

Spectrum Sharing between S-band Radar and LTE Cellular System: A Spatial Approach

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Abstract—Spectrum sharing is a new way forward to solve spectrum scarcity problem. In this paper, we propose a spatial approach for spectrum sharing between a MIMO radar and an LTE cellular system with N_{BS} base stations (BS). The MIMO radar and LTE share N_{BS} interference channels i.e. $H_i, i = 1, 2, \dots, N_{BS}$. We propose projecting the radar signal onto the null space of interference channel between the MIMO radar and LTE using our proposed interference-channel-selection algorithm, in order to have zero-interference from the MIMO radar. We select interference channel with the maximum null space i.e. $\arg \max_{1 \leq i \leq N_{BS}} \dim[N(H_i)]$ and project the radar signal onto the null space of this channel. Our proposed spatial spectrum-sharing algorithm is radar-centric such that it causes minimum loss in radar performance by carefully selecting the interference channel and at the same time protects the i^{th} LTE BS from the radar interference. Through our analytical and simulation results we show that the loss in the radar performance is less when the proposed interference-channel-selection algorithm is used to select the channel onto which radar signals are projected.

I. INTRODUCTION

In order to provide cellular broadband, operators need more spectrum. However, the demand for spectrum far exceeds the availability for commercial utilization. In order to satisfy operator requirements, the Federal Communications Commission (FCC), in the United States (US), is considering a number of options including incentive auctioning and sharing of spectrum. Of these, spectrum sharing is quite promising due to the large number of spectrum bands that can be shared. Yet it is challenging because incumbents need to be shielded from harmful interference that can arise due to the operation of other systems in the shared bands.

Recent studies by the FCC and the National Telecommunications and Information Administration (NTIA), the spectrum manager for the federal agencies in the US, have noticed that huge chunks of spectrum held by the federal agencies have low average utilization, especially in the urban areas. On the other hand, spectrum held by commercial operators, including cellular operators, have heavy utilization in the urban areas than the federal agencies. The demands for bandwidth from wireless operators have led to proposals to reduce or share spectrum allocated for federal radar operations [1]. In order to efficiently utilize spectrum currently in-use by the federal agencies, the US government is exploring ways to share the federal spectrum with commercial operators. More spectrum

for commercial operators will result in huge economic and social prospects for the nation. However, such sharing should not compromise sensitive or classified information or operations of the federal incumbents. Thus, innovative approaches need to be sought in order to enable sharing of the federal spectrum.

The NTIA, recently, evaluated the sharing of radar band with WiMAX and found that in order to protect the WiMAX system from radar interference huge exclusion zones upto tens of kilometers are required [2]. This is due to high signal power used by radars and high-peak sidelobes which saturate communication system receivers, which are traditionally designed to handle power levels in watts rather than kilo watts or mega watts. Such high peak powers are typical of airport surveillance radars, weather radars, and military phased array radars such as SPY-1 radar of Aegis system. On the other hand, due to highly sensitive radar receivers, designed to detect even the faintest of returned signal, has in the past mandated for exclusive rights to radio spectrum allocations since its operation can be affected by commercial wireless system interference [2], [3].

The heterogeneous nature of devices sharing an RF band, radar and cellular system in our case, dictates the need for electromagnetic interference (EMI) mitigation tactics for both systems since traditional interference mitigation tactics are meant for the other exclusive users of the same RF band. The emission pattern, both in space and time, of a radar is significantly different from a communication system. This point is also validated from a study by the NTIA, showing that radar receivers handle noise like interference from communications systems differently than the interference from other radars with former having detrimental effect on radar due to its continuous wide-band nature than the low duty cycle radar waveforms [2].

In the past, it has been made possible for wireless systems to share government bands such that they operate under a low-power constraint in order to protect incumbents from interference. Example includes: Wi-Fi and Bluetooth in the 2450-2490 MHz band, wireless local area network (WLAN) in the 5.25-5.35 and 5.47-5.725 GHz radar bands [4], and recently the FCC has proposed small cells, i.e. wireless base stations operating on a low power, to operate in the 3550-3650 MHz radar band [5].

