2014 International Conference on Computing, Networking and Communications, Wireless Communications Symposium

Spectrum Sharing between Public Safety and Commercial Users in 4G-LTE

Haya Shajaiah, Ahmed Abdel-Hadi and Charles Clancy
Bradley Department of Electrical and Computer Engineering
Virginia Tech, Arlington, VA, 22203, USA
{hayajs, aabdelhadi, tcc}@vt.edu

***Abstract*—In this paper, we consider resource allocation optimization problem in fourth generation long term evolution (4G-LTE) for public safety and commercial users running elastic or inelastic traffic. Each mobile user can run delay-tolerant or real-time applications. In our proposed model, each user equipment (UE) is assigned a utility function that represents the application type running on the UE. Our objective is to allocate the resources from a single evolved node B (eNodeB) to each user based on the user application that is represented by the utility function assigned to that user. We consider two groups of users, one represents public safety users with elastic or inelastic traffic and the other represents commercial users with elastic or inelastic traffic. The public safety group is given priority over the commercial group and within each group the inelastic traffic is prioritized over the elastic traffic. Our goal is to guarantee a minimum quality of service (QoS) that varies based on the user type, the user application type and the application target rate. A rate allocation algorithm is presented to allocate the eNodeB resources optimally among public safety and commercial users. Finally, the simulation results are presented on the performance of the proposed rate allocation algorithm.**

***Index Terms*—Resource Allocation, Application Target Rate, Elastic Traffic, Inelastic Traffic**

I. Introduction

The public safety wide area wireless communication system is currently separate from the commercial cellular networks. Industries are willing to support both communities by providing a common technology. Release 12 of 3GPP LTE standards will enhance LTE to support public safety requirements. Advanced standards such as LTE provide multimedia capabilities and voice and messages services at multi-megabit per second. The services that public safety networks provide such as communications for police, fire and ambulance require systems development to meet the communication needs of emergency services.

A common technical standard for commercial and public safety users provides advantages for both. The public safety systems market is much smaller than the commercial cellular market which makes it unable to attract the level of investment that goes in to commercial cellular networks and this makes a common technical standards for both the best solution. The public safety community gains access to the technical advantages provided by the commercial cellular networks whereas the commercial cellular community gains enhancement in their systems and make it more attractive to consumers. The USA has reserved spectrum in the 700MHz band for an LTE based public safety network. The current public safety standards support medium speed data which drives the need of new technology.

In [1], the author presented a utility proportional fairness resource allocation approach, where fairness is in utility percentage, for 4G-LTE that optimally allocate one eNodeB resources based on the optimization problem that solves for elastic and inelastic utility functions. The rate allocation algorithm in [1] gives priority to real-time applications over delay-tolerant applications and guarantees a minimum QoS when allocating resources.

In this paper, we focus on finding an optimal solution for the resource allocation problem for two groups of users running two types of applications presented by logarithmic utility functions or sigmoidal-like utility functions. These utility functions are concave and non-concave utility functions respectively. The optimization problem allocates part of the bandwidth from one eNodeB to each user subscribing for a mobile service taking into consideration that each user is getting a minimum QoS. In addition, the public safety users in emergency mode are given priority over the commercial users and within each group the non concave functions that are approximated by sigmoidal-like functions and presenting real-time applications are given priority over the concave functions approximated by logarithmic functions and presenting delay tolerant applications. In our system model, each public safety subscriber has an assigned application target rate that varies based on the application type and assigned to the public safety subscriber by the network.

Our resource allocation algorithm first allocates the application target rate to each public safety UE when that UE is in emergency mode. It then allocates the remaining resources among the commercial UEs subscribing for resources.

A. Related Work

In [2], The authors introduced bandwidth proportional fairness (Frank Kelly algorithm). This algorithm is an iterative process for determining rate allocation as well as the price the network should charge for given sets of resources. The iterative nature of the solution allows users to bid for resources until the allocated rate matches the optimal rate based on bandwidth proportional fairness. In [3], the author presented a weighted aggregation of elastic and inelastic utility functions for each UE. The utility functions are then approximated to the

978-1-4799-2358-8/14/$31.00 ©2014 IEEE

nearest concave utility function from a set of functions using minimum mean square error. The resulting utility functions are solved using a modified version of the distributed rate allocation algorithm by Frank Kelly [2].

