MIMO Radar Waveform Design for Coexistence With Cellular Systems

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***Abstract*—The design of multiple-input multiple-output (MIMO) radar waveforms with specified properties has a number of applications including clutter suppression and interference mitigation. In this paper, we design radar waveforms with the aim to mitigate radar interference to communication system. We design constant envelope (CE) transmit beampattern of a MIMO radar, when it is sharing its spectrum with a cellular system, with the constraint that the waveform is in the null space of interference channel between radar and communication system. We design the desired beampattern by unconstrained nonlinear optimization of the desired covariance matrix. We consider waveform design problem for a stationary maritime MIMO radar when interference channels are changing slowly and thus they can be included in the beampattern matching optimization problem. In addition, we also consider the case when the maritime MIMO radar is moving and thus experiences interference channels that are changing fast enough to be not included in the optimization problem. For this case, we design the CE waveform first and then project it onto the null space of interference channel, before transmission, in order to have zero interference to the communication system. We demonstrate, through simulations, the effect of including the constraint of spectrum sharing or the waveform be in the null space of interference channel on the CE waveform design.**

***Index Terms*—MIMO Radar, Null Space Projection, Cellular Systems Coexistence, Constant Envelope Waveform**

I. Introduction

The problem of waveform design for MIMO radars has gained considerable attention in the last few years. This is due to the fact that MIMO radars allow to transmit a very diverse set of waveforms through each transmit element independently, thus providing extra degrees of freedom (DoF). In contrast, a phased array radar transmits waveforms that are correlated in phase or time. This capability of MIMO radar gives it superiority over phased array radars when it comes to target's spatial diversity and resolution [1]. Moreover, with the help of extra DoF an efficient transmit beampattern or waveform can be designed.

One way to design desired MIMO radar waveform is by synthesis of covariance matrix of waveform. This is also known as beampattern matching problem where the actual beams are matched to the desired beams. A practical constraint in the design of radar waveform is for them to have constant envelope (CE) property in order to allow radio frequency amplifiers operate at maximum efficiency. Therefore, in practical systems, waveforms with the CE property are desirable.

The Federal Communications Commission (FCC) along with the National Telecommunications and Information Administration (NTIA), in the United States (US), are working on an initiative to share some of the spectrum, held by the federal agencies, with commercial wireless operators. The 3550-3650 MHz band, currently used for military radar operations, is identified for spectrum sharing between military radars and broadband wireless access (BWA) communication systems such as LTE and WiMAX, according to the NTIA's 2010 Fast Track Report [2].

This spectrum sharing initiative is believed to be one of the approaches to solve the problem of spectrum congestion. It will allow cellular operators access to more spectrum in order to satisfy the ever growing bandwidth demands of commercial users. However, spectrum sharing will create electromagnetic interference (EMI) to military radar operations and commercial cellular systems since both will be operating in the same frequency band. Thus, innovative solutions are required for interference mitigation in order for both the systems to achieve their performance objectives.

A. Related Work

Recently, several initiatives have been taken to address the problem of interference mitigation. To mitigate radar interference to communication systems, Lackpour et. al. [3] have proposed radar interference mitigation schemes, for WiMAX systems, in four domains namely space, time, frequency, and system-level design. Shabnam et al. [4] have proposed projection of MIMO radar waveform onto the null space of interference channel between radar and communication system to mitigate radar interference to communication system. Alternately, MIMO radar waveforms can also be designed that not only have finite alphabet constant envelope property but are also non-interfering with the communication system by being in the null space of interference channel [5].

On the other hand, radar systems due to their receiver sensitivity are also susceptible to interference from communication systems. In the past, communication systems have been allowed to use radar band opportunistically with the conditions that the radar channel should be vacant and the transmission power, of communication system, should not exceed an already established interference threshold [6]–[8]. With the advancements of MIMO radars, beamforming approaches, to mitigate

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interference from wireless communication systems to MIMO radar, have also been made possible [9].

B. Our Contributions

Our contributions in this paper are summarized as:

- The problem of waveform design for MIMO radars to coexist with a single communication system is considered in [5]. We extend this approach and design MIMO radar waveforms that can coexist with a cellular system, i.e. waveforms that support coexistence with many communication systems.
- We focus on the problem of waveform design for stationary maritime MIMO radar which experiences a stationary or a slowly moving interference channels. Due to the tractability of interference channels, null space projection (NSP) is included in the unconstrained nonlinear optimization problem for waveform design.
- In addition, we also design waveform for a moving maritime MIMO radar which experiences interference channels that are fast enough not to be included in the optimization problem due to their intractability [10]. For this case, we first design CE waveforms and then project them onto null space of interference channel before transmission.
- We study the impact of radar spectrum sharing via NSP of radar waveform in the optimization problem and NSP after optimization problem on CE waveform design.

