Utility Proportional Fairness Resource Allocation with Carrier Aggregation in 4G-LTE

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Abstract—In this paper, we consider a resource allocation optimization problem with carrier aggregation in fourth generation long term evolution (4G-LTE). In our proposed model, each user equipment (UE) is assigned a utility function that represents the application type running on the UE. Our objective is to allocate the resources from two carriers to each user based on its application that is represented by the utility function assigned to that user. We consider two groups of users, one with elastic traffic and the other with inelastic traffic. Each user is guaranteed a minimum resource allocation. In addition, a priority resource allocation is given to the UEs running adaptive real time applications. We prove that the optimal rate allocated to each UE by the single carrier resource allocation optimization problem is equivalent to the aggregated optimal rates allocated to the same user by the primary and secondary carriers when their total resources is equivalent to the single carrier resources. Our goal is to guarantee a minimum quality of service (QoS) that varies based on the user application type. We present a carrier aggregation rate allocation algorithm to allocate two carriers resources optimally among users. Finally we present simulation results with the carrier aggregation rate allocation algorithm.

Index Terms—Resource Allocation, Carrier Aggregation, Proportional Fairness

I. INTRODUCTION

In [1], a new feature was added to the 3GPP 4G-LTE advanced standard, Release 10. A single user is allowed to employ multiple carriers to achieve higher bandwidth. This is essential because smart phones users are so limited to the carrier bandwidths provided by the network. Smart phones are now required to run multiple numbers of applications that require a higher bandwidth and make users so limited to the carrier resources. Additionally, the problem of having a highly segmented frequency can be reduced by using the carrier aggregation approach.

This new feature will be of a great benefit for military. The naval section of the military has announced that three of its ships will be outfitted with 4G-LTE connectivity; this will make it possible for sailors to get access to their email, stream video, or do anything for personal use with high speed using their smart phones. While Ships in the US Navy fleet are generating more data, the new added feature of carrier aggregation will make it possible to achieve higher bandwidth. The 4G-LTE will serve as a localized platform for feeding wireless data to sailors, as well as a way for the enlisted to connect to the outside world and this service is improved when using carrier aggregation.

In [2], the authors introduced bandwidth proportional fair resource allocation with logarithmic utilities. The algorithms at the links are based on Lagrange multiplier methods of optimization theory, so the concavity assumption is satisfied.

In [3], the author presented a utility proportional fairness resource allocation approach, where fairness is in utilization, for 4G-LTE that optimally allocate one eNodeB resources based on the optimization problem that solves for elastic and inelastic utility functions.

In this paper, we focus on finding an optimal solution for the carrier aggregation resource allocation problem for a group of users running two types of applications presented by a logarithmic utility functions or a sigmoidal-like utility functions. These utility functions are concave and non-concave utility functions respectively. The optimization problem is to assign part of the bandwidth from two carriers to each user subscribing for a mobile service taking into consideration that each user is getting a minimum QoS. In addition, a non concave functions that are approximated by sigmoidal-like functions and presenting real-time applications are given priority over the concave functions.

A. Related Work

In [4], the author presented a weighted aggregation of elastic and inelastic utility functions in each UE. The aggregated utility functions are then approximated to the nearest concave utility function from a set of functions using minimum mean square error. That approximate utility function is solved using a modified version of the distributed rate allocation algorithm by Frank Kelly [2].

In [5], a Round Robin packet scheduling method is used to distribute the load across the network. From a network perspective, this approach seems to be fair as it assigns new user to a carrier that has the least number of current users. This method does not seem to be fair for resource allocation as the network could be inefficient in bandwidth and throughput.

B. Our Contributions

Our contributions in this paper are summarized as:

- We present a resource allocation optimization problem with carrier aggregation that gives priority to real-time application users when allocating resources.
- We prove that the optimal rate allocated by the two carriers to each user when using carrier aggregation is



equivalent to the optimal rate allocated to the same user by one carrier that has resources equivalent to the total resources in the two carriers. We present a carrier aggregation rate allocation algorithm to solve the optimization problem and its corresponding simulation results.