A rate allocation with carrier aggregation is presented in [4]. The authors used two stage modified Frank Kelly algorithm to allocate two carriers resources optimally among UEs with real time applications or delay tolerant applications. One of the carriers is primary carrier and is used in the first stage of the algorithm whereas the other is secondary carrier and is used in the second stage. A priority is given to the real time applications presented by a sigmoidal-like utility functions while allocating resources in each stage. In [5], the authors presented two stage resource allocation algorithm to allocate optimal rates to users running multiple applications from one eNodeB. The proposed algorithm achieves the optimal rates without eNodeB knowledge of the UEs utilities.

In [6], a Round Robin packet scheduling method is used to distribute the load across the network. This method is not fair for resource allocation as the network could be inefficient in bandwidth and throughput. In [7] and [8], the authors used elastic and sigmoidal-like utility functions in a non-convex optimization problem to maximize utility functions in wireless networks. Using max-min architecture, the authors in [9] proposed a utility proportional fair optimization approach for high SINR wireless networks.

B. Our Contributions

Our contributions in this paper are summarized as:

- We present a resource allocation optimization problem to allocate the eNodeB resources optimally among public safety and commercial users. The eNodeB and the UE collaborate to allocate an optimal rate to each UE with priority given to public safety users. Within the same group of users, a priority is given to real time applications presented by sigmoidal-like utility functions.
- We show that each of our two cases resource allocation (RA) optimization problems has a unique tractable global optimal solution.

The remainder of this paper is organized as follows. Section II presents the problem formulation. In section III, we present the two cases resource allocation optimization problems. Section IV presents our distributed carrier aggregation rate allocation algorithm for the utility proportional fairness optimization problem. In section V, we discuss simulation setup and provide quantitative results along with discussion. Section VI concludes the paper.

II. Problem Formulation

We consider single cell 4G-LTE mobile system with a single eNodeB, N commercial UEs and M public safety UEs. The user i is allocated certain bandwidth r_i based on the type of application the UE is running. Each user is assigned a utility function $U_i(r_i)$ based on the application running on the UE and whether it is a commercial or public safety user. Our goal is to determine the optimal bandwidth that needs to be allocated to each user by the eNodeB.

The utility functions $U_i(r_i)$ are assumed to be a strictly concave or a sigmoidal-like functions. The utility functions $U_i(r_i)$ have the following properties:

- $U_i(0) = 0$ and $U_i(r_i)$ is an increasing function of r_i.
- $U_i(r_i)$ is twice continuously differentiable in r_i and bounded above.

In our model, we use the normalized sigmoidal-like utility function presented in [10], that is

$$U_i(r_i) = c_i\Big(\frac{1}{1+e^{-a_i(r_i-b_i)}} - d_i\Big) \tag{1}$$

where $c_i = \frac{1+e^{a_i b_i}}{e^{a_i b_i}}$ and $d_i = \frac{1}{1+e^{a_i b_i}}$ that satisfies $U(0) = 0$ and $U(\infty) = 1$. The inflection point of the normalized sigmoidal-like function is at $r_i^{\text{inf}} = b_i$. Additionally, we use the normalized logarithmic utility function used in [9] to represent a delay tolerant application, this utility function can be expressed as

$$U_i(r_i) = \frac{\log(1+k_i r_i)}{\log(1+k_i r_{\text{max}})} \tag{2}$$

where r_{max} gives 100% utility percentage for any user and k_i is the slope of the curve of the logarithmic utility function that varies from user to user. So, it satisfies $U(0) = 0$ and $U(r_{\text{max}}) = 1$. The inflection point of normalized logarithmic function is at $r_i^{\text{inf}} = 0$

The basic formulation of the resource allocation problem is given by the following optimization problem:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \prod_{i=1}^{M} U_i(r_{i,s}) \prod_{j=1}^{N} U_j(r_{j,c}) \\ \text{subject to} \quad & \sum_{i=1}^{M} r_{i,s} + \sum_{j=1}^{N} r_{j,c} \leq R, \\ & r_{i,s} \geq r_{i,s}^{t}, \quad i = 1,2,...,M \\ & r_{j,c} \geq 0, \quad j = 1,2,...,N. \end{aligned} \tag{3}$$