The remainder of this paper is organized as follows. Section II discusses problem formulation. Section III presents the synthesis of Gaussian covariance matrix for beampattern matching design problem. Section IV solves the waveform design optimization problem for spectrum sharing. Section V discusses simulation setup and results. Section VI concludes the paper.

Notation: Bold upper case letters, $\mathbf{A}$, denote matrices while bold lower case letters, $\mathbf{a}$, denote vectors. The m^{th} column of matrix is denoted by $\mathbf{a}_m$. For a matrix $\mathbf{A}$, the conjugate and conjugate transposition are respectively denoted by $\mathbf{A}^\star$ and $\mathbf{A}^H$. The m^{th} row and n^{th} column element is denoted by $\mathbf{A}(m,n)$. Null space of $\mathbf{A}$ is denoted by $\mathcal{N}(\mathbf{A})$ and the dimension or number of linearly independent columns in null space of $\mathbf{A}$ is denoted by dim$[\mathcal{N}(\mathbf{A})]$.

II. Problem Formulation

We consider the problem of designing MIMO radar waveform to match a given beampattern in the presence of a cellular system. We modify the classical problem of beampattern matching to include the constraint that the designed waveform should not cause interference to the cellular system. So in addition to maximizing the received power at a number of given target locations and minimizing at all other locations we also seek to null out interference to cellular system through our waveform design.

A. Constant Envelope Beampattern Matching

In this paper, we design waveforms with constant envelope property. We consider a uniform linear array of M transmit antennas with inter-element spacing of half-wavelength. Then, the transmit signal is given as

$$\mathbf{x}(n) = \begin{bmatrix} x_1(n) & x_2(n) & \cdots & x_M(n) \end{bmatrix} \tag{1}$$

where $x_m(n)$ is the baseband signal from the m^{th} transmit element at time index n. Then the received signal from a target at location θ_k is given as

$$r_k(n) = \sum_{m=1}^{M} e^{-j(m-1)\pi \sin\theta_k} x_m(n), k = 1, 2, ..., K. \tag{2}$$

The above received signal can be represented compactly as

$$r_k(n) = \mathbf{a}^H(\theta_k)\mathbf{x}(n) \tag{3}$$

where $\mathbf{a}(\theta_k)$ is the steering vector defined as

$$\mathbf{a}(\theta_k) = \begin{bmatrix} 1 & e^{-j\pi\sin\theta_k} & e^{-j2\pi\sin\theta_k} & \cdots & e^{-j(M-1)\pi\sin\theta_k} \end{bmatrix} \tag{4}$$

Now, the power received from the target at location θ_k is given as

$$\begin{aligned} P(\theta_k) &= \mathbb{E}\{\mathbf{a}^H(\theta_k)\,\mathbf{x}(n)\,\mathbf{x}^H(n)\,\mathbf{a}(\theta_k)\} \\ &= \mathbf{a}^H(\theta_k)\,\mathbf{R}\,\mathbf{a}(\theta_k) \end{aligned} \tag{5}$$

where $\mathbf{R}$ is the correlation matrix of the transmitted signal. The desired beampattern $\phi(\theta_k)$ is formed by minimizing the square of the error between $P(\theta_k)$ and $\phi(\theta_k)$ through a cost function defined as

$$J(\mathbf{R}) = \frac{1}{K}\sum_{k=1}^{K}\Big(\mathbf{a}^H(\theta_k)\,\mathbf{R}\,\mathbf{a}(\theta_k) - \phi(\theta_k)\Big)^2. \tag{6}$$

It is important to realize that we don't have freedom to choose $\mathbf{R}$ freely since it is a covariance matrix of the transmitted waveform and thus it must be positive semidefinite. In addition, we are interested in constant envelope waveform, i.e., all antennas are required to transmit at same power level which translates to same diagonal elements of $\mathbf{R}$. Thus, $\mathbf{R}$ is subject to two constraints, namely,

$$\begin{aligned} &C_1 : \mathbf{v}^H\mathbf{R}\mathbf{v} \geq 0, && \forall\,\mathbf{v} \\ &C_2 : \mathbf{R}(m,m) = c, && m = 1, 2, \ldots, M. \end{aligned}$$