The remainder of this paper is organized as follow. Section III presents the problem formulation. In Section III, we prove that the optimal rate provided by one carrier to each user is equivalent to the sum of optimal rates provided to the same user by two carriers when the two carriers have a total amount of resources equivalent to the resources in the single carrier case. In section IV, we present our distributed carrier aggregation rate allocation algorithm for the utility proportional fairness optimization problem. In section V we discuss simulation setup and provide quantitative results along with discussion. Section VI concludes the paper.

II. PROBLEM FORMULATION

We consider two eNodeBs that have the same coverage area with M UEs. One of the eNodeBs is considered to be the primary carrier and the other one is the secondary carrier. Each user is allocated certain bandwidth r_i based on the type of application the UE is running. Our goal is to determine the optimal bandwidth that needs to be allocated to each user by the two eNodeBs.

We assume the utility functions $U_i(r_i)$ to be a strictly concave or a sigmoidal-like functions. These utility functions have the following properties:

- $U_i(0) = 0$ and $U_i(r_i)$ is an increasing function of r_i .
- $U_i(r_i)$ is twice continuously differentiable in r_i and bounded above.

We use the normalized sigmoidal-like utility function in our model, same as the one presented in [6], that is

$$U_i(r_i) = c_i \left(\frac{1}{1 + e^{-a_i(r_i - b_i)}} - d_i \right)$$
 (1)

where $c_i=\frac{1+e^{a_ib_i}}{e^{a_ib_i}}$ and $d_i=\frac{1}{1+e^{a_ib_i}}$ so it satisfies U(0)=0 and $U(\infty)=1$. The inflection point of the normalized sigmoidal-like function is at $r_i^{\rm inf}=b_i$. Additionally, we use the normalized logarithmic utility function, used in [7], that can be expressed as

$$U_{i}(r_{i}) = \frac{\log(1 + k_{i}r_{i})}{\log(1 + k_{i}r_{\max})}$$
(2)

where $r_{\rm max}$ gives 100% utilization and k_i is the slope of the curve that varies from user to user. So, it satisfies U(0)=0 and $U(r_{\rm max})=1$.

A. Single Carrier Optimization Problem

The basic formulation of a single carrier resource allocation problem is given by the following optimization problem:

$$\max_{\mathbf{r}_{single}} \qquad \prod_{i=1}^{M} U_i(r_{i,single})$$

$$\text{subject to} \qquad \sum_{i=1}^{M} r_{i,single} \le R,$$

$$r_{i,single} > 0, \qquad i = 1, 2, ..., M.$$
(3)

where R is the maximum achievable rate of the eNodeB, $r_{i,single}$ is the rate for user i and M is the number of UEs.

The optimization problem (3) is a convex optimization problem and there exists a unique tractable global optimal solution [3]. The objective function in the optimization problem (3) is equivalent to $\max_{\mathbf{r_{single}}} \sum_{i=1}^{M} \log U_i(r_{i,single})$. The solution of this optimization problem is the global optimal solution for the resource allocation problem when resources are allocated by one eNodeB.

For the carrier aggregation resource allocation case, the optimization problem is divided into two stages as shown in section III.

III. TWO CARRIERS OPTIMIZATION PROBLEM

A. Primary Carrier

The two carriers optimization problem is done in two stages, primary and secondary stages.

The optimization problem for the first carrier can be written as:

$$\max_{\mathbf{r_p}} \qquad \prod_{i=1}^{M} U_i(r_{i,p})$$
 subject to
$$\sum_{i=1}^{M} r_{i,p} \leq R_p,$$

$$r_{i,p} \geq 0, \quad i = 1, 2, ..., M.$$
 (4)

where $\mathbf{r_p} = \{r_{1,p}, r_{2,p}, ..., r_{M,p}\}$ and M is the number of UEs in the coverage area of primary user eNodeB and R_p is the maximum achievable rate of the primary carrier. The resource allocation objective function is to maximize the total system utility when allocating resources to each user. Furthermore, it provides proportional fairness among utilities. Users running real-time applications are allocated more resources in this approach.