The 3550-3650 MHz band, currently used for military

and satellite operations, is a possible candidate for spectrum sharing between military radars and broadband wireless access (BWA) communication systems such as LTE and WiMAX, according to the NTIA's 2010 Fast Track Report [6]. Electro-magnetic interference to military radar operations is expected from spectrum sharing. However, one simply can't relocate these federal radar systems to other bands since the nature of the said band contains many frequencies which work best for highly sensitive fixed, airborne, and maritime radar systems and are essential for superior performance. Moreover, cost to relocate can be unbearable. The problem of EMI mitigation is possible due to advancements in transmitter and receiver design technology, of cellular systems, which has made real-time spectrum reassignment possible.

A. Related Work

In spectrum sharing perspective between radars and communication systems, EMI needs to be mitigated at both the systems. Communication systems due to their advancements give more freedom to mitigate interference from radar systems. For example, in order to counter radar interference on WiMAX systems, interference mitigation in four domains namely space, time, frequency, and system-level modification is proposed by Lackpour et. al. [7]. Radar systems due to their sensitivity are more susceptible to interference from communication systems. So far, as previously discussed, in order to protect radar operations, communication systems operate on a low-power basis to avoid interference to radars or operate by sensing the availability of radar channel at a power level which doesn't exceed the allowed interference limit [4], [8].

Radar systems are also evolving and with the recent trend in the design of MIMO radars and cognitive radars, radar systems are becoming more resilient in handling interference and jamming as they are more aware about their radio environment map (REM). This has motivated researchers to propose beamforming approaches to mitigate interference from wireless communication systems to MIMO radar [9]. In addition, spatial domain can also be used to mitigate MIMO radar interference to wireless communication system. One such technique was proposed by Shabnam et al. [10] which projected radar signal onto the null space of interference channel between MIMO radar and MIMO communication system. Moreover, radar waveforms can also be designed such that they don't cause interference to cellular systems, in addition to meeting their mission objectives [11].

B. Our Contributions

In this paper, we extend the previous work of radar signal projection onto the null space of interference channel between radar and communication systems in order to avoid interference to communication system [10], [12], [13]. The previous work considers coexistence scenario of a MIMO radar with a MIMO communication system where only one interference channel exists. We extend this approach and consider a more realistic scenario of an LTE cellular system sharing spectrum with a MIMO radar.

The idea of projecting signals onto the null space of interference channel, in order to avoid interference, is a well studied topic in cognitive radio research community [14], however, for MIMO radar systems this idea was first proposed and its effect on radar performance was studied in [10]. Interference channel's null space is calculated at transmitter, radar in this case, either by exploiting channel reciprocity, if communication system is a Time Division Duplex (TDD) system and using its second order statistics as in [14]; or by blindly estimating null space, using say the method of [15], if no cooperation exists between radar and communication systems.

Our contribution in this paper are as follows:

- We extend the spectrum sharing scenario of [10] between a MIMO radar and single communication system to spectrum sharing between a MIMO radar and an LTE system with many base stations. The extension to a cellular system gives rise to more than one interference channel, since a cellular system has many base stations. This makes the problem of interference mitigation challenging for radar system since now it has to cope with many interference channels instead of one, as in the setup of [10].
- In order to cope with many interference channels, we propose an interference-channel-selection algorithm which selects the best channel for radar's signal projection assuring minimum degradation in the radar performance while assuring zero interference to the LTE BS.
- We propose an improved null space projection algorithm by removing redundancies of the algorithm proposed in [10] to make it computationally more robust.

The remainder of this paper is organized as follows. Section II discusses spectrum sharing architecture between MIMO radar and LTE cellular system. Section III contains three subsections which cover our radar-centric spectrum-sharing methodology. Section III-A briefly discusses the idea of colocated MIMO radar. Section III-B presents our interference-channel-selection algorithm. In Section III-C, we explain our modified null-space projection algorithm. Section IV discusses simulation setup and provides quantitative results along with the discussion. Section V concludes the paper.

II. RADAR-LTE SPECTRUM SHARING ARCHITECTURE

In this paper, we consider a MIMO LTE cellular system, with N_{BS} base stations, each equipped with N_T^{BS} transmit antennas and N_R^{BS} receive antennas, with i^{th} BS supporting K_i^{UE} user equipments (UEs). The UEs are also multi-antenna systems with N_T^{UE} transmit antennas and N_R^{UE} receive antennas. MIMO radar under consideration is a colocated MIMO radar mounted on a ship. The MIMO radar has M_T transmit antennas and M_R receive antennas. We use colocated radars because they have improved spatial resolution over widely-spaced radars [16].