Thus, under the given constraints, a constrained nonlinear optimization problem can be setup to solve beampattern matching problem

$$\begin{aligned} \min_{\mathbf{R}} \quad & \frac{1}{K}\sum_{k=1}^{K}\Big(\mathbf{a}^H(\theta_k)\,\mathbf{R}\,\mathbf{a}(\theta_k) - \phi(\theta_k)\Big)^2 \\ \text{subject to} \quad & \mathbf{v}^H\mathbf{R}\mathbf{v}, \qquad \forall\,\mathbf{v} \\ & \mathbf{R}(m,m) = c, \quad m = 1, 2, ..., M. \end{aligned} \tag{7}$$

For radar waveform design, this constrained nonlinear optimization problem can be transformed into an unconstrained nonlinear optimization problem by bounding the variables using multidimensional spherical coordinates [11]. Once $\mathbf{R}$ is synthesized, the waveform matrix $\mathbf{X}$ with N samples defined as

$$\mathbf{X} = \begin{bmatrix}\mathbf{x}(1) & \mathbf{x}(2) & \cdots & \mathbf{x}(N)\end{bmatrix}^T \tag{8}$$

can be realized from

$$\mathbf{X} = \mathcal{X}\mathbf{\Lambda}^{1/2}\mathbf{W}^H \tag{9}$$

where $\mathcal{X} \in \mathcal{C}^{N\times M}$ is a matrix of zero mean and unit variance Gaussian random variables, $\mathbf{\Lambda} \in \mathcal{R}^{M\times M}$ is the diagonal matrix of eigenvalues and $\mathbf{W} \in \mathcal{C}^{M\times M}$ is the matrix of eigenvectors of $\mathbf{R}$ [12]. Due to the distribution of $\mathcal{X}$, the distribution of the random variables in the columns of $\mathbf{X}$ is also Gaussian but the waveform produced is not guaranteed to have the CE property. In Section III, we discuss the construction of waveforms with the CE property.

B. Selective Null Space Projection Algorithm

In the previous section, we introduced preliminaries of a beampattern matching design problem with constraints that the covariance matrix should be positive semidefinite and the requirement that all antennas transmit at the same power level, for constant envelope waveform design. These constraints are particular to MIMO radar waveform design and guarantee a constant envelope waveform, as we will see in Section III. However, the above mentioned constraints don't take into account interference caused by MIMO radar to cellular system. In order to ensure zero interference to the cellular system we put an additional constraint in waveform design problem that the waveform should also be in null space of the selected communication channel.

In this paper, we consider a MIMO cellular system, with N_{BS} base stations (BS). Each BS is equipped with N_T^{BS} transmit antennas and N_R^{BS} receive antennas. In addition the capacity of each BS is to support L^{UE} user equipments (UE). The UEs are also MIMO systems with N_T^{UE} transmit antennas and N_R^{UE} receive antennas. Under this configuration, the MIMO radar is sharing N_{BS} interference channels $\mathbf{H}_i^{N_R^{\text{BS}}\times M}$ with the cellular system. If $\mathbf{x}(t)$ and $\mathbf{x}_j^{\text{UE}}(t)$ are the signals transmitted from the MIMO radar and the j^{th} UE in the i^{th} cell, respectively, then the received signal at the i^{th} BS can be written as

$$\mathbf{y}_i(t) = \mathbf{H}_i^{N_R^{\text{BS}}\times M}\mathbf{x}(t) + \sum_j \mathbf{H}_j^{N_R^{\text{BS}}\times N_T^{\text{UE}}}\mathbf{x}_j^{\text{UE}}(t) + \mathbf{w}(t)$$
$$\text{for } 1 \leq i \leq N_{\text{BS}} \text{ and } 1 \leq j \leq L_i^{\text{UE}}$$

where $\mathbf{w}(t)$ is the additive white Gaussian noise. Our goal is to design MIMO radar waveform that do not interfere with the cellular system, i.e. in addition to already mentioned constraints it should also be in the null-space of $\mathbf{H}_i^{N_R^{\text{BS}}\times M}$ in order to avoid interference to the i^{th} cellular BS.

