair assumption since in macro cellular systems BS locations are subject to network planning.

The wireless propagation environment that exists between shipborne radars and BSs of cellular systems is significantly different than that of a typical mobile in a cellular system. This is due to the fact that radar is located far away and only path loss, which is significant over large distances, and shadow fading, due to blockage of signals from large obstacles, affect the received signal strength at radar receiver. Due to the same reason, radar receiver is insensitive to the effect of small scale fading, due to mulipath propagation. These factors bring in novelty in the problem of aggregate cellular interference analysis for radar system and makes it different from analyzing interference at a particular receiver inside a cellular system. Thus, in order to consider a realistic interference scenario, we consider mean path loss and log-normal shadow fading models, which are commonly used in interference analysis treatments [14]. Then, the interference power received from the ith BS is

$$I_i = P_i r_i^{-\alpha} e^{X_i}, \qquad i = 1, 2, \cdots, N, \qquad (1)$$

where P_i denotes the power transmitted, $r_i^{-\alpha}$ is the path loss exponent, and e^{X_i} is the log-normal random variable, where X_i is the transmitted signal from the ith BS. We consider X_i's that are jointly correlated Gaussian random variables with mean, μ_{X_i}, and variance, $\sigma_{X_i}^2$. We consider that the jointly correlated Gaussian random variables have a specific correlation structure, as specified in [15],

$$\rho_{ij} = \frac{\mathbb{E}\left[(X_i - \mu_{X_i})(X_j - \mu_{X_j})\right]}{\sigma_{X_i}\sigma_{X_j}} = \beta_i\beta_j. \qquad (2)$$

The model under consideration has some other practical applications. It can be used to characterize interference from a secondary cellular system to a primary TV system where the cellular system is opportunistically using the TV white spaces to enhance its capacity [1]. Another application is the IEEE 802.22 digital TV (DTV) [16] scenario in which a cellular system is used to broadcast a TV signal and the interest is
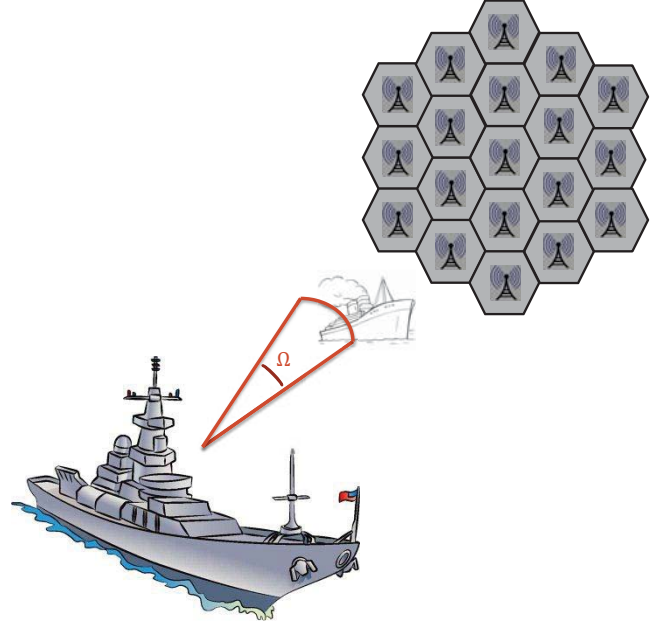

Fig. 1. Shipborne electrically-steered phased array radar is experiencing interference from an onshore cellular system while detecting a seaborne target that is a ship. The radar's main beam is subject to interference from cellular system.

in the interference from this system to cognitive radios using spectrum opportunistically.

III. AGGREGATE INTERFERENCE DISTRIBUTION

In order to evaluate the impact of cellular interference on radar's performance it is necessary to characterize the statistic of the interference. The interference from N BSs, at radar, is the sum of individual interference powers and can be written as

$$I = \sum_{i=1}^{N} I_i$$
$$= \sum_{i=1}^{N} P_i r_i^{-\alpha} e^{X_i}. \qquad (3)$$

Then, the distribution of aggregate interference, described by equation (3), follows the log-normal distribution according to the following theorem.