The MIMO radar and LTE systems are the co-primary users of the 3550-3650 MHz band under consideration. The LTE system operates in its licensed band and in order to increase

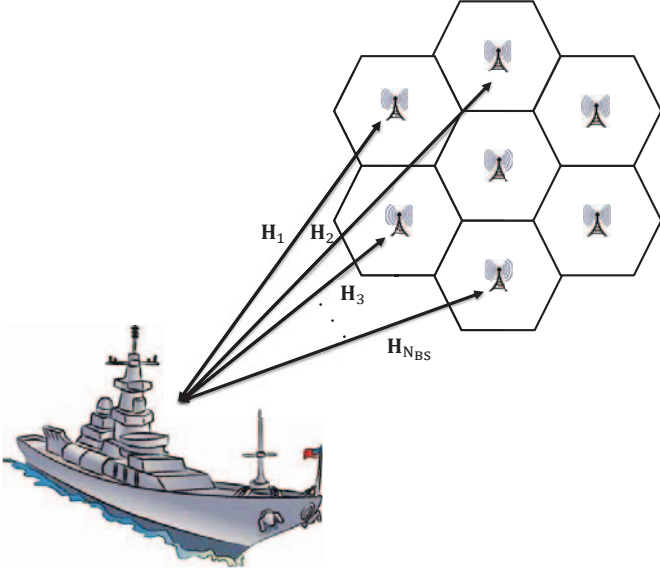


Fig. 1. Spatial spectrum-sharing scenario. An LTE cellular system with a maritime MIMO radar.

its capacity it operates in the 3550-3650 MHz radar band on a sharing basis under an agreement that both systems will employ tactics that will not cause harmful interference to each other.

A typical coexistence scenario is shown in Fig. 1 where the maritime MIMO radar is sharing N_{BS} interference channels $\mathbf{H}_i^{N_{BS} \times M_T}$ with the LTE system. If $\mathbf{x}_{\text{Radar}}(t)$ and $\mathbf{x}_j^{\text{UE}}(t)$ are the signals transmitted from the MIMO radar and the j^{th} UE in the i^{th} cell, respectively, then the received signal at the i^{th} BS receiver can be written as

$$\mathbf{y}_i(t) = \mathbf{H}_i^{N_{BS} \times M_T} \mathbf{x}_{\text{Radar}}(t) + \sum_j \mathbf{H}_j^{N_{BS} \times N_T^{\text{UE}}} \mathbf{x}_j^{\text{UE}}(t) + \mathbf{w}(t)$$

for $1 \leq i \leq N_{BS}$ and $1 \leq j \leq K_i^{\text{UE}}$

where $\mathbf{w}(t)$ is the additive white Gaussian noise and the goal of the MIMO radar is to map $\mathbf{x}_{\text{Radar}}(t)$ onto the null-space of $\mathbf{H}_i^{N_{BS} \times M_T}$ in order to avoid interference to the i^{th} LTE BS.

III. RADAR-CENTRIC DESIGN: A SPATIAL APPROACH

In this section, we describe briefly colocated MIMO radars and then present our interference-channel-selection algorithm along with null space projection algorithm.

A. Colocated MIMO Radar

In this section, we design radar waveform along with introducing the preliminaries of a colocated MIMO radar. The MIMO radar we consider in this paper is a colocated MIMO radar with M_T transmit antennas and M_R receive antennas. Let $\mathbf{x}_{\text{Radar}}(t)$ be the signal transmitted from the MIMO radar, defined as

$$\mathbf{x}_{\text{Radar}}(t) = [x_1(t)e^{j\omega_c t} \quad x_2(t)e^{j\omega_c t} \quad \dots \quad x_{M_T}(t)e^{j\omega_c t}]^T$$

where $x_k(t)$ is the baseband signal from the k^{th} transmit element, ω_c is the carrier angular frequency, $t \in [0, T_o]$, with