The optimization problem (4) is a convex optimization problem and there exists a unique tractable global optimal solution [3]. The objective function in the optimization problem (4) is equivalent to $\max_{\mathbf{r_p}} \sum_{i=1}^M \log U_i(r_{i,p})$. The solution of this optimization problem is the first optimal solution that gives each of the M users the optimal rate $r_{i,p}^{\text{opt}}$ only from the primary carrier and not yet the final optimal rate.

B. Secondary Carrier

As mentioned before, we consider a secondary carrier eNodeB located in the same coverage area of the same mobile system. Again, M is the number of mobile users in the coverage area. Once the primary carrier finishes allocating its resources to the M users, the secondary carrier starts to allocate its resources to the same users while ensuring a minimum user QoS. Therefore, we assume again that the secondary carrier will allocate the resources based on utility proportional fairness.

The optimization problem for the secondary carrier can be written as:

$$\max_{\mathbf{r}_{s}} \qquad \prod_{i=1}^{M} U_{i}(r_{i,s} + r_{i,p}^{\text{opt}})$$

$$\text{subject to} \qquad \sum_{i=1}^{M} r_{i,s} \leq R_{s},$$

$$r_{i} > 0, \qquad i = 1, 2, \dots, M.$$

$$(5)$$

where $\mathbf{r_s} = \{r_{1,s}, r_{2,s}, ..., r_{M,s}\}$ is the rate for user i, R_s is the maximum achievable rate by the secondary carrier and $r_{i,p}^{\text{opt}}$ is the first optimal rate allocated to user i by the primary carrier and estimated in (4). The optimization problem here gives priority to the real-time application users and ensures a minimum rate for each user equals to the first optimal rate $r_{i,p}^{\text{opt}}$ estimated in (4).

The optimization problem (5) is a convex optimization problem and there exists a unique tractable global optimal solution [3]. The objective function in the optimization problem (5) is equivalent to $\max \sum_{i=1}^{M} \log U_i(r_{i,s} + r_{i,p}^{\text{opt}})$. The global optimal rate for each user is obtained by the sum of the solution given by (4) $r_{i,p}^{\rm opt}$ and the solution given by (5) $r_{i,s}^{\rm opt}$ for user i and is equal $r_{i,agg}^{\rm opt} = r_{i,s}^{\rm opt} + r_{i,p}^{\rm opt}$, such that $r_{i,agg}^{\rm opt}$ is the global optimal solution that gives each of the M users the optimal rate from both the primary and secondary carriers and considered the final optimal rate.

C. Equivalence

In this section, we show the equivalence of the optimal rate $r_{i,agg}^{opt}$ given to each user by the primary and secondary eNodeBs to the optimal rate given to the same user by a single eNodeB, given by the single carrier optimization problem (3), when its available resources are equivalent to the resources available in both the primary and secondary eNodeBs in the carrier aggregation case.

Theorem III.1. The optimal rate $r_{i,agg}^{opt}$ allocated to user i by the two carriers from optimization problem (4) and optimization problem (5) is equivalent to the optimal rate allocated to the same user by the single carrier optimization problem (3) when $R = R_p + R_s$.

Proof. From the optimization problem (4), we have the La-

$$L_p(r_{i,p}) = \left(\sum_{i=1}^{M} \log U_i(r_{i,p})\right) - P_p\left(\sum_{i=1}^{M} r_{i,p} - R_p - z_p\right)$$
 (6)

where $z_p \geq 0$ is the slack variable and P_p is the Lagrange multiplier which is equivalent to the shadow price that corresponds to the total price per bandwidth for the M channels as in [3]. So we have

$$\frac{\partial L_p(r_{i,p})}{\partial r_{i,p}} = \frac{U_i'(r_{i,p})}{U_i(r_{i,p})} - P_p = 0 \tag{7}$$

solving for $r_{i,p}$ we obtain $r_{i,p}^{\text{opt}}$. From optimization problem (5), we have the Lagrangian:

$$L_s(r_{i,s}) = \left(\sum_{i=1}^{M} \log U_i(r_{i,s} + r_{i,p}^{\text{opt}})\right) - P_s\left(\sum_{i=1}^{M} r_{i,s} - R_s - z_s\right)$$
(8)

where $z_s \geq 0$ is the slack variable and P_s is the Lagrange multiplier. So we have

$$\frac{\partial L_s(r_{i,s})}{\partial r_{i,s}} = \frac{U_i'(r_{i,s} + r_{i,p}^{\text{opt}})}{U_i(r_{i,s} + r_{i,p}^{\text{opt}})} - P_s = 0$$
 (9)

solving for $r_{i,s}$ we obtain $r_{i,s}^{\text{opt}}$

replacing $r_{i,s} + r_{i,p}^{\text{opt}}$ in equation (8) by a new variable $r_{i,agg}$ such that $r_{i,agg} = r_{i,s} + r_{i,p}^{\text{opt}}$ and rewrite the Lagrangian in terms of $r_{i,agg}$ we obtain

$$L_{agg}(r_{i,agg}) = (\sum_{i=1}^{M} \log U_i(r_{i,agg}))$$

$$- P_s(\sum_{i=1}^{M} (r_{i,agg} - r_{i,p}^{\text{opt}}) - R_s - z_s)$$
(10)

where $r_{i,agg} \geq r_{i,p}^{\text{opt}}$. From the primary carrier we have $\sum_{i=1}^{M} r_{i,p}^{\text{opt}} = R_p$. So equation (10) is equivalent to

$$L(r_{i,agg}) = \left(\sum_{i=1}^{M} \log U_i(r_{i,agg}) - P_s(\sum_{i=1}^{M} r_{i,agg} - R - z_s)\right)$$
(11)

From problem (3) we have

$$L_{single}(r_{i,single}) = (\sum_{i=1}^{M} \log U_i(r_{i,single}))$$

$$-P(\sum_{i=1}^{M} (r_{i,single} - R - z))$$
(12)

equivalent to (10) for $r_i \geq r_{i,p}^{\text{opt}}$. Therefore, the optimal solution $r_{i,agg}^{\text{opt}}$ given by (10) is equivalent to the optimal solution $r_{i,single}^{\text{opt}}$ given by (12) when $R = R_p + R_s$.

IV. ALGORITHM

We use the same approach used in [3] for bandwidth proportional fairness. Our algorithm is divided into two stages. In first stage (stage1), algorithm 1 and algorithm 2 are the UE and the eNodeB algorithms, respectively. In stage 1, each UE transmits an initial bid $w_{i,p}(1)$ to the primary eNodeB. The eNodeB checks whether the difference between the current received bid and the previous one is less than a threshold δ , if so it exits. Otherwise, if the difference is greater than δ , eNodeB calculates the shadow price $P_p(n) = \frac{\sum_{i=1}^{M} w_{i,p}(n)}{R_n}$. The shadow price does not depend on the number of users competing for some resources, it only depends on the users bids and the eNodeB's available resources. The estimated $P_p(n)$ is then sent to the UE where it is used to calculate the rate $r_{i,p}(n)$ which is the solution of the optimization problem $r_{i,p}(n) = \arg\max_{r_{i,p}} \left(\log U_i(r_{i,p}) - P_p(n)r_{i,p} \right)$. A new bid $w_{i,p}(n)$ is calculated using $r_{i,p}(n)$ where $w_{i,p}(n)$ $P_p(n)r_{i,p}(n)$. All UEs send their new bids $w_{i,p}(n)$ to the primary eNodeB. Stage 1 of the Algorithm is finalized by the primary eNodeB. Each UE then calculates its allocated rate $r_{i,p}^{\text{opt}} = \frac{w_{i,p}(n)}{P_p(n)}$.