where R is the maximum achievable rate of the eNodeB, $\mathbf{r} = \{r_{1,s},...,r_{M,s},r_{1,c},...,r_{N,c}\}$ where $r_{i,s}$ is the rate for public safety user i, $r_{j,c}$ is the rate for commercial user j, $r_{i,s}^t$ is the application target rate for public safety user i which is the mnimum rate that the user wants to achieve, M and N are the numbers of the public safety and commercial UEs, respectively. The resource allocation objective function maximizes the product of users utilities system utility when allocating resources to each user. Therefore, it provides a proportional fairness among utilities. Public safety users that are running real-time applications are given the priority when allocating resources by the eNodeB. The next priority is given to the elastic traffic running by public safety users. Once each public safety user satisfies its application target rate the eNodeB starts allocating resources to commercial users giving priority to users running real time applications. We assume that the public safety users are in an emergency mode, therefore these users are given a higher priority over the commercial users. The optimization problem (3) has a unique tractable global optimal solution [1] that will be discussed in the next section.

We used utility proportional fairness model because non-zero rate allocation is guaranteed to all users. So it is impossible to set a users allocation to zero without setting the efficiency of the network to zero. Because this resource does not disenfranchise any given user, it will be considered as an appropriate fairness model for this problem.

III. Resource Allocation Optimization Problem

The resource allocation for public safety and commercial users is divided into two cases. The first case is when the maximum available resources R for the eNodeB is less than the sum of the total application target rates of the public safety UEs subscribing for a service from that eNodeB and the second case is when R is greater than that total. The two cases are two different optimization problems that will be solved by our proposed algorithm to obtain the optimal rate for each UE.

A. The First Case RA Optimization Problem when $\sum_{i=1}^{M} r_{i,s}^t \geq R$

As mentioned before the first case optimization problem is applied in the case of $\sum_{i=1}^{M} r_{i,s}^t \geq R$. In this case the eNodeB only allocates resources to the public safety users because they are considered more important and the eNodeB's available resources doesn't exceed their need. The commercial users will not be given any of the eNodeB resources in this case. This optimization problem can be written as:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \prod_{i=1}^{M} U_i(r_{i,s}) \\ \text{subject to} \quad & \sum_{i=1}^{M} r_{i,s} \leq R, \\ & 0 \leq r_{i,s} \leq r_{i,s}^t, \quad i = 1, 2, ..., M. \end{aligned} \tag{4}$$

where U_i is the public safety i^{th} utility function and $\mathbf{r} = \{r_{1,s}, ..., r_{M,s}\}$ and M is the number of public safety UEs in the coverage area of the eNodeB. The solution of the optimization problem (4) is the optimal solution when $\sum_{i=1}^{M} r_{i,s}^t \geq R$. This solution will guarantee the public safety users priority when allocating the eNodeB resources. The optimal rate for each public safety UE is less than or equal to the application target rate for each public safety UE. The public safety users running real time applications will be given priority over public safety users with elastic traffic.

The objective function in the optimization problem (4) is equivalent to $\max_{\mathbf{r}} \sum_{i=1}^{M} \log U_i(r_{i,s})$. The optimization problem (4) is a convex optimization problem and there exists a unique tractable global optimal solution as shown in Theorem (III.1) [1]. This optimal solution gives each of the M users an optimal rate $r_{i,s}^{\text{opt}}$.

B. The Second Case RA Optimization Problem when $\sum_{i=1}^{M} r_{i,s}^t < R$

The second case optimization problem is applied in the case of $\sum_{i=1}^{M} r_{i,s}^t < R$. The eNodeB collaborate with the UEs to solve this optimization problem. The eNodeB allocates resources to both the public safety and commercial users because its available resources exceed the minimum need of the public safety UEs expressed by the application target rates. As mentioned before, the eNodeB gives priority to the public safety users and within the public safety group the priority is given to the UEs running inelastic traffic. This optimization problem can be written as:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \prod_{i=1}^{M} U_i(r_{i,s}) \prod_{j=1}^{N} U_j(r_{j,c}) \\ \text{subject to} \quad & \sum_{i=1}^{M} r_{i,s} + \sum_{j=1}^{N} r_{j,c} \leq R, \\ & r_{i,s} \geq r_{i,s}^t, \quad i = 1, 2, ..., M \\ & r_{j,c} \geq 0, \quad j = 1, 2, ..., N. \end{aligned} \tag{5}$$

This optimization problem is same as the one discussed in the problem formulation (section II). First, the eNodeB allocates the application target rate to each public safety UE. It then starts allocating its remaining resources both to the public safety and commercial UEs based on utility proportional fairness. The solution of the optimization problem (5) is the global optimal solution that gives an optimal rate $r_{i,s}^{\text{opt}}$ to each public safety UE and an optimal rate $r_{i,c}^{\text{opt}}$ to each commercial user UE.