T_o being the observation time. We choose to design finite alphabet constant envelope radar waveforms because of their property to allow radio frequency amplifiers to operate at a maximum power efficiency [12]. In order to write the received signal, let us further define the transmit steering vector as

$$\mathbf{a}_T(\theta) \triangleq [e^{-j\omega_c \tau_{T1}(\theta)} \quad e^{-j\omega_c \tau_{T2}(\theta)} \quad \dots \quad e^{-j\omega_c \tau_{TM_T}(\theta)}]^T$$

the receive steering vector as

$$\mathbf{a}_R(\theta) \triangleq [e^{-j\omega_c \tau_{R1}(\theta)} \quad e^{-j\omega_c \tau_{R2}(\theta)} \quad \dots \quad e^{-j\omega_c \tau_{RM_R}(\theta)}]^T$$

and the transmit-receive steering matrix as

$$\mathbf{A}(\theta) \triangleq \mathbf{a}_R(\theta) \mathbf{a}_T^T(\theta).$$

Then, the signal received from a single point target at an angle θ is

$$\mathbf{y}_{\text{Radar}}(t) = \alpha \mathbf{A}(\theta) \mathbf{x}_{\text{Radar}}(t - \tau(t))$$

where $\tau(t) = \tau_{T_k}(t) + \tau_{R_l}(t)$, denoting the sum of propagation delays between the target and the k^{th} transmit element and the l^{th} receive element, respectively; and α represents the complex path loss including the propagation loss and the coefficient of reflection.

We choose the Cramér Rao bound (CRB) and maximum likelihood (ML) estimate of the target's angle of arrival as our performance metric for the MIMO radar system. We are interested in studying the degradation in the estimate of the target's angle of arrival due to null space projection of the radar waveform. The CRB for a single target, no-interference case, is given as in [17],

$$\text{CRB}(\theta) = \frac{1}{2 \text{SNR}} \left(M_R \dot{\mathbf{a}}_T^H(\theta) \mathbf{R}_{\mathbf{x}_{\text{Radar}}}^T \dot{\mathbf{a}}_T(\theta) + \mathbf{a}_T^H(\theta) \mathbf{R}_{\mathbf{x}_{\text{Radar}}}^T \mathbf{a}_T(\theta) \|\dot{\mathbf{a}}_R(\theta)\|^2 - \frac{M_R |\mathbf{a}_T^H(\theta) \mathbf{R}_{\mathbf{x}_{\text{Radar}}}^T \dot{\mathbf{a}}_T(\theta)|^2}{\mathbf{a}_T^H(\theta) \mathbf{R}_{\mathbf{x}_{\text{Radar}}}^T \mathbf{a}_T(\theta)} \right)^{-1} \quad (1)$$

and the ML for the case of no interference and a single target can be written as in [17],

$$(\hat{\theta}, \hat{\tau}_r, \hat{\omega}_D)_{\text{ML}} = \arg \max_{\theta, \tau_r, \omega_D} \frac{|\mathbf{a}_R^H(\theta) \mathbf{E}(\tau_r, \omega_D) \mathbf{a}_T^*(\theta)|^2}{M_R \mathbf{a}_T^H(\theta) \mathbf{R}_{\mathbf{x}_{\text{Radar}}}^T \mathbf{a}_T(\theta)} \quad (2)$$

where

$$\begin{aligned} \dot{\mathbf{a}}_R(\theta) &= \frac{d\mathbf{a}_R(\theta)}{d\theta} \\ \dot{\mathbf{a}}_T(\theta) &= \frac{d\mathbf{a}_T(\theta)}{d\theta} \\ \mathbf{R}_{\mathbf{x}_{\text{Radar}}} &= \int_{T_0} \mathbf{x}_{\text{Radar}}(t) \mathbf{x}_{\text{Radar}}^H(t) dt \\ \mathbf{E}(\tau_r, \omega_D) &= \int_{T_0} \mathbf{y}_{\text{Radar}}(t) \mathbf{x}_{\text{Radar}}^H(t - \tau_r) e^{j\omega_D t} dt \end{aligned}$$

τ_r is the propagation delay, two-way, between the target and the reference point, and ω_D is the Doppler frequency shift as defined in Table I.