Theorem 1. *The probability distribution of the normalized aggregate interference, at a radar system, has a limit distribution which is log-normal with probability 1, i.e.*

$$I = \frac{\sum_{i=1}^{N} I_i}{N} \xrightarrow{w.p.1} \ln \mathcal{N}(\mu_I, \sigma_I^2)$$

where the parameters μ_I and σ_I^2 are given as

$$\mu_I = \frac{1}{N} \sum_{i=1}^{N} P_i r_i^{-\alpha} e^{\left(\mu_{X_i} + \frac{\sigma_{X_i}^2}{2}\right)}$$

and

$$\sigma_I^2 = \mu_I^2 \left(e^{\zeta^2} - 1\right)$$

where

$$\zeta^2 = \sigma_{X_i}\sigma_{X_j}\beta_i\beta_j.$$

The cumulative probability density function (cdf) is given as

$$\mathbb{F}_I(i;\mu_I,\zeta) = \mathbb{P}(I \leq i) = 1 - \mathbb{Q}\left(\frac{\ln i - \ln \mu_I}{\zeta} + \frac{\zeta}{2}\right)$$

where $\mathbb{Q}(\cdot)$ is the Marcum's Q-function defined as

$$\mathbb{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt.$$

A. Proof of the Aggregate Interference Distribution

Let $Y_i, i = 0,1,\cdots,N$, be independent Gaussian random variables with mean μ_{Y_i} and variance $\sigma_{Y_i}^2$. The first step is to create jointly correlated Gaussian random variables $X_i, i = 1,\cdots,N$, with a transformation

$$X_i = \zeta Y_0 + Y_i \tag{4}$$

where ζ is a positive real number and we set $\mu_{Y_0} = 0$ and $\sigma_{Y_0}^2 = 1$. With this transformation $\mu_{X_i} = \mu_{Y_i}$ and $\sigma_{X_i}^2 = \zeta^2 + \sigma_{Y_i}^2$.

In order to proceed with the proof, we first define the independent log-normal random variables

$$Z_i = P_i r_i^{-\alpha} e^{Y_i} \quad i = 1,2,\cdots,N \tag{5}$$

with mean

$$\mu_{Z_i} = P_i r_i^{-\alpha} e^{\mu_{Y_i}+\sigma_{Y_i}^2/2}$$

and variance

$$\sigma_{Z_i}^2 = \left(P_i r_i^{-\alpha}\right)^2 e^{\left(2\mu_{Y_i}+\sigma_{Y_i}^2\right)}\left(e^{\sigma_{Y_i}^2}-1\right).$$

Second, we define jointly correlated log-normal random variables

$$I_i = P_i r_i^{-\alpha} e^{X_i} \quad i = 1,2,\cdots,N$$

with mean

$$\mu_{I_i} = \mu_{Z_i} e^{\zeta^2/2} \tag{6}$$

and variance

$$\sigma_{I_i}^2 = \left(P_i r_i^{-\alpha}\right)^2 e^{\left(2\mu_{Y_i}+\sigma_{Y_i}^2\right)}\left(e^{\sigma_{Y_i}^2}-1\right).$$

Using (3) we can write the normalized aggregate interference as

$$I = \frac{1}{N}\sum_{i=1}^N I_i$$

$$= \frac{1}{N}\sum_{i=1}^N P_i r_i^{-\alpha} e^{X_i}. \tag{7}$$

Substituting X_i from equation (4) in (7) we get

$$I = \frac{1}{N}\sum_{i=1}^N P_i r_i^{-\alpha} e^{(\zeta Y_0 + Y_i)}$$

$$= \frac{1}{N}e^{\zeta Y_0}\sum_{i=1}^N P_i r_i^{-\alpha} e^{Y_i}$$

$$= e^{\zeta Y_0}\frac{\sum_{i=1}^N Z_i}{N}. \tag{8}$$

Applying the strong law of large numbers on equation (8) yields

$$\lim_{N\to\infty} I = e^{\zeta Y_0}\lim_{N\to\infty}\frac{\sum_{i=1}^N Z_i}{N}$$

$$= e^{\zeta Y_0}\frac{\sum_{i=1}^N \mathbb{E}[Z_i]}{N}$$

$$= e^{\zeta Y_0}\frac{\sum_{i=1}^N \mu_{Z_i}}{N}$$

$$= e^{\zeta Y_0}\bar{\mu}_{Z_i}$$

$$= e^{(\zeta Y_0 + \ln \bar{\mu}_{Z_i})} \triangleq \widetilde{I} \tag{9}$$

where $\widetilde{I}$ is a log-normal random variable with mean $\mu_{\widetilde{I}}$ and variance $\sigma_{\widetilde{I}}^2$. Note that equation (9) follows with probability one if Z_i's are independent, this follows from our definition of Z_i's in equation (5), and if the series $\sum_{i=0}^\infty \sigma_{Z_i}^2/i^2$ converges. To prove that, let

$$\sigma_{max}^2 = \max_i \sigma_{Z_i}^2$$

$$= \max_i\left\{\left(P_i r_i^{-\alpha}\right)^2 e^{\left(2\mu_{Y_i}+\sigma_{Y_i}^2\right)}\left(e^{\sigma_{Y_i}^2}-1\right)\right\}.$$