The MIMO radar is sharing spectrum with a cellular system which has N_{BS} base stations, thus, there exist N_{BS} interference channels, i.e. $\mathbf{H}_i, i = 1, 2, \ldots, N_{\text{BS}}$, between the MIMO radar and the cellular system. Among N_{BS} interference channels we select the interference channel for waveform design which has the maximum null space in order for the waveform to have optimum performance. For interference channel selection, two algorithms, Algorithms (1) and (2), are presented in [13] which first estimate the channel state information of interference channels. This is followed by the calculation of null space of interference channels and interference channel with the maximum null space is selected as the candidate channel. For our beampattern matching problem, we seek to select the best interference channel, defined as

$$i_{\text{max}} \triangleq \underset{1\leq i\leq N_{BS}}{\arg\max}\ \dim[\mathcal{N}(\mathbf{H}_i)]$$
$$\mathbf{H}_{\text{Best}} \triangleq \mathbf{H}_{i_{\text{max}}}$$

and we seek to avoid the worst channel, defined as

$$i_{\text{min}} \triangleq \underset{1\leq i\leq N_{BS}}{\arg\min}\ \dim[\mathcal{N}(\mathbf{H}_i)]$$
$$\mathbf{H}_{\text{Worst}} \triangleq \mathbf{H}_{i_{\text{min}}}$$

for MIMO radar waveform design.

Algorithm 1 Interference-Channel-Selection Algorithm [13]

loop
  for $i = 1 : N_{\text{BS}}$ **do**
    Estimate CSI of $\mathbf{H}_i$.
    Send $\mathbf{H}_i$ to Algorithm (2) for null space computation.
    Receive dim$[\mathcal{N}(\mathbf{H}_i)]$ from Algorithm (2).
  end for
  Find $i_{\text{max}} = \arg\max_{1\leq i\leq N_{BS}} \dim[\mathcal{N}(\mathbf{H}_i)]$.
  Set $\breve{\mathbf{H}} = \mathbf{H}_{i_{\text{max}}}$ as the candidate interference channel.
  Send $\breve{\mathbf{H}}$ to Algorithm (2) to get NSP radar waveform.
end loop

Once $\mathbf{H}_{\text{Best}}$ or $\breve{\mathbf{H}}$ is selected the next step is to construct a projection matrix via singular value decomposition (SVD) theorem, which is given as

$$\breve{\mathbf{H}}^{N_R^{\text{BS}}\times M} = \mathbf{U}\mathbf{\Sigma}^{N_R^{\text{BS}}\times M}\mathbf{V}^H$$
$$= \mathbf{U}\begin{pmatrix}\sigma_1 & & & \\ & \sigma_2 & & \mathbf{0} \\ & & \ddots & \\ \mathbf{0} & & & \sigma_{j\in\min(N_R^{\text{BS}},M)}\end{pmatrix}\mathbf{V}^H$$

where $\mathbf{U}$ is the complex unitary matrix, $\mathbf{\Sigma}$ is the diagonal matrix of singular values, and $\mathbf{V}^H$ is the complex unitary matrix. If SVD results in non-zero singular values, we calculate null space numerically via Algorithm (2). A threshold is defined and all the vectors in $\mathbf{V}^H$ corresponding to singular values below the threshold are collected in $\breve{\mathbf{V}}$. Then, the projection matrix is formulated as in [4], [14]

$$\mathbf{P}_{\breve{\mathbf{V}}} = \breve{\mathbf{V}}\breve{\mathbf{V}}^H. \tag{10}$$

III. Gaussian Covariance Matrix Synthesis for Desired Beampattern

An algorithm to directly synthesize covariance matrix of Gaussian random variables to generate finite alphabet constant

Algorithm 2 Modified Null-Space Projection (NSP) [13]

```
if H_i received from Algorithm (1) then
    Perform SVD on H_i (i.e. H_i = U_i Σ_i V_i^H)
    if σ_j ≠ 0 (i.e. j^th singular value of Σ_i) then
        dim[N(H_i)] = 0
        Use pre-specified threshold δ
        for j = 1 : min(N_R^BS, M) do
            if σ_j < δ then
                dim[N(H_i)] = dim[N(H_i)] + 1
            else
                dim[N(H_i)] = 0
            end if
        end for
    else
        dim[N(H_i)] = The number of zero singular values
    end if
    Send dim[N(H_i)] to Algorithm (1).
end if
if Ȟ received from Algorithm (1) then
    Perform SVD on Ȟ = UΣV
    if σ_j ≠ 0 then
        Use pre-specified threshold δ
        σ_Null = {} {An empty set to collect σs below threshold δ}
        for j = 1 : min(N_R^BS, M) do
            if σ_j < δ then
                Add σ_j to σ_Null
            end if
        end for
        V̌ = σ_Null corresponding columns in V.
    end if
    Setup projection matrix P_V̌ = V̌ V̌^H.
    Get NSP radar signal via Z_NSP = Z P_V̌^H.
end if
```