In addition to performance metrics like CRB and ML, we are also interested in the changes in beampatterns of the

MIMO radar due to null space projection of the radar waveform. Beampattern is a measure of beamformer's response to a target at direction θ given by, as in [17],

$$G(\theta, \theta_D) = \Gamma \frac{|\mathbf{a}_T^H(\theta) \mathbf{R}_{\mathbf{x}_{\text{Radar}}}^T \mathbf{a}_T(\theta_D)|^2}{|\mathbf{a}_T^H(\theta_D) \mathbf{R}_{\mathbf{x}_{\text{Radar}}}^T \mathbf{a}_T(\theta_D)|^2} \frac{|\mathbf{a}_R^H(\theta) \mathbf{a}_R(\theta_D)|^2}{M_R} \quad (3)$$

where Γ is the normalization constant and θ_D represents the digital steering direction of the main beam.

B. Interference-Channel-Selection Algorithm

In this section, we propose our interference-channel-selection algorithm, shown in Algorithm (1), which selects interference channel onto which radar signals are projected using NSP method, i.e. Algorithm (2). We assume there exist N_{BS} interference channels, i.e. $\mathbf{H}_i, i = 1, 2, \dots, N_{\text{BS}}$, between the MIMO radar and the LTE system and we seek to select the best interference channel, defined as

$$i_{\max} \triangleq \arg \max_{1 \leq i \leq N_{\text{BS}}} \dim[\mathcal{N}(\mathbf{H}_i)]$$

$$\mathbf{H}_{\text{Best}} \triangleq \mathbf{H}_{i_{\max}}$$

and we seek to avoid the worst channel, defined as

$$i_{\min} \triangleq \arg \min_{1 \leq i \leq N_{\text{BS}}} \dim[\mathcal{N}(\mathbf{H}_i)]$$

$$\mathbf{H}_{\text{Worst}} \triangleq \mathbf{H}_{i_{\min}}$$

where null space of $\mathbf{H}_i^{N_{\text{BS}} \times M_T}$ is defined as

$$\mathcal{N}(\mathbf{H}_i) \triangleq \{\mathbf{x} \in \mathbb{C}^{M_T} : \mathbf{H}_i \mathbf{x} = \mathbf{0}\}$$

and then null of $\mathbf{H}_i^{N_{\text{BS}} \times M_T}$ is defined as

$$\text{null } \mathbf{H}_i \triangleq \dim[\mathcal{N}(\mathbf{H}_i)]$$

where 'dim' is the number of linearly independent columns in null space of $\mathbf{H}_i^{N_{\text{BS}} \times M_T}$.

At the MIMO radar, we first estimate the channel state information (CSI) of the N_{BS} interference channels using a blind null space learning algorithm [15]. This is followed by the calculation of null space of these N_{BS} interference channels via Algorithm (2). Once Algorithm (1) receives null space of interference channels, it selects channel with the maximum null space as the candidate channel i.e. $\check{\mathbf{H}}$ and sends it to Algorithm (2) for NSP of radar signals. Our interference-channel-selection algorithm i.e. Algorithm (1) guarantees minimum degradation in radar performance and at the same time assures zero interference to the candidate BS.

C. Modified Null-Space Projection (NSP) Algorithm

In this section, we explain the projection of radar signals onto null space of interference channel selected using Algorithm (1). As mentioned earlier, the CSI of N_{BS} interference channels is estimated using a blind null space learning algorithm [15]. After getting CSI estimates of N_{BS} interference channels, from Algorithm (1), the next step is to find null space of each $\mathbf{H}_i^{N_{\text{BS}} \times M_T}$ using Algorithm (2). This step is performed using the singular value decomposition (SVD)

Algorithm 1 Interference-Channel-Selection Algorithm

loop

for $i = 1 : N_{\text{BS}}$ **do**

Estimate CSI of \mathbf{H}_i .

Send \mathbf{H}_i to Algorithm (2) for null space computation.

Receive $\dim[\mathcal{N}(\mathbf{H}_i)]$ from Algorithm (2).

end for

Find $i_{\max} = \arg \max_{1 \leq i \leq N_{\text{BS}}} \dim[\mathcal{N}(\mathbf{H}_i)]$.

Set $\check{\mathbf{H}} = \mathbf{H}_{i_{\max}}$ as the candidate interference channel.