Since $P_i, r_i^{-\alpha}, \mu_{Y_i}$, and $\sigma_{Y_i}^2$ are all bounded so $\sigma_{max}^2 < \infty$. Then

$$\sum_{i=0}^\infty \frac{\sigma_{Z_i}^2}{i^2} \leq \sum_{i=0}^\infty \frac{\sigma_{max}^2}{i^2}$$

$$= \sigma_{max}^2\sum_{i=0}^\infty \frac{1}{i^2} = \sigma_{max}^2\frac{\pi^2}{6} < \infty$$

where the last equality follows from [15].

B. Parameters of the Aggregate Interference Distribution

In this section we find the parameters for the aggregate interference in terms of our signal parameters, $P_i, r_i^{-\alpha}, \mu_{X_i}, \sigma_{X_i}^2$, as described by equation (3). In the last section we showed that the aggregate interference has a joint log-normal distribution with parameters μ_I and σ_I^2 which can be calculated from equation (9) by first noting

$$\mu_{\widetilde{I}} = e^{\zeta^2/2}\bar{\mu}_{Z_i}. \tag{10}$$

Using definition of $\bar{\mu}_{Z_i}$ we can write

$$\bar{\mu}_{Z_i} = \frac{1}{N}\sum_{i=1}^N \mu_{Z_i} = e^{-\zeta^2/2}\frac{1}{N}\sum_{i=1}^N \mu_{I_i} \tag{11}$$

where (11) follows from (6). Substituting (11) in (10) yields

$$\mu_{\widetilde{I}} = \frac{1}{N}\sum_{i=1}^N \mu_{I_i}$$

$$= \frac{1}{N}\sum_{i=1}^N P_i r_i^{-\alpha} e^{\left(\mu_{X_i}+\frac{\sigma_{X_i}^2}{2}\right)} \triangleq \mu_I. \tag{12}$$

Similarly, the variance of the aggregate interference is given as

$$\sigma_{\widetilde{I}}^2 = \mathbb{E}\left[\left(\widetilde{I}-\mu_{\widetilde{I}}\right)^2\right] = \bar{\mu}_{Z_i}^2 e^{\zeta^2}\left(e^{\zeta^2}-1\right). \tag{13}$$

Now, using equations (10) to (12), we can write (13) in terms of our interference parameters as

$$\sigma_{\tilde{I}}^2 = \mu_I^2 \left(e^{\zeta^2} - 1\right) \triangleq \sigma_I^2.$$

Next, in order to determine ζ^2 we proceed using equations (4) and (7), i.e.,

$$
\begin{aligned}
\rho_{ij} &= \frac{\mathbb{E}\left[(X_i - \mu_{X_i})(X_j - \mu_{X_j})\right]}{\sigma_{X_i}\sigma_{X_j}} \\
&= \frac{\mathbb{E}\left[(\zeta Y_0 + Y_i - \mu_{Y_i})(\zeta Y_0 + Y_j - \mu_{Y_j})\right]}{\sigma_{X_i}\sigma_{X_j}} \\
&= \frac{\zeta^2}{\sigma_{X_i}\sigma_{X_j}} = \beta_i\beta_j.
\end{aligned} \tag{14}
$$

The last equality follows from our assumptions, introduced at the beginning of this proof, i.e. $\mu_{Y_0} = 0$, $\sigma_{Y_0}^2 = 1$, and $Y_i, i = 0, 1, \cdots, N$, being independent Gaussian random variables. Then ζ^2 follows from (14) as

$$\zeta^2 = \sigma_{X_i}\sigma_{X_j}\beta_i\beta_j.$$

The cdf of aggregate interference follows from the definition of cdf of log-normal random variables and the definitions of μ_I and σ_I^2 after some algebraic manipulations.