After allocating rates from primary carrier, stage 2 starts performing. Each UE transmits an initial bid $w_{i,s}(1)$ to the secondary eNodeB. The secondary eNodeB checks whether the

Algorithm 1 UE Stage 1 of Carrier Aggregation

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Send initial bid w_{i,p}(1) to eNodeB egin{align*} \textbf{loop} \\ \textbf{Receive shadow price } P_p(n) \text{ from eNodeB} \\ \textbf{if STOP from eNodeB then} \\ \textbf{Calculate allocated rate } r_{i,p}^{\text{opt}} = \frac{w_{i,p}(n)}{P_p(n)} \\ \textbf{else} \\ \textbf{Solve } r_{i,p}(n) = \arg\max_{r_{i,p}} \left( \log U_i(r_{i,p}) - P_p(n) r_{i,p} \right) \\ \textbf{Send new bid } w_{i,p}(n) = P_p(n) r_{i,p}(n) \text{ to eNodeB} \\ \textbf{end if} \\ \textbf{end loop} \\ \end{align*}
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Algorithm 2 eNodeB Stage 1 of Carrier Aggregation

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\begin{aligned} & \textbf{loop} \\ & \text{Receive bids } w_{i,p}(n) \text{ from UEs } \{ \text{Let } w_{i,p}(0) = 0 \ \forall i \} \\ & \textbf{if } |w_{i,p}(n) - w_{i,p}(n-1)| < \delta \ \forall i \ \textbf{then} \\ & \text{STOP and allocate rates (i.e } r_{i,p}^{\text{opt}} \text{ to user } i ) \\ & \textbf{else} \\ & \text{Calculate } P_p(n) = \frac{\sum_{i=1}^M w_{i,p}(n)}{R_p} \\ & \text{Send new shadow price } P_p(n) \text{ to all UEs} \\ & \textbf{end if} \\ & \textbf{end loop} \end{aligned}
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difference between the current received bid and the previous one is less than a threshold $\delta,$ if so it exits. Otherwise, if the difference is greater than $\delta,$ the secondary eNodeB calculates the shadow price $P_s(n) = \frac{\sum_{i=1}^M w_{i,s}(n)}{R_s}$. The estimated $P_s(n)$ is then sent to the UE where it is used to calculate the rate $r_{i,s}(n)$ which is the solution of the optimization problem $r_{i,s}(n) = \arg\max_{r_{i,s}} \left(\log U_i(r_{i,s} + r_{i,p}^{\text{opt}}) - P_s(n)r_{i,s}\right)$. A new bid $w_{i,s}(n)$ is calculated using $r_{i,s}(n)$ where $w_{i,s}(n) = P_s(n)r_{i,s}(n)$. All UEs send their new bids $w_{i,s}(n)$ to the secondary eNodeB. Stage 2 of the Algorithm is finalized by the seconded eNodeB. Each UE then calculates its allocated rate $r_{i,s}^{\text{opt}} = \frac{w_{i,s}(n)}{P_s(n)}$.

Algorithm 3 UE Stage 2 of Carrier Aggregation

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Send initial bid w_{i,s}(1) to eNodeB egin{align*} \textbf{loop} \\ \textbf{Receive shadow price} \ P_s(n) \ \text{from eNodeB} \\ \textbf{if STOP from eNodeB then} \\ \textbf{Calculate allocated rate} \ r_{i,s}^{\text{opt}} = \frac{w_{i,s}(n)}{P_s(n)} \\ \textbf{else} \\ \textbf{Solve} \ \ r_{i,s}(n) \ \ = \ \arg\max_{r_{i,s}} \Big( \log U_i(r_{i,s} \ + \ r_{i,p}^{\text{opt}}) \ - \ P_s(n)r_i \Big) \\ \textbf{Send new bid} \ w_{i,s}(n) = P_s(n)r_{i,s}(n) \ \text{to eNodeB} \\ \textbf{end if} \\ \textbf{end loop} \\ \end{align*}
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V. SIMULATION RESULTS

As shown in Figure 1, we consider two eNodeBs with the same coverage area and six UEs. One of the eNodeBs is the