Send $\check{\mathbf{H}}$ to Algorithm (2) to get NSP radar waveform.

end loop

theorem according to our modified-NSP projection algorithm, as shown in Algorithm (2). For the complex i^{th} interference channel matrix the SVD is given as

$$\mathbf{H}_i^{N_{\text{BS}} \times M_T} = \mathbf{U}_i \Sigma_i^{N_{\text{BS}} \times M_T} \mathbf{V}_i^H$$

$$= \mathbf{U}_i \begin{pmatrix} \sigma_1 & & & & \\ & \sigma_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \sigma_{j \in \min(N_{\text{BS}}, M_T)} \\ & & & & & 0 \end{pmatrix} \mathbf{V}_i^H$$

where \mathbf{U}_i is the complex unitary matrix, Σ_i is the diagonal matrix of singular values, and \mathbf{V}_i^H is the complex unitary matrix. If the SVD analysis don't yield any zero singular values we resort to a numerical approach to calculate null space. In order to do that, in Algorithm (2), we set a threshold δ and select singular values below the threshold value. Then the number of singular values below the threshold serves as the dimension of null space.

Once the null space of all interference channels is determined we seek to find the best channel $\check{\mathbf{H}}$, the one with the maximum null space, which according to our Algorithm (1) is given as

$$i_{\max} = \arg \max_{1 \leq i \leq N_{\text{BS}}} \dim[\mathcal{N}(\mathbf{H}_i)]$$

$$\check{\mathbf{H}} = \mathbf{H}_{i_{\max}}$$

Algorithm (1) sends $\check{\mathbf{H}}$ to Algorithm (2) for null space computation where after SVD the right singular vectors corresponding to vanishing singular are collected in $\check{\mathbf{V}}$ for the formation of projection matrix. Once this is done, we project the radar signal onto the null space of \mathbf{H}_{Best} via a modified version of our projection algorithm [10], [12]. The proposed NSP algorithm removes redundancy from previous algorithm and is computationally efficient. The modified NSP algorithm is given as

$$\mathbf{P}_{\check{\mathbf{V}}} = \check{\mathbf{V}} \check{\mathbf{V}}^H.$$

The radar waveform projected onto null space of $\check{\mathbf{H}}$ can be written as

$$\check{\mathbf{x}}_{\text{Radar}} = \mathbf{P}_{\check{\mathbf{V}}} \mathbf{x}_{\text{Radar}}. \quad (4)$$

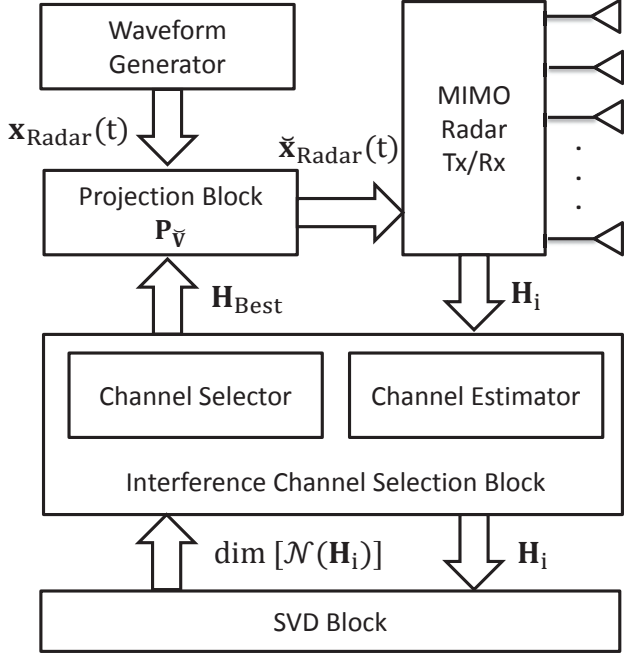


Fig. 2. Block diagram of the spatial spectrum sharing approach between MIMO radar and LTE systems. MIMO radar projects its waveform onto the interference channel with the maximum null space, i.e. \mathbf{H}_{Best} . The selection of interference channel and NSP is done using Algorithms (1) and (2). This process mitigates radar interference to the selected LTE BS.