IV. RADAR SYSTEM AND TARGET MODEL

In this paper, we consider a shipborne electronically-steered phased array radar with four phased arrays, each capable to carry a $45°$ azimuth scan. The phased array radars are capable of performing multiple functions at the same time. Some of the functions include complete search of hemisphere, track multiple targets; illuminate multiple targets and guide missiles towards them; they also have flexible search and track rates and frequency agility.

We consider a seaborne target which is a ship. The detection of fluctuating target signals, in the presence of Gaussian noise, is a well studied problem in radar literature [17]. Usually it is assumed that the amplitude of the fluctuating signal has a Rayleigh distribution. This assumption is justified since radar returns from a target are composed of numerous and diverse reflecting elements. Then, statistically, the sum of these large number of independent random vectors, each having a uniform phase and a Rayleigh amplitude distribution, is a Rayleigh vector [18]. However, the Rayleigh assumption is generally true for targets having small radar cross section (RCS) because targets with high RCS exhibit a heavier tail than the Rayleigh distribution [19].

It is observed that certain target signals are best modeled by a log-normal distribution, for example, targets like ships, satellites and space vehicles. Assuming a Rayleigh distributed signal for such targets lead to significant errors in the probability of detection [20]. Therefore, we use a log-normal target model, which is constant within a scan but fluctuates log-normally from scan to scan. Since, the maximum effect of interference on radar will be during the azimuth search/track/detect operation and that is where the target is located it is appropriate to use a log-normal target model. Any

other target model for such a scenario may not give accurate results due to mismatch in target models. Thus, the reflected signal from the target ship has a log-normal distribution and can be expressed as

$$x = P_r e^{-\psi}$$

where P_r is the power of the received signal from the target, ψ is normally distributed with mean μ_ψ and variance σ_ψ^2. The probability of detection for such a target, no-interference case, is given as [20]

$$\mathbb{P}_D(\text{SNR}, \Gamma, n_p, \psi) = \int_0^\infty \mathbb{P}_{n_p}(x, \Gamma)f(x|\text{SNR}, \psi)\,dx \quad (15)$$

where

$$
\begin{aligned}
\mathbb{P}_{n_p}(\text{SNR}, \Gamma) = \int_\gamma^\infty & (v|\text{SNR})^{(n_p-1)/2} e^{-(v-n_p\text{SNR})} \\
& \mathbb{I}_{n_p-1}\left(2\sqrt{\text{SNR}\,n_p v}\right)\,dv
\end{aligned}
$$

is the probability of detection for n_p integrated pulses with some signal to noise ratio (SNR) at some detector threshold Γ, $\mathbb{I}_K(\cdot)$ is the modified Bessel function of order K, and

$$f(x|\text{SNR}, \psi) = \frac{1}{\sqrt{2\pi}\sigma_\psi x}\exp\left(-\frac{\ln^2(\psi x/\text{SNR})}{2\sigma_\psi^2}\right), \quad \psi \geq 1$$

is the probability density function for the log-normally fluctuating signal with ψ being the fluctuation parameter and $\sigma_\psi^2 = 2\ln\psi$.

The radar range equation is a useful metric for estimating the range of a radar, as a function of radar parameters. We are interested in the maximum radar range, $R_{\max}$, that can be achieved in a cellular interference scenario. The simple form of the radar equation is given as [21],

$$R_{\max}^4 = \frac{P_{t,\max}G_t G_r \bar{\sigma}\lambda^2}{(4\pi)^3 P_{r,\min}} \tag{16}$$

where $P_{t,\max}$ is the peak transmitted power from the transmit antenna of gain G_t, G_r is the gain of the receive antenna, λ is the wavelength, $\bar{\sigma}$ is the RCS of the target ship, and $P_{r,\min}$ is the smallest received power that can be detected by the radar. Since we are considering our target to be a ship its RCS can be approximated by the ship's displacement in tons. An empirical relation to calculate RCS is given by [21]

$$\bar{\sigma} = 52 f^{1/2} D^{3/2}\,\text{m}^2$$

where f is the radar operating frequency in megahertz (MHz), and D is the ship's (full load) displacement in kilotons.

