

# On the Impact of Mobility on Multicast Capacity of Wireless Networks

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**Abstract**—Analogous to the beneficial impact that mobility has on the throughput of unicast networks, this paper establishes that mobility can provide a similar gain in the order-wise growth-rate of the throughput for multicast networks. This paper considers an all-mobile multicast network, and characterizes its multicast capacity scaling. The scaling result shows that the growth-rate of the throughput in the all-mobile multicast network is order-wise higher compared to the all-static multicast network. Further, the paper considers a static-mobile hybrid multicast network, and establishes that, if there are sufficient number of mobile nodes (that is order-wise smaller than the total number of nodes) in the network, then mobile nodes can enhance the order behavior of the multicast throughput.

## I. INTRODUCTION

Characterizing the capacity scaling for large-scale wireless networks with number of nodes under varying assumptions and constraints is now a fairly well-established field [1], [2], [3], [4], [5], [6], [7], [8]. As noted in the existing body of work, assumptions on the channel model (such as protocol, physical) and requirements (such as unicast and multicast) can greatly impact the overall order of growth of throughput. Over the past decade, we have gained a fairly in-depth understanding of the capacity of unicast networks. In [1], [3], the authors present some of the early capacity results for unicast networks. For physical channel models, the authors of [4], [5] determine the impact of cooperation on network capacity. In particular, in [5], the authors show that a total throughput of  $\Theta(n^{1-\epsilon})$  with any  $\epsilon > 0$  is achievable for unicast transmission over dense random networks using a hierarchical cooperation scheme. Such results have been generalized to arbitrary networks in [7]. More recently, models inspired by physics and electromagnetics have been introduced to determine capacity scaling results for unicast networks in [8].

Although a majority of literature in the domain studies capacity scaling laws for static networks, there is a growing body of work on mobile networks. Pioneering work on impact of mobility on unicast capacity, where each node in the network is mobile is in [2]. In [6], the authors introduce new restricted mobility models to represent delay tolerant networks, and determine the impact of this restricted mobility on the capacity of unicast networks. Note that the existing body of work focuses primarily on unicast networks, but, as we show in this paper, multicast network capacity is also impacted considerably by node mobility.

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Capacity scaling laws for multicast networks (with static nodes) have been analyzed much more recently [9], [10], [11], [12]. In [11], [12], the authors show that the multicast requirement can significantly decrease the achievable throughput in a wireless network. In particular, for a multicast group per source of size  $n_d$ , it is shown that a total throughput of at most  $O(\sqrt{n/n_d})$  is sustainable (under some conditions). The achievable schemes are based on constructing a (approximate) multicast tree from the source to the destinations in the network. Thus, large multicast group size places a fairly stringent constraint on the number of possible multicast transmissions in a network. As there are multiple applications in which multicasting plays an important role, it is useful to determine the impact that mobility has on improving the performance of multicast communication in networks.

## A. Main Results

We consider a dense<sup>1</sup> wireless network with  $n$  nodes placed randomly inside a unit square. The mobile nodes can move freely on the entire area. There are  $n_s$  multicast sources and each source has a multicast destination group of size  $n_d$ . These sources and destinations are picked randomly among traffic associations that satisfy certain conditions<sup>2</sup> for uniform traffic among nodes.

- To ascertain the benefits of mobility to multicast networks, we first consider a network configuration where all  $n$  nodes are mobile. For this setting, we extend and generalize the results of [2] to multicast networks. We show that the total multicast throughput increases to  $\Theta(\min\{n_s, n/n_d\})$  from  $O(\min\{n_s, \sqrt{n/n_d}\})$  for the static case [11], [12]. Thus, we establish that the growth-rate of the throughput in the all-mobile multicast network is order-wise higher compared to the all-static multicast network.
- Next, we consider a hybrid setting where a subset  $n^\gamma$  ( $\gamma < 1$ ) nodes in the network are mobile while the remainder  $\Theta(n)$  nodes are static. In this setting, we show that the following total throughput is achievable:

$$\Omega\left(\min\left\{n_s, \max\left\{\frac{n^\gamma}{n_d \log n}, \sqrt{\frac{n}{n_d \log n}}\right\}\right\}\right). \quad (1)$$

<sup>1</sup>In this paper, we do not consider extended networks where area scale with the number of nodes. However, most results derived for dense networks carry over to extended networks.