This whole process is also illustrated via a radar-centric system-level block diagram in Figure 2. By inserting the projected signal, as in equation (4), into the Cramer Rao bound (CRB) for the single target no interference case, equation (1), we get the CRB for the NSP projected radar waveform as

$$\text{CRB}_{\text{NSP}}(\theta) = \frac{1}{2 \text{SNR}} \left(M_R \dot{\mathbf{a}}_T^H(\theta) \mathbf{R}_{\tilde{\mathbf{x}}_{\text{Radar}}}^T \dot{\mathbf{a}}_T(\theta) + \mathbf{a}_T^H(\theta) \mathbf{R}_{\tilde{\mathbf{x}}_{\text{Radar}}}^T \mathbf{a}_T(\theta) \|\dot{\mathbf{a}}_R(\theta)\|^2 - \frac{M_R |\mathbf{a}_T^H(\theta) \mathbf{R}_{\tilde{\mathbf{x}}_{\text{Radar}}}^T \dot{\mathbf{a}}_T(\theta)|^2}{\mathbf{a}_T^H(\theta) \mathbf{R}_{\tilde{\mathbf{x}}_{\text{Radar}}}^T \mathbf{a}_T(\theta)} \right)^{-1}. \quad (5)$$

Similarly, equation (4) can be substituted in (2) to get the ML estimate of angle arrival for the NSP projected radar waveform as

$$(\hat{\theta}, \hat{\tau}_r, \hat{\omega}_D)_{\text{ML-NSP}} = \arg \max_{\theta, \tau_r, \omega_D} \frac{|\mathbf{a}_R^H(\theta) \mathbf{E}(\tau_r, \omega_D) \mathbf{a}_T^*(\theta)|^2}{M_R \mathbf{a}_T^H(\theta) \mathbf{R}_{\tilde{\mathbf{x}}_{\text{Radar}}}^T \mathbf{a}_T(\theta)}. \quad (6)$$

In order to analyze the beampattern of the NSP projected waveform we can substitute equation (4) in equation (3) to get

$$G_{\text{NSP}}(\theta, \theta_D) = \Gamma \frac{|\mathbf{a}_T^H(\theta) \mathbf{R}_{\tilde{\mathbf{x}}_{\text{Radar}}}^T \mathbf{a}_T(\theta_D)|^2}{\mathbf{a}_T^H(\theta_D) \mathbf{R}_{\tilde{\mathbf{x}}_{\text{Radar}}}^T \mathbf{a}_T(\theta_D)} \frac{|\mathbf{a}_R^H(\theta) \mathbf{a}_R(\theta_D)|^2}{M_R}. \quad (7)$$

Algorithm 2 Modified Null-Space Projection (NSP)