V. RECEIVER OPERATING CURVES (ROC)

Assume the target is located in the same azimuth as that of cellular system. We are interested in finding the distribution of signal-to-interference-plus-noise ratio (SINR) so as to evaluate the radar's ROC. The SINR at the radar can be written as

$$
\begin{aligned}
\text{SINR} &= \frac{P_r e^{-\psi}}{P_n + \sum_{i=1}^N P_i r_i^{-\alpha} e^{X_i}} \\
&= \frac{e^{-\psi}}{\frac{P_n}{P_r} + \frac{1}{P_r}\sum_{i=1}^N P_i r_i^{-\alpha} e^{X_i}}.
\end{aligned}
$$

Let us define $\gamma \triangleq P_r/P_n$, so that

$$\text{SINR} = \frac{e^{-\psi}}{\frac{1}{\gamma} + \frac{1}{P_r}\sum_{i=1}^{N} P_i r_i^{-\alpha} e^{X_i}}. \quad (17)$$

We are interested in the probability of detection $\mathbb{P}_D$ which can be expressed as

$$\mathbb{P}_D = 1 - \mathbb{P}_{\text{Miss}} = 1 - \mathbb{P}(\text{SINR} < \Gamma) \quad (18)$$

where $\mathbb{P}_{\text{Miss}}$ is the probability of miss detection. The analog to $\mathbb{P}_{\text{Miss}}$ in wireless communications is the outage probability. In order to characterize $\mathbb{P}_{\text{Miss}}$ and consequently $\mathbb{P}_D$ we are interested in the distribution of SINR for which no closed form expression exists and only approximations are used. However, a lower bound on the $\mathbb{P}_D$ of radar can be achieved by using similar arguments as in [22].

In order to simplify equation (17) let us define

$$\Delta \triangleq \frac{1}{\gamma} + \frac{1}{P_r}\sum_{i=1}^{N} P_i r_i^{-\alpha}$$

$$\xi_i \triangleq \frac{P_i r_i^{-\alpha}}{P_r \Delta}$$

$$\xi_0 \triangleq \frac{1}{\gamma \Delta}$$

where $\sum_{i=0}^{N} \xi_i = 1$. The SINR, in equation (17), can be rewritten using above definitions as

$$\text{SINR} = \frac{1}{\Delta}\frac{e^{-\psi}}{\xi_0 + \sum_{i=1}^{N} \xi_i e^{X_i}}.$$

Without loss of generality we assume $\mu_\psi = 0$ and $\mu_{X_i} = 0$. Then, the lower bound on the probability of miss detection can be calculated by utilizing arithmatic-geometric mean inequality, as in [22]:

$$\mathbb{P}_{\text{Miss}} = \mathbb{P}(\text{SINR} < \Gamma)$$

$$= \mathbb{P}\left(e^{-\psi} < \Gamma\Delta\left(\xi_0 + \sum_{i=1}^{N}\xi_i e^{X_i}\right)\right)$$

$$\geq \mathbb{P}\left(e^{-\psi} < \Gamma\Delta\prod_{i=1}^{N} e^{\xi_i X_i}\right)$$

$$= \mathbb{P}\left(e^{-(\psi + \sum_{i=1}^{N}\xi_i X_i)} < \Gamma\Delta\right)$$

$$= \mathbb{P}\left(\psi + \sum_{i=1}^{N}\xi_i X_i > -\left(\ln\Gamma + \ln\Delta\right)\right)$$

$$= 1 - \mathbb{Q}\left(\frac{\ln\Gamma + \ln\Delta}{\sigma}\right). \quad (19)$$

The variance σ^2 can be evaluated as

$$\sigma^2 = \mathbb{E}\left[\left(\psi + \sum_{i=1}^{N}\xi_i X_i\right)^2\right]$$

$$= \sigma_\psi^2 + \sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i\xi_j\rho_{ij}\sigma_{X_i}\sigma_{X_j}$$

$$= \sigma_\psi^2 + \zeta^2\sum_{i=1}^{N}\sum_{j=1}^{N}\xi_i\xi_j \quad (20)$$

where equation (20) follows from the definition of jointly correlated Gaussian random variables X_i and X_j, see equation (4), the definition of ζ^2 in Theorem 1, and the assumption that target reflections and interference are independent. Then, the bound on probability of detection $\mathbb{P}_D$ can be evaluated from equation (18) by using the result of equation (19).