Algorithm 4 eNodeB Stage 2 of Carrier Aggregation

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loop Receive bids w_{i,s}(n) from UEs {Let w_{i,s}(0) = 0 \ \forall i} if |w_{i,s}(n) - w_{i,s}(n-1)| < \delta \ \forall i then STOP and allocate rates (i.e r_{i,s}^{\mathrm{opt}} to user i) else Calculate P_s(n) = \frac{\sum_{i=1}^M w_{i,s}(n)}{R_s} Send new shadow price P_s(n) to all UEs end if end loop
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primary carrier and the other one is the secondary carrier with a coverage area that is almost the same for the two carriers. In Figure 2, we show three normalized sigmoidal-like utility

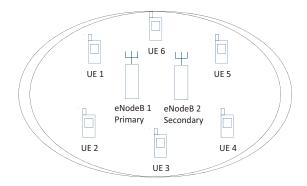


Fig. 1: System Model.

functions expressed in equation (1), each one is corresponding to one user. We use different parameters a and b for each one where a=5, b=10 for the first user, a=3, b=20 for the second user and a=1, b=30 for the third user. Each sigmoidal-like function is an approximation to a step function at rate b. We also show three logarithmic functions expressed in equation (2), which represent delay tolerant applications, with $k=\{15,3,0.5\}$ for user four, five and six, respectively. We set $r_{max}=120$.

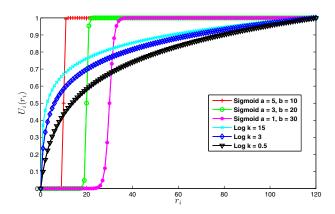


Fig. 2: The users utility functions $U_i(r_i)$.

A. Convergence Dynamics for $R_p=70$ in stage 1 of the Algorithm

We applied algorithm 1 and 2 of stage 1 in C++ to the sigmoidal-like and logarithmic utility functions shown in Figure 2. We set $R_p = 70$ and $\delta = 10^{-2}$. In Figure 3, we show the simulation results for the rate of different users and the number of iterations. As mentioned before the sigmoidal-like utility functions are given priority over the logarithmic utility functions for rate allocation and this explain the results we got in Figure 3 where the steady state rate of each sigmoidallike function exceeds the inflection point b_i . In Figure 4, we show the bids of the six users with the number of iterations. As expected, the higher the user bids the higher the allocated rate is for that user. The algorithm allows users with real-time applications, presented in sigmoidal-like utility functions, to bid higher than the other users until each one of them reaches its inflection point then the elastic traffic starts dividing the remaining resources among them based on their parameters. The first optimal rates for the six users are obtained at the end when running Algorithm 1 and 2 of stage 1. The first optimal rates are used in the next simulation that is performed for the secondary eNodeB and the same six UEs.

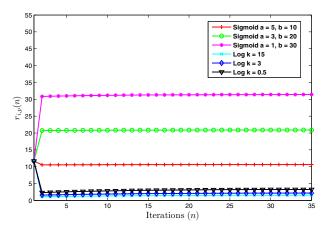


Fig. 3: The rates $r_{i,p}(n)$ with the number of iterations n for different users and $R_p = 70$.

B. Convergence Dynamics for the carrier aggregation $R_s = 50$ in stage 2 of the Algorithm

We applied algorithm 3 and 4 of stage 2 in C++ to the sigmoidal-like and logarithmic utility functions. We set $R_s=50$ and $\delta=10^{-2}$.

In Figure 5, we show the simulation results for the rate of the six users and the number of iterations. Again, the sigmoidal-like utility functions are given priority over the logarithmic utility functions for rate allocation, but since each sigmoidal-like function reached its steady state in stage 1 of the Algorithm most of R_s is distributed among the logarithmic functions. In stage 2 the optimal rates for the real time applications users $r_{i,s}^{\rm opt}$ slightly increased from the first optimal rate $r_{i,p}^{\rm opt}$ as they were given priory to reach their optimal rates

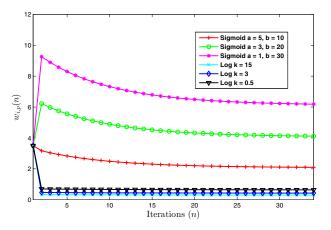


Fig. 4: The bids convergence $w_{i,p}(n)$ with the number of iterations n for different users and $R_p = 70$.