envelope binary phase shift keying (BPSK) waveform for a desired beampattern was proposed by Ahmed et al [11]. Using the same approach, the Gaussian random variables with zero mean and unit variance, x_m, can be mapped onto BPSK symbol, z_m, through a simple relation

$$z_m = \text{sign}(x_m), \quad m \in \{1, 2, ..., M\}. \tag{11}$$

Using results from [11], we have

$$\begin{aligned}\mathbb{E}(z_p z_q) &= \mathbb{E}\Big(\text{sign}(x_p)\text{sign}(x_q)\Big) \\ &= \frac{2}{\pi}\sin^{-1}\big(\mathbb{E}(x_p x_q)\big)\end{aligned} \tag{12}$$

where x_p and x_q are Gaussian random variables and z_p and z_q are BPSK random variables. Therefore, the relation between real covariance matrix of beampattern $\mathbf{R}$ and Gaussian covariance matrix $\mathbf{R}_g$ is given by

$$\mathbf{R} = \frac{2}{\pi}\sin^{-1}(\mathbf{R}_g). \tag{13}$$

The Gaussian covariance matrix $\mathbf{R}_g$ is generated by the matrix $\mathbf{X}$ of Gaussian random variables using (9). Then BPSK random variables are generated directly by

$$\mathbf{Z} = \text{sign}(\mathbf{X}). \tag{14}$$

In [12], the authors propose to synthesize $\mathbf{R}_g$ as $\mathbf{R}_g = \mathbf{U}^H\mathbf{U}$,

$$\min_{\psi_{ij}} \frac{1}{K}\sum_{k=1}^{K}\Big(\frac{2}{\pi}\mathbf{a}^H(\theta_k)\sin^{-1}(\mathbf{U}^H\mathbf{U})\mathbf{a}(\theta_k) - \phi(\theta_k)\Big)^2 \tag{15}$$

where ψ_{ij} are the variables of the optimization problem and $\mathbf{U}$ is given by equation (16).

IV. Waveform Design for Spectrum Sharing

In this section, we consider the design of MIMO radar waveforms for spectrum sharing. We consider two waveform design approaches: one in which we include the spectrum sharing constraint in the optimization problem and in the other we don't. The motivation and reasons for these two approaches and their impact on radar waveform performance is discussed in the next sections.

A. NSP included in the optimization

Consider the case of a maritime MIMO radar when a ship is docked or is stationary and thus radar platform is stationary. In this case, interference channels have little to no variations and thus it is feasible to include the constraint of NSP into the optimization problem. The new optimization problem is formulated by the combination of projection matrix, equation (10), into the optimization problem in equation (15) as

$$\min_{\psi_{ij}} \frac{1}{K}\sum_{k=1}^{K}\Big(\frac{2}{\pi}\mathbf{a}^H(\theta_k)\mathbf{P}_{\check{\mathbf{V}}}\sin^{-1}(\mathbf{U}^H\mathbf{U})\mathbf{P}_{\check{\mathbf{V}}}^H\mathbf{a}(\theta_k) - \phi(\theta_k)\Big)^2. \tag{17}$$

This optimization problem does not guarantee to generate constant envelope radar waveform but guarantees that the designed waveform is in the null space of the interference channel or the designed waveform does not cause interference to the communication system. In addition, it is an evaluation of the impact of the NSP on the CE radar waveforms. The waveform generation process is shown using the block diagram of Figure 1. The waveform generated by solving the optimization problem in equation (17) and then using equation (9) is denoted by $\mathbf{X}_{\text{opt}}$. The corresponding BPSK waveform is denoted by $\mathbf{Z}_{\text{opt}}$ which is obtained using equation (14). Then, the output NSP waveform is given by

$$\mathbf{Z}_{\text{opt-NSP}} = \mathbf{Z}_{\text{opt}}\mathbf{P}_{\check{\mathbf{V}}}^H. \tag{18}$$