VI. SIMULATION AND DISCUSSION

In this section, we present our quantitative results to complement the analytical results. We employ simplified assumption on radar and cellular system parameters and their deployments in order to facilitate quantitative analysis. The military radar under consideration, SPY-1 of Aegis system, has a bandwidth of 10 MHz and has approximately 6 dB more gain on the transmit array. Moreover, they are capable to transmit at peak power levels of up to 6 MW. Some other parameters, including carrier frequency and noise floor are not necessarily exact, due
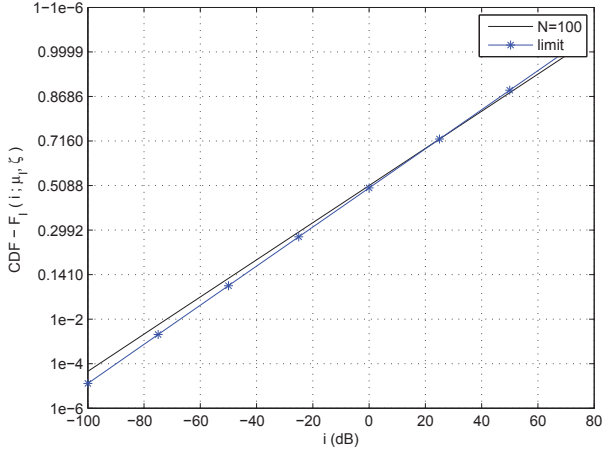
Fig. 2. The cdf, $F_I(i; \mu_I, \zeta)$, of the aggregate interference for $N = 100$ cellular base stations and the limit distribution. The limit distribution is reached for a large number of BSs but it is possible for radars which employ large beamwidths to scan azimuth for targets and can easily cover hundreds of BSs.
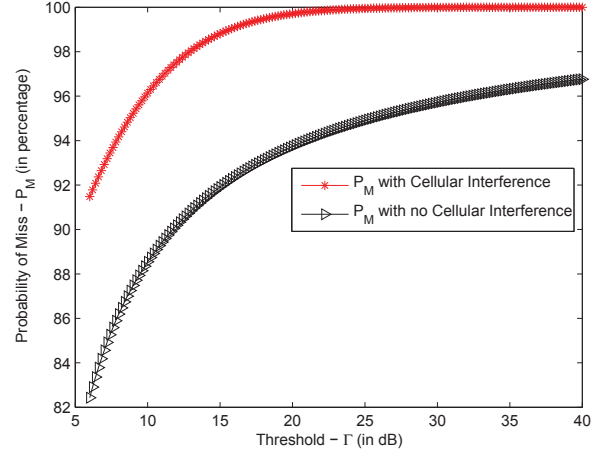

Fig. 3. Comparison of probability of miss with and without interference. The probability of miss curve without interference is an exact curve but with interference is a lower bound on the probability of miss targets.

to the unavailability of such parameters, but this does not affect in any way the results and the conclusions drawn. The radar and cellular system simulation parameters are mentioned in Tables I and II, respectively.

For the cellular system, we assume that the BS signals, X_i's, are zero-mean i.e. $\mu_{X_i} = 0$ for $i = 1, 2, \ldots, N$, and has the following variance and correlation structure:

$$\sigma^2_{X_i} = \begin{cases} 6 \, \text{dB}, & \text{for } 1 \le i \le \left\lfloor \frac{N}{2} \right\rfloor \\ 12 \, \text{dB}, & \text{for } \left\lfloor \frac{N}{2} \right\rfloor + 1 \le i \le N \end{cases}$$

and

$$\rho_{ij} = \begin{cases} 0.80, & \text{for } 1 \le i, j \le \left\lfloor \frac{N}{2} \right\rfloor \text{ and } i \ne j \\ 0.20, & \text{for } \left\lfloor \frac{N}{2} \right\rfloor + 1 \le i, j \le N \text{ and } i \ne j \\ 0.40, & \text{elsewhere.} \end{cases}$$

For the radar system, we assume, without loss of generality, the log-normal target has zero mean, $\mu_\psi = 0$, and the variance is determined by the fluctuating parameter ψ.

In Figure 2, we use a log-normal probability paper to compare our results with the limiting distribution. It is a useful tool to compare two distributions especially for log-normal distributions which form a straight line when plotted on a log-normal paper [15]. The absicca is transformed into $log(\text{absicca})$ and we plot the corresponding probabilities on vertical axis. We observe that the curve for 100 BSs is close to the limit distribution for values of cdf between 10^{-6} and 1. This value is very realistic for a search radar which employs large beamwidths, i.e. larger Ω in Figure 1, and thus is capable to scan a large azimuth which can cover areas containing hundreds of BSs.