Proposition III.1. *The optimization problem (5) is a convex optimization problem and there exists a unique tractable global optimal solution.*

Proof. We introduce a new parameter c_i where c_i is the application target rate for the public safety UE whereas it is 0 for the commercial UE, the optimization problem (5) can be rewritten as follows:

$$\begin{aligned} \max_{\mathbf{r}} \quad & \prod_{i=1}^{M+N} U_i(r_i + c_i) \\ \text{subject to} \quad & \sum_{i=1}^{M+N} (r_i + c_i) \leq R, \\ & r_i \geq 0, \quad i = 1, 2, ..., M + N. \end{aligned} \tag{6}$$

$$c_i = \begin{cases} r_{i,s}^t & \text{if public safety UE} \\ 0 & \text{if commercial UE} \end{cases}$$

where R is the maximum achievable rate of the eNodeB, $\mathbf{r} = \{r_1, ..., r_M, r_{M+1}, ..., r_{M+N}\}$ where the first M rates are for the M public safety users and the last N rates are for the N commercial users, $U_i(r_i + c_i)$ is the UE utility function, this optimization problem guarantees an optimal rate that is at least equal to the application target rate for the public safety UE. The objective function in the optimization problem (6) can be written as $\sum_{i=1}^{M+N} \log U_i(r_i + c_i)$.

The utility function $U_i(r_i + c_i)$ for the UE is strictly concave or sigmoidal-like function as mentioned in section II. As shown in Theorem (III.1) [1], $\log U_i(r_i)$ is a strictly concave function for a strictly concave or sigmoidal-like utility function. It follows that the optimization problem 6 that is equivalent to (5) is convex. Therefore, there exists a tractable global optimal solution for the optimization problem (5). $\square$

IV. Algorithm

In our proposed iterative algorithm, the eNodeB and the UEs collaborate to allocate optimal rates for the public safety and commercial users subscribing for a mobile service. Algorithm 1 and algorithm 2 are the public safety UE and the commercial UE algorithms, respectively. Algorithm 3 is the eNodeB algorithm. The algorithm starts when each UE transmits an initial bid $w_i(1)$ to the eNodeB. Additionally, each public safety UE transmits its application target rate to the eNodeB. The eNodeB checks whether the $\sum_{i=1}^{M} r_{i,s}^t$ is less or greater than R and send a flag with this information to each UE. In the case of $\sum_{i=1}^{M} r_{i,s}^t \geq R$, the commercial UEs will not be allocated any of the resources and will not be sending any further bids to the eNodeB unless they receive a flag from the eNodeB with $\sum_{i=1}^{M} r_{i,s}^t < R$.

On the other hand, each public safety UE checks whether the difference between the current received bid and the previous one is less than a threshold δ, if so it exits. Otherwise, if the difference is greater than δ, eNodeB calculates the shadow price $p(n) = \frac{\sum_{i=1}^{M} w_i(n)}{R}$. The estimated $p(n)$ is then sent to the public safety UEs where it is used to calculate the rate $r_{i,s}(n)$ which is the solution of the optimization problem $r_{i,s}(n) = \arg\max_{r_{i,s}} \Big(\log U_i(r_{i,s}) - p(n) r_{i,s} \Big)$. A new bid $w_i(n)$ is calculated using $r_i(n)$ where $w_i(n) = p(n) r_{i,s}(n)$. All public safety UEs send their new bids $w_i(n)$ to the eNodeB. The Algorithm is finalized by the eNodeB. Each public safety UE then calculates its allocated rate $r_{i,s}^{\text{opt}} = \frac{w_i(n)}{p(n)}$.