<sup>2</sup>Even if the sources and destinations are picked randomly from the  $n$  nodes, the results remain same up to poly log factors.

Let  $n_d = \Theta(n^{1-\epsilon})$ . If  $\gamma \geq 1 - \frac{\epsilon}{2} + \delta$  for some  $\delta > 0$ , then we have  $n^{\gamma+\epsilon-1} \geq \sqrt{n^{\epsilon+2\delta}}$ . This ensures that the overall achievable throughput in (1) can increase with the number of mobile nodes in the network if the number of sources  $n_s$  were to grow as  $n^{\gamma+\epsilon-1}/\log n$ . Thus, we establish that mobility enhances the order of growth of throughput of the hybrid network when the order of growth of mobile nodes in the network is greater than a threshold.

## II. SYSTEM MODEL

We consider a dense wireless network with  $n$  nodes inside a square  $A$  of unit area. This network contains two families of nodes - static and mobile. There are  $n^\gamma$  ( $0 \leq \gamma \leq 1$ ) mobile nodes, and the remaining nodes are static nodes. The mobile nodes can move freely on the entire area  $A$ . The locations of the mobile nodes  $X_i(t)$  are considered to be stationary and ergodic processes with uniform stationary distribution over the entire area  $A$ . The locations  $X_i(t)$  of the static nodes are fixed over time, and are uniformly and independently distributed over the entire area  $A$ . The trajectories of all nodes are independent.

The multicast scenario is modeled as follows: there are  $n_s$  nodes that are multicast sources, and each multicast source is accompanied by  $n_d$  destination nodes. Note that the number of destination nodes is the same for each source. We consider  $n_s = \Theta(n^\mu)$  and  $n_d = \Theta(n^\nu)$  with  $0 \leq \mu \leq 1$  and  $0 \leq \nu \leq 1$ . Since the case with constant  $n_s$  is straightforward, we consider  $n_s \rightarrow \infty$  as  $n \rightarrow \infty$ . The sources and destinations are selected uniformly over all traffic associations such the following conditions for uniform traffic among nodes are satisfied. (i) The  $n_s$  multicast sources are distinct. (ii) For each multicast, the source node and destination nodes are distinct. (iii) Each node in the wireless network is the destination to at most  $\lceil n_s n_d / n \rceil$  multicasts.

We use the same definitions of feasible throughput and throughput capacity (with destination replaced by the set of destination nodes) as defined in [1]. Next, we describe the two different interference models we use in this paper - the so-called protocol model and the signal to interference and noise ratio (SINR) based physical model. These are based on models introduced in [1].

### A. Interference Models

In both the interference models described below, if a transmission is successful,  $W$  bits of data per unit time (bps) can be transferred.  $X_i$  denotes the location of node  $i$ .

- 1) *Protocol Model:* All nodes choose a common radius of transmission  $r$ . At time  $t$ , a transmission from node  $i$  to node  $j$  is successful if the following two conditions are satisfied. (a) The receiver is within the transmission range of the transmitter, i.e.,

$$|X_i(t) - X_j(t)| \leq r. \quad (2)$$

- (b) Every other transmitter  $X_k(t)$  ( $k \neq i$ ) simultaneously transmitting in the same time slot  $t$  does not cause

interference to node  $j$ , i.e.,

$$|X_k(t) - X_j(t)| \geq (1 + \Delta)r. \quad (3)$$

Here,  $\Delta > 0$  models a guard zone, and is a constant that does not depend on  $n$ .

- 2) *Physical Model:* At time  $t$ , the transmission from node  $i$  with power  $P_i(t)$  is successfully received at node  $j$  if the SINR satisfies

$$\frac{P_i(t)\eta_{ij}(t)}{N_0 + \sum_{k \neq i} P_k(t)\eta_{kj}(t)} > \zeta, \quad (4)$$

where  $N_0$  is the noise power (same at all nodes),  $\zeta$  is the minimum SINR (constant that does not depend on  $n$ ) for successful transmission, and  $\eta_{ij}(t)$  is the channel gain from node  $i$  to node  $j$ . The channel gain  $\eta_{ij}(t)$  depends on the distance between node  $i$  and node  $j$ , and is given by

$$\eta_{ij}(t) = |X_i(t) - X_j(t)|^{-\alpha_g}, \quad (5)$$

where  $\alpha_g \geq 2$  is the power decay parameter.