B. NSP after the optimization

Consider the case of a maritime radar which is moving and thus experiences interference channels that change too fast. In this case, it is not feasible to include the NSP in the optimization problem. Alternately, we can design CE waveforms by solving the optimization problem in equation (15) and then projection the waveform onto the null space

$$\mathbf{U} = \begin{pmatrix} 1 & \sin(\psi_{21}) & \sin(\psi_{31})\sin(\psi_{32}) & \cdots & \prod_{m=1}^{M-1}\sin(\psi_{Mm}) \\ 0 & \cos(\psi_{21}) & \sin(\psi_{31})\cos(\psi_{32}) & \cdots & \prod_{m=1}^{M-2}\sin(\psi_{Mm})\cos(\psi_{M,M-1}) \\ 0 & 0 & \cos(\psi_{31}) & \ddots & \vdots \\ \vdots & \vdots & \ddots & \cdots & \sin(\psi_{M1})\cos(\psi_{M2}) \\ 0 & 0 & \cdots & \cdots & \cos(\psi_{M1}) \end{pmatrix} \tag{16}$$

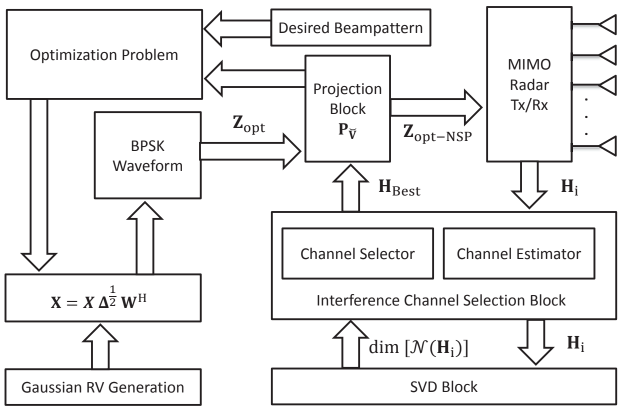

Fig. 1. Block diagram of the transmit beampattern design problem for a stationary maritime MIMO radar. The desired waveform is generated by including the projection matrix $\mathbf{P}_{\check{\mathbf{V}}}$, for the candidate interference channel $\mathbf{H}_{\text{Best}}$, in the optimization process. For this waveform constant envelope property is not guaranteed. The candidate interference channel is selected by Algorithms (1) and (2).

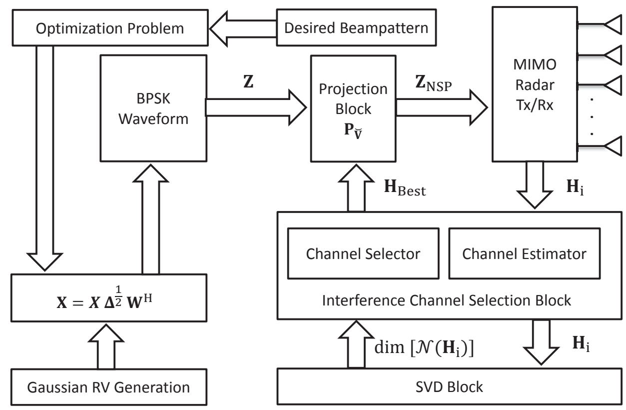

Fig. 2. Block diagram of the transmit beampattern design problem for a moving maritime MIMO radar. The desired waveform is generated with constant envelope property and then projected onto the candidate interference channel $\mathbf{H}_{\text{Best}}$ via projection matrix $\mathbf{P}_{\check{\mathbf{V}}}$. The candidate interference channel is selected by Algorithms (1) and (2).

of interference channel. The waveform generation process is shown using the block diagram of Figure 2. Thus, the CE waveform is generated according to the method explained in Section III and then projected onto the null space of the interference channel according to

$$\mathbf{Z}_{\text{NSP}} = \mathbf{Z}\mathbf{P}_{\check{\mathbf{V}}}^{H}. \tag{19}$$

This formulation projects the CE waveform onto the null space of the interference channel and we study its impact on the radar waveform performance.