In the case of $\sum_{i=1}^{M} r_{i,s}^t < R$, the eNodeB sends a flag with this information to each UE. Each public safety and commercial UE checks whether the difference between the current received bid and the previous one is less than a threshold δ, if so it exits. Otherwise, if the difference is greater than δ, eNodeB calculates the shadow price $p(n) = \frac{\sum_{i=1}^{M+N} w_i(n)}{R}$. The estimated $p(n)$ is then sent to the public safety and commercial UEs where it is used by the public safety UE to calculate the rate $r_{i,s}(n) = r_i + r_{i,s}^{\text{t}}$ which is the solution of the optimization problem $r_{i,s}(n) = \arg\max_{r_{i,s}} \Big(\log U_i(r_i + c_i) - p(n)(r_i + c_i) \Big)$. A new bid $w_i(n)$ is calculated by the public safety UE using $r_i(n)$ where $w_i(n) = p(n)(r_i(n) + c_i)$. All public safety UEs send their new bids $w_i(n)$ to the eNodeB. On the other hand, the commercial UEs receive $p(n)$ and use it to calculate the rate $r_{i,c}(n)$ which is the solution of the optimization problem $r_{i,c}(n) = \arg\max_{r_{i,c}} \Big(\log U_i(r_{i,c}) - p(n) r_{i,c} \Big)$. A new bid $w_i(n)$ is calculated by the commercial UE using $r_{i,c}(n)$ where $w_i(n) = p(n) r_i(n)$. All public safety UEs send their new bids $w_i(n)$ to the eNodeB. The Algorithm is finalized by the eNodeB. Each public safety UE then calculates its allocated rate $r_{i,s}^{\text{opt}} = \frac{w_i(n)}{p(n)}$ and each commercial UE calculates its allocated rate $r_{i,c}^{\text{opt}} = \frac{w_i(n)}{p(n)}$.

V. Simulation Results

We consider one eNodeB with four public safety UEs and another four commercial UEs in its coverage area. We use multiple sigmoidal-like and logarithmic utility functions in our simulations and present two cases, one when the eNodeB resources R is less than the total application target rates of the public safety UEs and the other when R is greater than that total. We applied algorithm 1, 2 and 3 in C++ to the sigmoidal-like and logarithmic utility functions. The simulation results showed convergence to the optimal global point in both cases. We present the simulation results for eight utility functions that correspond to public safety and commercial UEs running real time application or delay tolerant applications. We use two normalized utility functions expressed in equation (1) with different parameters a and b for each utility function, $a = 3$, $b = 20$ for the first public safety user, $a = 1$, $b = 30$ for the

Algorithm 1 Public Safety UE Algorithm

Send initial bid $w_i(1)$ to eNodeB
Send the application target rate $r_{i,s}^{\text{t}}$ to eNodeB
loop
  while Flag $\sum_{i=1}^{M} r_{i,s}^t \geq R$ from eNodeB **do**
    Receive shadow price $p(n)$ from eNodeB
    if STOP from eNodeB **then**
      Calculate allocated rate $r_{i,s}^{\text{opt}} = \frac{w_i(n)}{p(n)}$
    else
      Solve $r_{i,s}(n) = \arg\max_{r_{i,s}} \Big(\log U_i(r_{i,s}) - p(n) r_{i,s} \Big)$
      Send new bid $w_i(n) = p(n) r_{i,s}(n)$ to eNodeB
    end if
  end while
  while Flag $\sum_{i=1}^{M} r_{i,s}^t < R$ from eNodeB **do**
    Receive shadow price $p(n)$ from eNodeB
    if STOP from eNodeB **then**
      Calculate allocated rate $r_{i,s}^{\text{opt}} = \frac{w_i(n)}{p(n)}$
    else
      Solve $r_{i,s}(n) = r_i + r_{i,s}^{\text{t}} = \arg\max_{r_i} \Big(\log U_i(r_i + c_i) - p(n)(r_i + c_i) \Big)$
      Send new bid $w_i(n) = p(n)(r_i(n) + c_i)$ to eNodeB
    end if
  end while
end loop

Algorithm 2 Commercial UE Algorithm

Send initial bid $w_i(1)$ to eNodeB
loop
  while Flag $\sum_{i=1}^{M} r_{i,s}^t \geq R$ from eNodeB **do**
    Allocated rate $r_{i,c}^{\text{opt}} = 0$
  end while
  while Flag $\sum_{i=1}^{M} r_{i,s}^t < R$ from eNodeB **do**
    Receive shadow price $p(n)$ from eNodeB
    if STOP from eNodeB **then**
      Calculate allocated rate $r_{i,c}^{\text{opt}} = \frac{w_i(n)}{p(n)}$
    else
      Solve $r_{i,c}(n) = \arg\max_{r_{i,c}} \Big(\log U_i(r_{i,c}) - p(n) r_{i,c} \Big)$
      Send new bid $w_i(n) = p(n) r_{i,c}(n)$ to eNodeB
    end if
  end while
end loop