## III. MULTICAST THROUGHPUT WITH MOBILITY

### A. Upper bounds

We provide two simple upper bounds that hold for the entire range of  $0 \leq \gamma \leq 1$ . (i) Since the sources can transmit at a maximum rate of  $W$  bps, the total throughput is upper bounded by  $n_s W$  bps. (ii) Let  $\lambda(n)$  be any feasible throughput per multicast source. Then, there has to be at least  $n_s n_d \lambda(n)$  bps of data transfer in the network. Even when all nodes obtain full rate of  $W$  bps, the total data transfer is  $nW$  bps. Therefore,  $nW/n_d$  bps is an upper bound to the total throughput. The above results are summarized in the following lemma.

*Lemma 1:* Let  $0 \leq \gamma \leq 1$ . Then, an upper bound on the total multicast throughput is  $\min\{n_s, n/n_d\}W$  bps.

### B. Achievable Schemes

The most natural setting to begin our analysis is one where all nodes in the network are mobile, i.e.,  $\gamma = 1$  (similar to that in [2]). Further, we consider the protocol model for interference. Once we gain a good understanding of this, we extend our analysis to other settings in the next section.

1)  $n_s n_d \leq n$ : The achievable scheme  $\Pi_1$  proposed next is a generalization of the *two phase scheme* introduced in [2]. In the *two phase scheme*, a data packet traverses at most two hops - one hop from its source node to a relay node or its destination node, and one hop from the relay node or its source node to its destination node. The main difference from the achievable scheme in [2] is that, in the first phase of  $\Pi_1$ , the sources simultaneously transmit the packets to multiple nodes per source. An outline of the achievable scheme  $\Pi_1$  is as follows.

- 1) In the first phase,  $\Theta(n_s)$  sources are selected such that, using a common radius of transmission of  $\Theta(1/\sqrt{n_s})$ , each of these sources can successfully transmit a packet to  $\Omega(n/n_s)$  nodes.

- 2) The nodes receiving packet can be relays or destinations for that packet. Since there are multiple destinations for each packet, both destination nodes and relay nodes store the packet in its buffer for the second phase.
- 3) In the second phase,  $\Theta(n_s n_d)$  nodes are selected such that, using a common radius of transmission of  $\Theta(1/\sqrt{n_s n_d})$ , each of these nodes can successfully transmit a packet to a destination node in the network. We assume that every transmitter has the knowledge of the packet IDs already received by the corresponding receiver. A packet is removed from the buffer of a transmitter when it learns that this packet is received by all its destinations.

In the above description of  $\Pi_1$ , we consider  $n/n_s \rightarrow \infty$ . Otherwise, the first phase is scheduled in the same way as the second phase. For this reason, we do not address it separately.

In the first phase,  $K_1 n_s$  sources are selected such that the minimum distance between these selected sources is greater than  $K_2/\sqrt{n_s}$ . This is based on the following result.

**Lemma 2:** Let  $m$  points be uniformly and identically distributed inside the unit square. For large  $m$ , there exists  $K_1 > 0$  and  $K_2 > 0$  such that  $K_1 m$  points can be selected with high probability (w.h.p) such that the minimum distance between all these selected points is greater than  $K_2/\sqrt{m}$ .

*Proof:* Divide the unit square into  $\frac{1}{\sqrt{m}} \times \frac{1}{\sqrt{m}}$  square grid. Next, we consider the probability that at least  $c_1 m$  squares are empty. Let  $\bar{c}_1 = 1 - c_1$ . Using the union bound and the Stirling's approximation, we get

$$\begin{aligned} \Pr\{c_1 l \text{ or more squares empty}\} &\leq \frac{m!}{(c_1 m)!(\bar{c}_1 m)!} \bar{c}_1^m \\ &\leq \frac{(1 + O(1/m))}{\sqrt{2\pi m c_1 \bar{c}_1}} \left(\frac{\bar{c}_1}{c_1}\right)^{c_1 m} \\ &\rightarrow 0 \quad \text{as } m \rightarrow \infty \end{aligned}$$

if  $c_1 > 1/2$ . Now, from all the filled (non-empty) squares, we can pick at least  $1/9$  of these such that they are at least  $2/\sqrt{m}$  apart. We form the set of nodes by picking one node from each of such filled squares. This set has at least  $\bar{c}_1 m / 9$  nodes that are at least  $2/\sqrt{m}$  apart. This completes the proof with  $K_1 = \bar{c}_1 / 9 > 0$  and  $K_2 = 2 > 0$ . ■