It is expected that the radar performance degrades when subject to interference from a cellular system operating in the same band. The lower bound on the probability of miss

detection for a log-normal target is given by equation (19). This is used to compare the performance in Figure 3 for a log-normal target with and without interference with different detector threshold levels. These threshold levels are selected based on a desired probability of false alarm. The results indicate that under cellular interference a radar's performance is degraded considerably. This serves as a motivation to introduce tolerable interference levels at the radar receiver and design considerations for the deployment of cellular systems in order to protect the radar from cellular interference.

In Figure 4, we analyze the probability of miss when the radar engineer varies its transmit power, $P_{t,\max}$, from 1 MWs to 4 MWs, in order to counter the cellular interference. As the radar's transmit power is increased, the SINR increases, and the performance increases. It is also noted from Figure 4 that when the transmit power increases the probability of miss drops. This can be used to counter interference from cellular systems.

In Figure 5, we analyze the probability of miss for targets at different ranges. As expected, by increasing the target range from the radar the SINR decreases and the probability of miss increases. Hence, detection of far away targets will be a challenge for radar in cellular interference. One way to counter this is to increase transmit power in order to improve SINR so as to get better detection performance.

VII. Conclusion

We investigate the performance of S-band seaborne military radar when the incumbent is sharing its frequency band with a commercial cellular system. The radar is subject to interference from the cellular system and we evaluate the radar's performance under these new operating conditions.

First, we derive the aggregate distribution of cellular interference and prove that it is log-normal with probability 1. Our method does not rely on approximations and moment matching methods which are commonly used to characterize
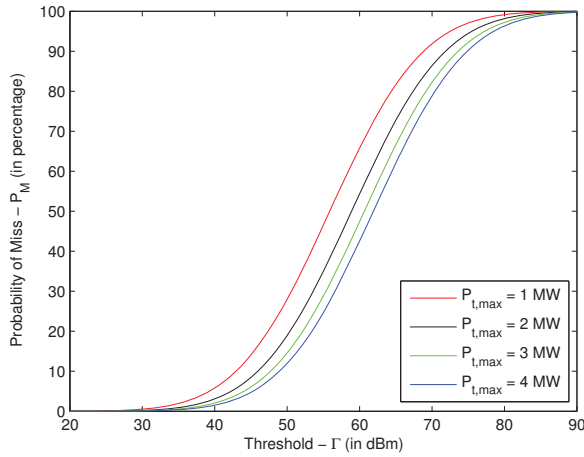
Fig. 4. Impact of cellular interference on the radar's detection performance when the radar has freedom to choose transmitted power in order to counter interference. The target is present at a distance of 100 Kms.
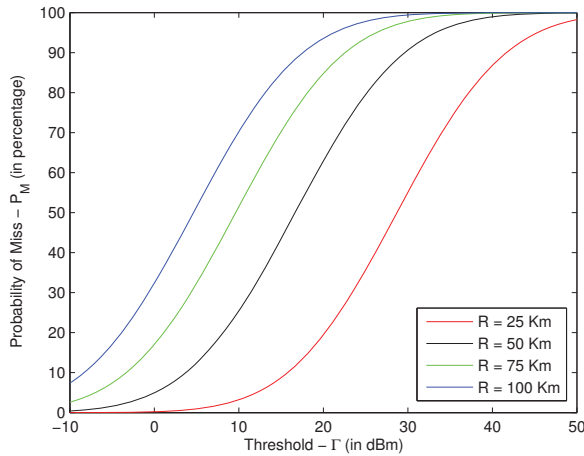

Fig. 5. Impact of cellular interference on the radar's detection performance when targets are present at different distances. The simulation parameters of Tables I and II are used except that the radar transmits a 4 MW signal.

aggregate interference distribution. Second, we derive bounds on the probability of detection and the probability of miss detection under cellular interference.

The analytical results are complemented with the simulation results where we access the detection performance of radar under cellular interference. We show that for smaller SINR values the difference in performance of radar detector with and without interference is much more than at higher SINR values. In addition, the targets that are far away are more hard to detect due to low SINR in interference regimes. We show the impact of interference can be minimized by using higher transmit powers at the radar terminal to increase SINR and thus have better detection performance.

The results presented serve as a motivation for further exploration of spectrum sharing between radar and commercial cellular systems so that both the systems can perform in an optimum way. In addition to innovative interference mitigation methods, policy level decisions, from regulators and cellular operators, are required in order to make such a sharing possible.

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