Algorithm 3 eNodeB Algorithm

loop
  Receive bids $w_i(n)$ from UEs $\{\text{Let } w_i(0) = 0\ \forall i\}$
  Receive application target rates from public safety UES
  while $\sum_{i=1}^{M} r_{i,s}^{t} \geq R$ **do**
    Send flag $\sum_{i=1}^{M} r_{i,s}^{t} \geq R$ to all UEs
    if $|w_i(n) - w_i(n-1)| < \delta,\ i = \{1, ..., M\}$ **then**
      STOP and allocate rates (i.e $r_{i,s}^{\text{opt}}$ to public safety user i)
    else
      Calculate $p(n) = \frac{\sum_{i=1}^{M} w_i(n)}{R}$, $i = \{1, ..., M\}$
      Send new shadow price $p(n)$ to public safety UEs
    end if
  end while
  while $\sum_{i=1}^{M} r_{i,s}^{t} < R$ **do**
    Send flag $\sum_{i=1}^{M} r_{i,s}^{t} < R$ to all UEs
    if $|w_i(n) - w_i(n-1)| < \delta\ \forall i$ **then**
      STOP and allocate rates (i.e $r_{i,s}^{\text{opt}}$ or $r_{i,c}^{\text{opt}}$ to user i)
    else
      Calculate $p(n) = \frac{\sum_{i=1}^{M+N} w_i(n)}{R}$
      Send new shadow price $p(n)$ to all UEs
    end if
  end while
end loop

second public safety user. We set the application target rate $r_{i,s}^{t}$ for these two users to equal b that is 20 and 30 respectively. Another two normalized utility functions are used with the same a and b parameters to represent two commercial users running real time applications. Each sigmoidal-like function is an approximation to a step function at rate b. We also use two logarithmic functions expressed in equation (2) with different parameters $k = 3$ for one public safety UE and $k = 0.5$ for second public safety UE running delay tolerant application. We set the application target rate $r_{i,s}^{t}$ for each these two users to equal 15. Another two logarithmic utility functions are used with the same k parameters to represent two commercial users running delay tolerant applications.

A. *Convergence Dynamics for $R = 70$ where $\sum_{i=1}^{M} r_{i,s}^{t} \geq R$*

This represents the first case where $\sum_{i=1}^{M} r_{i,s}^{t} \geq R$. We set $R = 70$ and $\delta = 10^{-2}$. As mentioned before, in this case the commercial UEs will not be allocated any of the eNodeB resources because R does not exceed the public safety application target rates which need to be satisfied before the eNodeB starts allocating resources to the commercial users. In Figure 1, we show the simulation results for the rate of different public safety users and the number of iterations. The sigmoidal-like utility functions are given priority over the logarithmic utility functions for rate allocation. This explain the results we got in Figure 1. In this case the final optimal rate does not exceed the user application target rate. In Figure 2, we show the bids of the four public safety users with the number of iterations. As expected, user rates are proportional to the user bids. The algorithm allows users with real-time applications to bid higher than the other users until each one of them reaches its inflection point, which is equivalent to their application target rates, then users with elastic traffic start dividing the remaining resources among them based on their parameters while not exceeding their application target rates. In Figure 3, we show the shadow price $p(n)$ with the number of iterations where the convergence behavior of the shadow price with the number of iterations is shown.

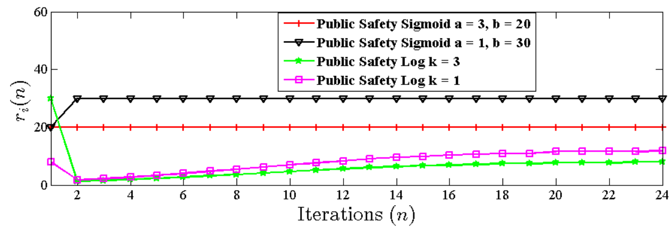

Fig. 1: The rates $r_i(n)$ with the number of iterations n for different users and $R = 70$.