These selected sources transmit with a common radius of transmission of  $\frac{K_2}{(2+\Delta)\sqrt{n_s}}$ . Let  $S_k$  denote the location of these source nodes and  $A_k$  be the circular area with  $S_k$  as the center and  $\frac{K_2}{(2+\Delta)\sqrt{n_s}}$  as the radius. From Lemma 2, it is clear that all the nodes inside  $A_k$  can decode the data packet transmitted by source  $S_k$  as (2) and (3) are satisfied. The next result shows that in all  $A_k$  w.h.p there are  $\Omega(n/n_s)$  nodes, and thus the first phase can be scheduled as claimed.

**Lemma 3:** Let  $n/n_s \rightarrow \infty$ . Then, during the first phase of the achievable scheme  $\Pi_1$ ,  $\Theta(n_s)$  sources can simultaneously transmit to  $\Omega(n/n_s)$  nodes each w.h.p.

*Proof:* It is clear that at least a constant fraction  $K_3$  ( $1/4$ ) of the area  $A_k$  is within  $A$ . Therefore, edge effects can be easily handled by considering nodes in this fraction of the area alone. Let  $|A_k|$  denote the number of nodes in

this fraction of  $A_k$  other than the multicast source  $S_k$ , and  $K_4 = \frac{\pi K_2^2 K_3}{(2+\Delta)^2}$ . Note that  $|A_k|$  is binomial distributed with (at least)  $n - n_s$  number of trials and success probability of  $\frac{K_4}{n_s}$ . Using Chernoff's inequality applied to binomial distribution at  $\frac{K_4(n-n_s)}{2n_s}$ , we obtain

$$\Pr\left\{|A_k| < \frac{K_4}{2} \left(\frac{n}{n_s} - 1\right)\right\} \leq \exp\left(-\frac{K_4}{8} \left(\frac{n}{n_s} - 1\right)\right).$$

Hence,

$$\begin{aligned} \Pr\left\{\bigcap_k |A_k| \geq \frac{K_4}{2} \left(\frac{n}{n_s} - 1\right)\right\} \\ = 1 - \Pr\left\{\bigcup_k |A_k| < \frac{K_4}{2} \left(\frac{n}{n_s} - 1\right)\right\} \\ \geq 1 - K_1 n_s \exp\left(-\frac{K_4}{8} \left(\frac{n}{n_s} - 1\right)\right) \\ \rightarrow 1 \quad \text{as } n \rightarrow \infty \end{aligned}$$

if  $n/n_s \rightarrow \infty$ . This completes the proof. ■

In the second phase,  $K_1 \bar{n}$  nodes are selected among randomly chosen  $\bar{n} = n_s n_d / 2$  nodes (denoted by  $Z$ ) such that the minimum distance between these selected nodes is greater than  $K_2/\sqrt{\bar{n}}$ . This follows from Lemma 2. These selected nodes transmit with a common radius of transmission of  $\frac{K_2}{(2+\Delta)\sqrt{\bar{n}}}$  to other (i.e. in  $Z^c$ ) destination nodes. Note that there are at least  $n_s n_d / 2$  of these destination nodes. Every node buffers packets from all the multicast sources at an equal long-term rate in the first phase. Hence, in steady-state, every node will have packets for the chosen destination as shown in [2]. Next, we prove that the second phase can be scheduled as claimed.

**Lemma 4:** During the second phase of the achievable scheme  $\Pi_1$ ,  $\Theta(n_s n_d)$  nodes can simultaneously transmit a packet to a destination node in the network w.h.p.

*Proof:* As mentioned before, the edge effects can be handled by looking at the constant  $K_3$  fraction of the area within the area  $A$ . Let  $R_k$  denote the location of these selected nodes,  $B_k$  be the circular area with  $R_k$  as the center and  $\frac{K_2}{(2+\Delta)\sqrt{\bar{n}}}$  as the radius, and  $K_4 = \frac{\pi K_2^2 K_3}{(2+\Delta)^2}$ .