V. Simulation Results

In this section, we simulate our designed MIMO radar waveform for spectrum sharing. We use a uniform linear array (ULA) of ten elements, i.e., $M = 10$, with an interelement spacing of half-wavelength. In addition, all antennas transmit at the same power level which we fix to unity. Each antenna transmits a waveform with $N = 100$ symbols and the resulting beampattern is the average of 100 Monte-Carlo trials of BPSK waveforms. The mean-squared error (MSE) between the desired, $\phi(\theta_k)$, and actual beampatterns, $P(\theta_k)$, is given by

$$\text{MSE} = \frac{1}{K}\sum_{k=1}^{K}\Big(P(\theta_k) - \phi(\theta_k)\Big)^2.$$

The interference channels, $\mathbf{H}_i$, are modeled as a Rayleigh fading channels with Rayleigh probability density function (pdf) given by

$$f(h|\rho) = \frac{h}{\rho^2}e^{\frac{-h^2}{2\rho^2}}$$

where ρ is the mode of the Rayleigh distribution. The candidate interference channel, $\check{\mathbf{H}}$, for waveform design is selected using Algorithm (1) and its null space is computed using SVD according to Algorithm (2).

In Figure 3, the desired beampattern has two main lobes from $-60°$ to $-40°$ and from $40°$ to $60°$. It is the beampattern for stationary maritime MIMO radar obtained by solving the optimization problem in equation (17), as discussed in Section IV-A. The resulting waveform covariance matrix is given by

$$\mathbf{R}_{\text{opt-NSP}} = \frac{1}{N}\mathbf{Z}_{\text{opt-NSP}}^{H}\mathbf{Z}_{\text{opt-NSP}}$$

Note that the desired beampattern and the beampattern obtained by including the projection matrix inside the optimization problem match closely for interference channel $\mathbf{H}_{\text{Best}}$ than $\mathbf{H}_{\text{Worst}}$.

In Figure 4, the desired beampattern has two main lobes from $-60°$ to $-40°$ and from $40°$ to $60°$. It represents the beampattern of a moving maritime MIMO radar. Since interference channels are evolving fast, beampattern is obtained by

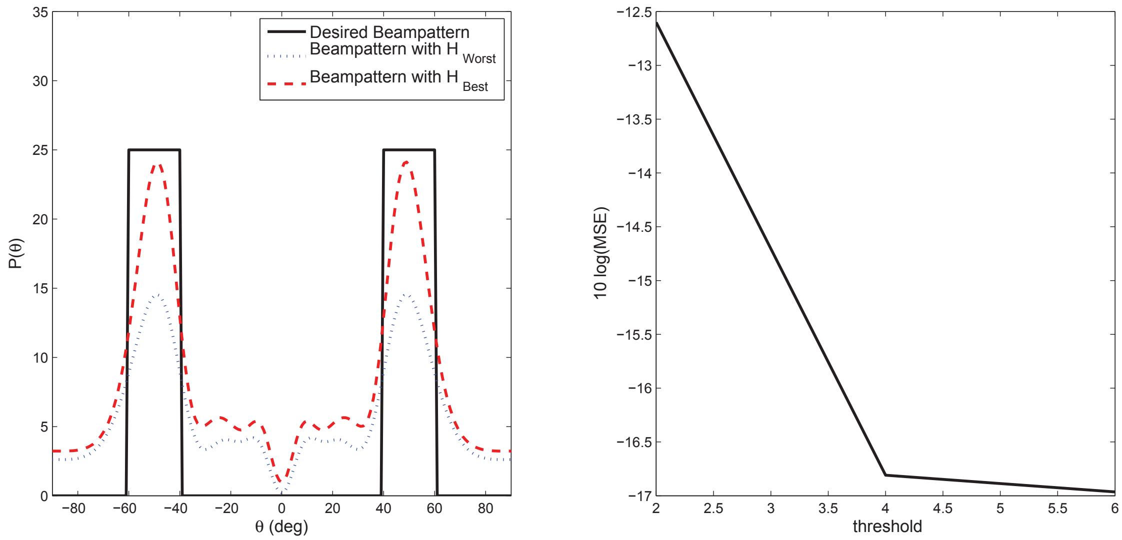

Fig. 3. Transmit beampattern and its MSE for a *stationary* maritime MIMO radar. The figure compares the desired beampattern with the average beampattern of BPSK waveforms for null space projection *included* in beampattern matching optimization problem for candidate interference channel $\mathbf{H}_{\text{Best}}$.