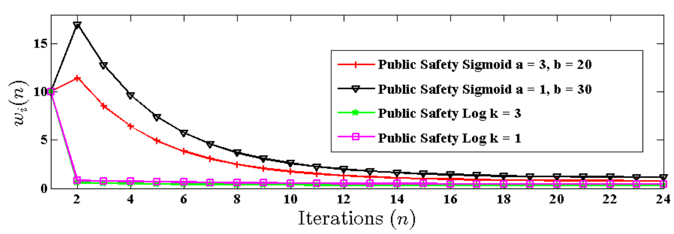

Fig. 2: The bids convergence $w_i(n)$ with the number of iterations n for different users and $R = 70$.

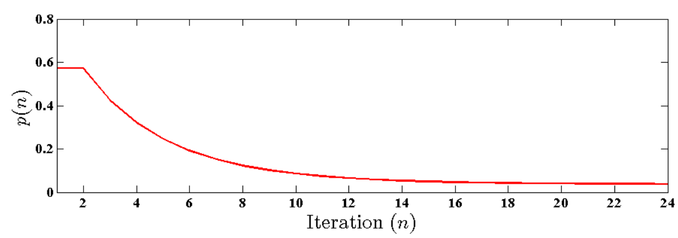

Fig. 3: The shadow price convergence with the number of iterations n.

B. *Convergence Dynamics for $R = 200$ where $\sum_{i=1}^{M} r_{i,s}^{t} < R$*

Figure 4 shows four public safety normalized sigmoidal-like utility functions expressed in equation (1) corresponding to two public safety users and another two commercial users. We also show four logarithmic functions expressed in equation (2), which represent delay tolerant applications for two public safety users and another two commercial users. We set $R = 120$ and $\delta = 10^{-2}$. This represents the second case where $\sum_{i=1}^{M} r_{i,s}^{t} < R$. In this case the public safety UES are given priority over the commercial UEs. In Figure 5, we show the simulation results for the rate of different public safety and commercial users and the number of iterations., first the algorithm allocates an equivalent amount of resources to the application target rate to each public safety user. It then starts allocating resources to each commercial UE with inelastic traffic until it reaches the inflection point of that user utility function. It then starts dividing the remaining resources among all users based on their parameters. In Figure 6, we show the bids of the eight users with the number of iterations. The algorithm allows public safety users to bid higher than the other users until each one of them reaches its application target rate. Commercial users with inelastic traffic then start bidding higher until they reach each utility function reaches its inflection point. In Figure 7, we show the shadow price $p(n)$ with the number of iterations where the convergence behavior of the shadow price with the number of iterations is shown.

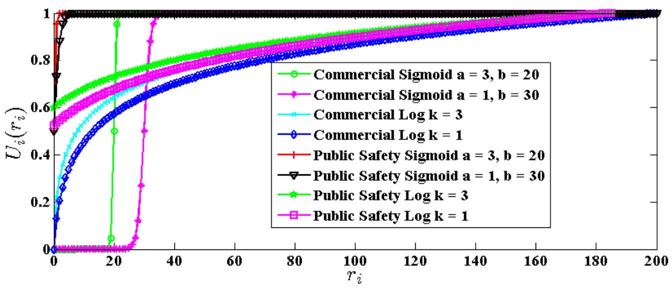

Fig. 4: The users utility functions $U_i(r_i + c_i)$.

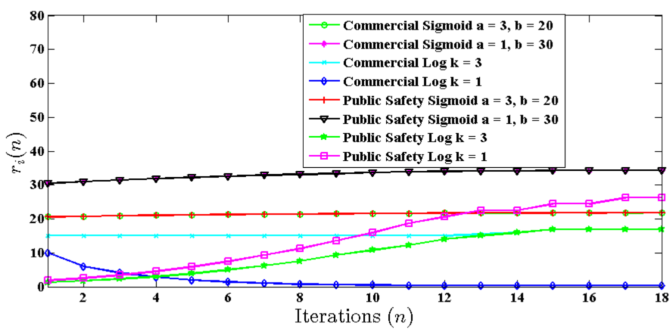

Fig. 5: The rates $r_i(n)$ with the number of iterations n for different users and $R = 200$.