The total number of destination nodes in  $\bigcup_k B_k$  is binomial distributed with  $\bar{n}$  number of trials and success probability of  $K_4$ . Applying Chebyshev's inequality, we have

$$\Pr\left\{\sum_k |B_k| < \frac{K_4 \bar{n}}{2}\right\} \leq \frac{4(1 - K_4)}{\bar{n} K_4} \rightarrow 0 \quad \text{as } n \rightarrow \infty.$$

In order to upper bound the number of selected nodes such that  $|B_k| = 0$ , we can upper bound the number of empty bins when  $k = K_4 \frac{\bar{n}}{2}$  balls are thrown randomly into  $l = K_1 \bar{n}$  ( $l = \frac{2K_1}{K_4} k$ ) bins. Let  $0 < \delta < 1$  and  $\bar{\delta} = 1 - \delta$ . Applying the union bound and the Stirling's approximation, we have

$$\Pr\{\delta l \text{ or more bins empty}\} \leq \frac{l!}{(\delta l)!(\bar{\delta} l)!} \bar{\delta}^l \leq \frac{\bar{\delta}^l (1 + O(1/l))}{\sqrt{2\pi l \delta \bar{\delta}} (\delta \bar{\delta})^{l/2}}.$$

Note that we can choose  $\delta$ , which does not depend on  $n$ , such that  $\log \bar{\delta} < \frac{2K_1}{K_4} (\delta \log \delta + \bar{\delta} \log \bar{\delta})$  for any  $K_1 > 0$  and

$K_4 > 0$ . For such a  $\delta$ , the above probability goes to zero as  $n \rightarrow \infty$ . Therefore,  $\bar{\delta}K_1 n_s n_d$  of the selected nodes can transmit a packet to a destination node w.h.p. ■

The two phases are alternatively scheduled, for instance, the first phase during odd time instants and the second phase during even time instants. Next, we show that this scheduling policy is order-wise throughput optimal.

*Lemma 5:* Let  $n_s n_d \leq n$  and  $\gamma = 1$ . For the above scheme, the total multicast throughput achieved is  $\Omega(n_s)$ .

*Proof:* The scheduling policy depends solely on the node locations, which are i.i.d., stationary and ergodic. Hence, all the source-destination pairs obtain the same long-term rate. Due to ergodicity, the long-term throughput between any source and any of its destinations is equal to the expected number of source-destination transmissions (directly or through relay nodes) that are scheduled. From Lemma 3 and Lemma 4, it follows that a multicast rate of  $\Omega(1)$  is achieved by all the sources. ■

2)  $n_s n_d > n$ : The achievable scheme  $\Pi_2$  is based on  $\Pi_1$  performed over multiple time slots. Both first and second phases consist of  $\lceil n_s n_d / n \rceil$  time slots.

- 1) First phase: During  $\lceil n_s n_d / n \rceil$  time slots, the  $n_s$  multicast sources are scheduled in a round robin manner with  $n/n_d$  sources in each time slot. For each selected subset of sources, the first phase of  $\Pi_1$  is applied.
- 2) Second phase: During  $\lceil n_s n_d / n \rceil$  time slots, each node is considered as the destination of one of the multicast sources it is associated with in a round robin manner. For each time slot,  $\Theta(n)$  transmissions are scheduled as in the second phase of  $\Pi_1$ .

Next, we show that the above scheduling policy is order-wise throughput optimal.

*Lemma 6:* Let  $n_s n_d > n$  and  $\gamma = 1$ . For the above scheme, the total multicast throughput achieved is  $\Omega(n/n_d)$ .

*Proof:* It follows from the description of the scheme that the achievable throughput (for each multicast source) in this setting is  $1/\lceil n_s n_d / n \rceil$  times smaller compared to the setting with  $n_s n_d = n$ . This completes the proof. ■

Summarizing the results obtained in this section (Lemma 1, Lemma 5 and Lemma 6), we obtain the following theorem on throughput capacity for the all-mobile multicast network.

*Theorem 7:* The total throughput capacity of the all-mobile multicast network with protocol model for interference is  $\Theta(\min\{n_s, n/n_d\})$ .

#### IV. EXTENSIONS

##### A. Hybrid Network

We consider the hybrid network where  $n^\gamma$  ( $\gamma < 1$ ) are mobile nodes and the rest are static nodes. The main goal is to determine a regime where the presence of mobile nodes can provide an order-wise better throughput compared to the all-static multicast network. For this, we focus on a hybrid scheme consisting of two achievable schemes: 1) The achievable scheme based on forming multicast trees given in [11], and 2) A simple extension of the achievable scheme proposed in Section III-B.