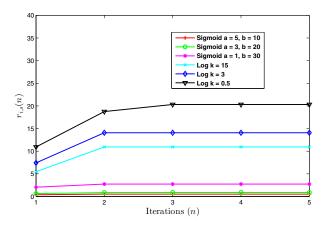


Fig. 5: The rates $r_{i,s}(n)$ with the number of iterations n for different users and $R_s=50$.

in stage 1 by the primary eNodeB, whereas the elastic traffic divided the remaining resources among them and showed a high increase in their second optimal rate $r_{i,s}^{\rm opt}$ from their first optimal rates obtained in stage 1.

In Figure 6, we show the bids of the six users with the number of iterations. As expected the higher the user bids the higher the allocated rate is for that user. The algorithm allows users with real-time applications, presented in sigmoidal-like utility functions, to bid higher than the other users until each one of them reaches its inflection point, but since these users reached their steady states in stage 1 of the Algorithm the elastic traffic users bid higher than the inelastic traffic users and share the secondary carrier's resources among them based on their parameters.

The final optimal rate for each user $r_{i,agg}^{\mathrm{opt}}$ is the sum of $r_{i,p}^{\mathrm{opt}}$ obtained at the end of stage 1 of the Algorithm and $r_{i,s}^{\mathrm{opt}}$ obtained at the end of stage 2 of the Algorithm. As expected the final optimal rates for the six users sum up to 120 which is the total rate of the primary and secondary maximum rates.

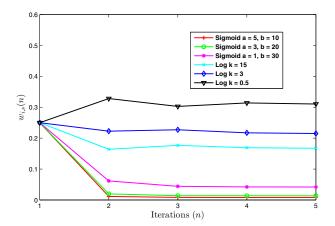


Fig. 6: The bids convergence $w_{i,s}(n)$ with the number of iterations n for different users and $R_s = 50$.

C. Equivalence of Optimal rate $r_{i,single}^{opt}$ with $r_{i,p}^{opt} + r_{i,s}^{opt}$ when $R = R_p + R_s$

Figure 7 shows the optimal rates obtained when we run Algorithm 1 and 2 of stage 1 for the same six users sharing resources of a single carrier with R=120. We made $R_p=R$, $r_{i,p}(n)=r_{i,single}(n)$, $w_{i,p}(n)=w_i(n)$ and $P_p(n)=P(n)$ when running Algorithm 1 and 2 of stage 1 for the single carrier case. The optimal rates obtained in this case are almost similar to the final optimal rates $r_{i,agg}^{\rm opt}$ in the carrier aggregation case when the same users share the resources of two carriers one being the primary and the other being the secondary with a total R_p and R_s of 120.

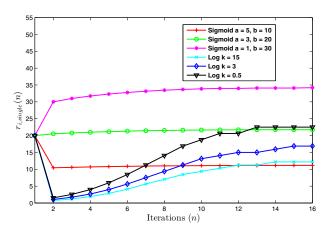


Fig. 7: The rates $r_{i,single}(n)$ with the number of iterations n for different users for the single carrier case with R=120.

VI. SUMMARY AND CONCLUSIONS

In this paper, we presented an optimal carrier aggregation resource allocation approach in 4G-LTE. We considered two utility functions based on the application type of the user one represents real time applications and the other represents delay tolerant application. The carrier aggregation resource allocation problem is divided into two optimization problems, one for the primary carrier and the other for the secondary carrier. The solution to this is characterized by utility proportional fairness. We proved that this resource allocation is equivalent to the single carrier resource allocation when it has equivalent amount of resources to the primary and secondary carriers. As a result of our analysis, we presented an iterative decentralized algorithm for the UEs and both the primary and secondary carriers. The algorithm provides a utility proportional fair resource allocation which guarantees a minimum QoS based on the users applications. We showed through simulations that our algorithm converges to the optimal rates.

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