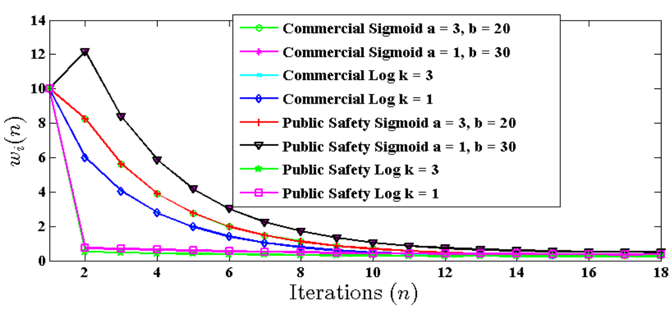

Fig. 6: The bids convergence $w_i(n)$ with the number of iterations n for different users and $R = 200$.

VI. Summary and Conclusions

In this paper, we presented a resource allocation approach to allocate a single eNodeB resources optimally among public safety and commercial users in 4G-LTE. We considered two

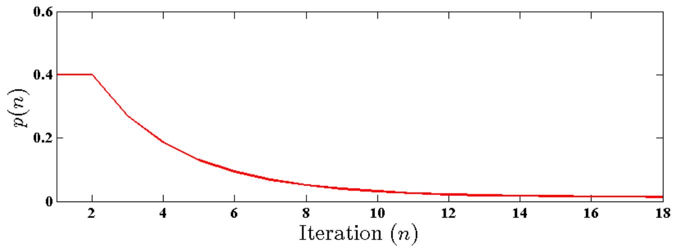

Fig. 7: The shadow price convergence with the number of iterations n.

utility functions based on the UE application type. One represents real time applications and the other represents delay tolerant applications. We considered two resource allocation optimization problems based on the amount of resources available by the eNodeB. One is when the eNodeB resources are less than or equal to the total application target rates of the public safety users subscribing for a service. The other is when the eNodeB resources greater than that total. The solution to each optimization problem is characterized by utility proportional fairness. We proposed an iterative decentralized algorithm for the eNodeB and both the public safety and commercial UEs. The algorithm provides a utility proportional fair resource allocation which guarantees a minimum QoS based on the public safety UEs application target rates, the group that the UE belongs to and the eNodeB available resources. The public safety users group is given priority over the commercial users group and within each group, users running real time applications are prioritized over those running delay tolerant applications. We showed through simulations that our algorithm converges to the optimal rates.

References

[1] A. Abdel-Hadi and C. Clancy, "A utility proportional fairness approach for resource allocation in 4G-lte," Submitted to ICNC, 2013.
[2] F. Kelly, A. Maulloo, and D. Tan, "Rate control in communication networks: shadow prices, proportional fairness and stability," in *Journal of the Operational Research Society*, vol. 49, 1998.
[3] R. L. Kurrle, "Resource allocation for smart phones in 4G lte advanced carrier aggregation," November 2012.
[4] H. Shajaiah, A. Abdel-Hadi, and C. Clancy, "Utility proportional fairness resource allocation with carrier aggregation in 4G-lte," Submitted to Milcom, 2013.
[5] A. Abdel-Hadi, C. Clancy, and J. Mitola, "A resource allocation algorithm for multi-application users in 4G-lte," Submitted to CRAB, 2013.
[6] Y. Wang, K. Pedersen, P. Mogensen, and T. Sorensen, "Resource allocation considerations for multi-carrier lte-advanced systems operating in backward compatible mode," in *Personal, Indoor and Mobile Radio Communications, 2009 IEEE 20th International Symposium on*, pp. 370–374, 2009.
[7] G. Tychogiorgos, A. Gkelias, and K. Leung, "A new distributed optimization framework for hybrid ad-hoc networks," in *GLOBECOM Workshops (GC Wkshps), 2011 IEEE*, pp. 293–297, 2011.
[8] G. Tychogiorgos, A. Gkelias, and K. Leung, "Towards a fair non-convex resource allocation in wireless networks," in *Personal Indoor and Mobile Radio Communications (PIMRC), 2011 IEEE 22nd International Symposium on*, pp. 36–40, 2011.
[9] G. Tychogiorgos, A. Gkelias, and K. K. Leung, "Utility-proportional fairness in wireless networks.," in *PIMRC*, pp. 839–844, IEEE, 2012.
[10] J.-W. Lee, R. R. Mazumdar, and N. B. Shroff, "Downlink power allocation for multi-class wireless systems," *IEEE/ACM Trans. Netw.*, vol. 13, pp. 854–867, Aug. 2005.