In [11], the authors consider the all-static multicast network, and obtain the following main result.

*Result 1:* Let  $n_d = n^{1-\epsilon}$ ,  $0 < \epsilon < 1$  and  $\log n/n_s \rightarrow 0$  as  $n \rightarrow \infty$ . Then, the total throughput of the all-static multicast network with protocol model for interference is

$$\Theta\left(\min\left\{n_s, \frac{\sqrt{n^\epsilon}}{\sqrt{\log n}}\right\}\right) \text{ w.h.p.} \quad (6)$$

The achievable scheme in this paper is based on a comb architecture for multicast trees (see [11] for details). For  $\gamma < 1$ , the above total throughput can be achieved as follows. The mobile sources transfer packets to nearby static nodes whereas the mobile destinations obtain packets from the multicast tree.

Next, we provide an achievable scheme  $\Pi_3$  for the hybrid network. Both first and second phases consist of  $\Theta(n_s n_d \log n/n^\gamma)$  time slots. The achievable scheme  $\Pi_3$  is as follows.

- 1) If  $n^\gamma \leq \sqrt{n n_d \log n}$ , then form multicast trees as described in [11].
- 2) Otherwise, perform two-phase scheme as described next.
  - (i) First phase: Let  $n_1 = \Theta(\min\{n_s, n^\gamma / n_d \log n\})$ . During each time slot,  $n_1$  sources are scheduled such that each of these sources can successfully transmit a packet to  $\Omega(n_d)$  mobile nodes. All sources are selected during  $\Theta(n_s n_d \log n/n^\gamma)$  time slots using time division. This follows from [1] and the proof of Lemma 3.
  - (ii) Second phase: Let  $n_2 = \Theta(\min\{n_s n_d, n^\gamma / \log n\})$ . During each time slot,  $n_2$  mobile nodes are scheduled such that each of these nodes can successfully transmit a packet to a destination node in the network. All destination nodes are selected during  $\Theta(n_s n_d \log n/n^\gamma)$  time slots using time division. As before, this follows from [1] and the proof of Lemma 4.

Based on the analysis in Section III-B, we obtain the following result on the total multicast throughput.

*Theorem 8:* An achievable total throughput for the hybrid multicast network with protocol model for interference is

$$\Omega\left(\min\left\{n_s, \max\left\{\frac{n^\gamma}{n_d \log n}, \sqrt{\frac{n}{n_d \log n}}\right\}\right\}\right).$$

##### B. Physical Model

We consider all-mobile multicast network and establish that, in this case, the total multicast throughput for the physical model is order-wise same as that for the protocol model. In this section, the key lemma (based on results in [2]) for the physical model is the following.

*Lemma 9:* Let  $m$  sources  $S_1, S_2, \dots, S_m$  and a receiver  $V$  be uniformly and independently distributed inside the unit square, and all the sources transmit with unit power. This node  $V$  can decode the packet transmitted from the nearest source with a non-vanishing probability.

*Proof:* Based on (4) and (5) related to the physical model, we define the received power by node  $V$  from  $S_k$  as  $Q_k = |S_k - V|^{-\alpha_g}$ . We are interested in the SINR for this node, which is

$$\text{SINR} = \frac{\max_k Q_k}{N_0 + I}$$

where  $I$  is the sum of interference from all other sources given by  $I = \sum_k Q_k - \max_k Q_k$ .

Conditioning on  $V = v$ , we have the probability of received power greater than  $q$ , for large  $q$ , at node  $V$  is given by

$$\begin{aligned}\Pr\{Q_k > q | V = v\} &= \Pr\left\{|S_k - V| < q^{\frac{-1}{\alpha_g}} | V = v\right\} \\ &= \Pr\left\{|S_k - v| < q^{\frac{-1}{\alpha_g}}\right\} = \pi q^{\frac{-2}{\alpha_g}}.\end{aligned}$$

Note that conditioning on  $V = v$ ,  $Q_k$  are i.i.d. random variables. From the above calculation, each random variable  $Q_k \equiv Q$  conditioned on random variable  $V \equiv V$  has conditional CDF satisfying

$$\lim_{x \rightarrow \infty} \frac{1 - F_{Q|V}(x)}{1 - F_{Q|V}(lx)} = l^{2/\alpha_g}.$$