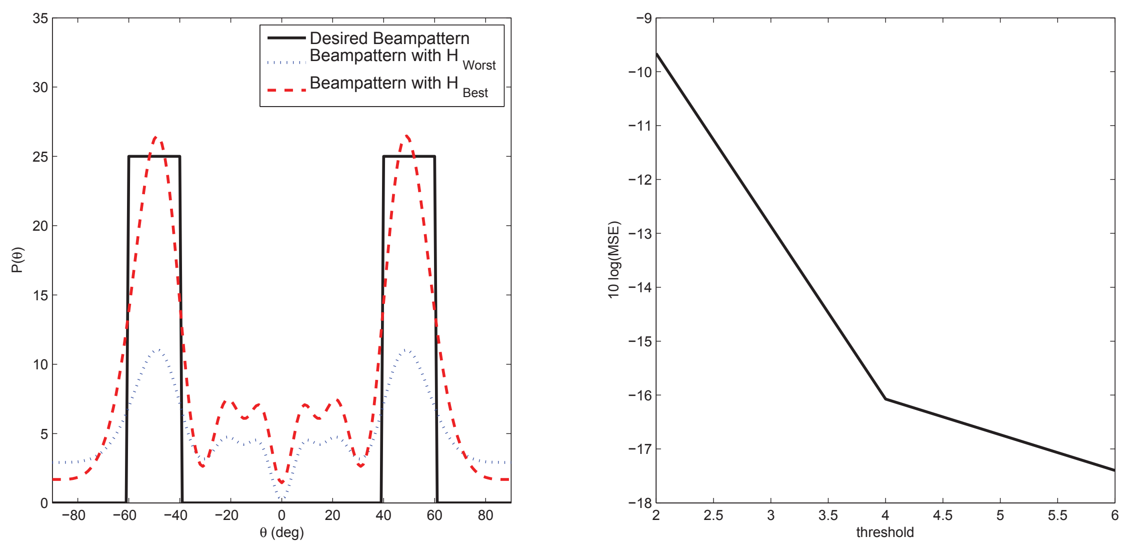

Fig. 4. Transmit beampattern and its MSE for a *moving* maritime MIMO radar. The figure compares the desired beampattern with the average beampattern of BPSK waveforms for null space projection *after* optimization for candidate interference channel $\mathbf{H}_{\text{Best}}$.

solving the optimization problem in equation (15) and then projecting the resulting waveform onto the null space of $\breve{\mathbf{H}}$ using the projection matrix in equation (10). The resulting waveform covariance matrix is given by

$$\mathbf{R}_{\text{NSP}} = \frac{1}{N}\mathbf{Z}_{\text{NSP}}^{H}\mathbf{Z}_{\text{NSP}}.$$

Note that the desired beampattern and the beampattern obtained by projecting the waveform onto the null space of interference channel match closely for interference channel $\mathbf{H}_{\text{Best}}$ than $\mathbf{H}_{\text{Worst}}$.

In Figures 3 and 4, we also plot the MSE of beampattern matching design problem. It shows that interference channel with the largest null space have the least MSE. This is in accordance with our methodology to select $\mathbf{H}_{\text{Best}}$ among $\mathbf{N}_{\text{BS}}$ interference channels using Algorithms (1) and (2).

In Figures 3 and 4, the desired beampattern match closely the actual beampattern when interference channel $\mathbf{H}_{\text{Best}}$ is used. Thus, by careful selection of interference channel using Algorithms (1) and (2), when sharing spectrum with a cellular system, we can obtain a beampattern which is very close to the desired beampattern and in addition do not interfere with the communication system

VI. Conclusion and Limitations

In this paper, we consider the MIMO radar waveform design from a spectrum sharing perspective. We consider a MIMO radar and a cellular system sharing spectrum and we design radar waveforms such that they are not interfering with the cellular system. A method to design MIMO radar waveforms is presented which matches the beampattern to a certain desired beam pattern with the constraints that the waveform should have constant envelope and belong to the null space of interference channel. We designed waveform for the case when the MIMO radar is stationary and thus NSP can be included in the optimization problem due to the tractability of interference channel. We also designed waveform for the case when the MIMO radar is moving and experiences rapidly changing interference channels. This problem didn't consider the inclusion of NSP in the optimization problem due to the intractability of interference channel but rather constructed a CE radar waveform and projected it onto null space of interference channel. The interference channel was selected using Algorithms (1) and (2) and results showed that for both type of waveforms the desired beampattern and NSP beampatterns matched closely.

In this work, radar waveform was designed to protect one cellular system from radar's interference. The other cellular systems, in the network, can be protected from radar interference by using resource allocation techniques at the cellular systems [15].

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