Let  $Q_{(1)}$  denote the extremum of all  $Q_k$ . From results of asymptotic distribution of extremum of i.i.d. random variables [13], we have  $\lim_{m \rightarrow \infty} \Pr\{Q_{(1)} \leq (\pi m)^{\alpha_g/2} x | V = v\} = \exp(-x^{-2/\alpha_g})$ . Now, since the right hand side of the above equation is independent of  $v$  and using dominated convergence theorem, we have  $\lim_{m \rightarrow \infty} \Pr\{Q_{(1)} \leq (\pi m)^{\alpha_g/2} x\} = \exp(-x^{-2/\alpha_g})$ . It follows that  $(\pi m)^{-\alpha_g/2} Q_{(1)} \rightarrow Q_\infty$  as  $m \rightarrow \infty$ , where  $Q_\infty$  has CDF given by  $F_{Q_\infty}(x) = \exp(-x^{-2/\alpha_g})$ ,  $x \geq 0$ .

The second random variable in SINR is the interference  $I = \sum_k Q_k - \max_k Q_k$ . We know that, for large  $q$ ,  $Q_k$  satisfies  $\Pr\{Q_k > q | V = v\} = \pi q^{\frac{-2}{\alpha_g}}$  and  $Q_k$  are i.i.d. random variables given  $V = v$ . Using the Theory of Stable Random Variables [14], we have

$$\left(\pi \Gamma\left(1 - \frac{2}{\alpha_g}\right)m\right)^{-\alpha_g/2} I \rightarrow I_\infty \text{ as } m \rightarrow \infty,$$

where  $\Gamma(s) = \int_0^\infty x^{s-1} e^{-x} dx$  is the gamma function, and  $I_\infty$  has CDF given by  $F_{I_\infty}(x) = \exp(-x^{-2/\alpha_g})$ ,  $x \geq 0$ .

Finally, we calculate SINR in the asymptotic limit using the above tail distributions. We have

$$\begin{aligned}\Pr\{\text{SINR} > \zeta\} &= \Pr\left\{\frac{Q_{(1)}}{N_0 + I} > \zeta\right\} \\ &\rightarrow \Pr\left\{\frac{Q_\infty}{I_\infty} > \zeta_\infty\right\} > 0 \text{ as } m \rightarrow \infty,\end{aligned}$$

where  $\zeta_\infty = \zeta \left(\Gamma\left(1 - \frac{2}{\alpha_g}\right)\right)^{\alpha_g/2}$ . ■

The scheduling policy  $\Pi_4$  for the case of physical model is similar to that of the case of protocol model. Note that the scheduling policy described next is for  $n_s n_d \leq n$ . It can be extended to  $n_s n_d > n$  similar to the case of protocol model. The two phases of  $\Pi_4$  are as follows.

- 1) All  $n_s$  sources transmit with unit power.  $\Theta(n_s)$  sources will have  $\Omega(n/n_s)$  nodes that decode successfully w.h.p.
- 2)  $\Theta(n_s n_d)$  nodes transmit with unit power.  $\Theta(n_s n_d)$  nodes will have a destination node that can decode successfully w.h.p.

This scheduling policy is based on the following lemma. In the lemma, the result for  $\beta > 1$  is required for the first phase,

and  $\beta = 1$  is required for the second phase. We omit the proof due to lack of space.

*Lemma 10:* Consider  $m$  transmitter nodes that are uniformly and independently distributed inside the unit square. Also, consider another  $m^\beta$  ( $\beta \geq 1$ ) receiver nodes distributed uniformly and independently. With a non-vanishing probability, there will be  $\Theta(n)$  transmitter nodes that have  $\max\{1, \Theta(n^{\beta-1})\}$  receiver nodes that are nearer to it than other transmitter nodes.

The multicast throughput capacity for the physical model follows from the proofs in Section III.

*Theorem 11:* The total throughput capacity of the all-mobile multicast network with physical model for interference is  $\Theta(\min\{n_s, n/n_d\})$ .

## V. CONCLUSION

In this paper, we analyze the impact of mobility on multicast communication in a wireless network. We pair an interference model (either protocol or physical) with a mobility model, and determine mechanisms for transferring information from sources to destinations. In general, we find that mobility enhances the order of growth of throughput in the network over static network configurations. This improvement comes from the ability of mobile nodes to relay information in order-wise smaller number of hops.

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