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if  $\mathbf{H}_i$  received from Algorithm (1) then
  Perform SVD on  $\mathbf{H}_i$  (i.e.  $\mathbf{H}_i = \mathbf{U}_i \mathbf{\Sigma}_i \mathbf{V}_i^H$ )
  if  $\sigma_j \neq 0$  (i.e.  $j^{\text{th}}$  singular value of  $\mathbf{\Sigma}_i$ ) then
     $\dim[\mathcal{N}(\mathbf{H}_i)] = 0$ 
    Use pre-specified threshold  $\delta$ 
    for  $j = 1 : \min(N_R^{\text{BS}}, M_T)$  do
      if  $\sigma_j < \delta$  then
         $\dim[\mathcal{N}(\mathbf{H}_i)] = \dim[\mathcal{N}(\mathbf{H}_i)] + 1$ 
      else
         $\dim[\mathcal{N}(\mathbf{H}_i)] = 0$ 
      end if
    end for
  else
     $\dim[\mathcal{N}(\mathbf{H}_i)] = \text{The number of zero singular values}$ 
  end if
  Send  $\dim[\mathcal{N}(\mathbf{H}_i)]$  to Algorithm (1).
end if
if  $\tilde{\mathbf{H}}$  received from Algorithm (1) then
  Perform SVD on  $\tilde{\mathbf{H}} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}$ 
  if  $\sigma_j \neq 0$  then
    Use pre-specified threshold  $\delta$ 
     $\sigma_{\text{Null}} = \{\}$  {An empty set to collect  $\sigma$ s below threshold  $\delta$ }
    for  $j = 1 : \min(N_R^{\text{BS}}, M_T)$  do
      if  $\sigma_j < \delta$  then
        Add  $\sigma_j$  to  $\sigma_{\text{Null}}$ 
      end if
    end for
     $\tilde{\mathbf{V}} = \sigma_{\text{Null}}$  corresponding columns in  $\mathbf{V}$ .
  end if
  Setup projection matrix  $\mathbf{P}_{\tilde{\mathbf{V}}} = \tilde{\mathbf{V}} \tilde{\mathbf{V}}^H$ .
  Get NSP radar signal via  $\tilde{\mathbf{x}}_{\text{Radar}} = \mathbf{P}_{\tilde{\mathbf{V}}} \mathbf{x}_{\text{Radar}}$ .
end if

```

IV. SIMULATION RESULTS

In this section, we simulate our MIMO radar-LTE sharing scenario and study its impact on the performance of radar. The simulation parameters used are listed in Table I.

The CRB for the target's angle of arrival is given by equations (1) and (5) for the original radar waveform and the NSP radar waveform, respectively. We are interested in understanding the effects of NSP on the radar waveform. In Figure 3, we compare the root-mean-square-error (RMSE) of different radar waveforms. We compare the performance of original radar waveform with the NSP waveform projected onto \mathbf{H}_{Best} and $\mathbf{H}_{\text{Worst}}$. Note that by using Algorithms (1) and (2) we are able to minimize degradation in the radar performance as the NSP waveform onto \mathbf{H}_{Best} is closer to the original radar waveform in RMSE sense than the NSP waveform onto $\mathbf{H}_{\text{Worst}}$. Thus, by an appropriate selection of the interference channel degradation in the radar performance, due to the NSP of its waveform, can be minimized.

Similar to the CRB, the ML estimate of the target's angle

TABLE I
MIMO RADAR SYSTEM PARAMETERS

Parameters	Notations	Values
Radar/LTE shared RF band	-	3550 – 3650 MHz
Radar waveform bandwidth	B	10 MHz
Radar transmit antennas	M_T	10
Radar receive antennas	M_R	10
Carrier frequency	f_c	3.55 GHz
Wavelength	λ	8.5 cm
Inter-element antenna spacing	$3\lambda/4$	6.42 cm
Radial velocity	v_r	2000 m/s
Speed of light	c	3×10^8 m/s
Threshold	δ	$M_T/3$
Target distance from the radar	r_0	5000 m
Target angle	θ	0°
Signal to noise ratio	SNR	20 dB
Doppler angular frequency	ω_D	$2\omega_c v_r / c$
Two way propagation delay	τ_r	$2r_0 / c$
Path loss	α	$\alpha_{ji} e^{-j\omega_c \tau_r}$

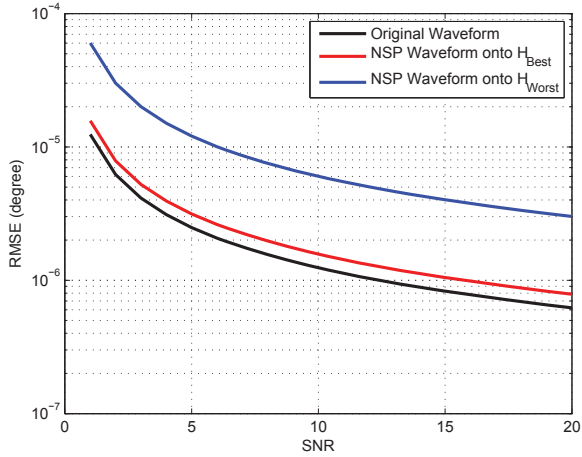


Fig. 3. CRB on target direction estimation RMSE as a function of the SNR. \mathbf{H}_{Best} and $\mathbf{H}_{\text{Worst}}$ channels are selected using Algorithms (1) and (2).

of arrival is given by equation (2) and (6) for the original radar waveform and the NSP radar waveform, respectively. We are interested in the estimation error of the angle due to the NSP of radar waveform. In Figure 4, we compare original angles and estimated angles using ML estimation for different radar waveforms. Using Algorithms (1) and (2) we can achieve almost similar ML results for original waveform and the NSP waveform which shows that by choosing \mathbf{H}_{Best} to project we can cause minimum degradation in radar performance. Note that the ML estimate for the NSP waveform onto $\mathbf{H}_{\text{Worst}}$ is much degraded from the original waveform and the NSP waveform onto \mathbf{H}_{Best} .

In Algorithm (2), we describe an approach to numerically

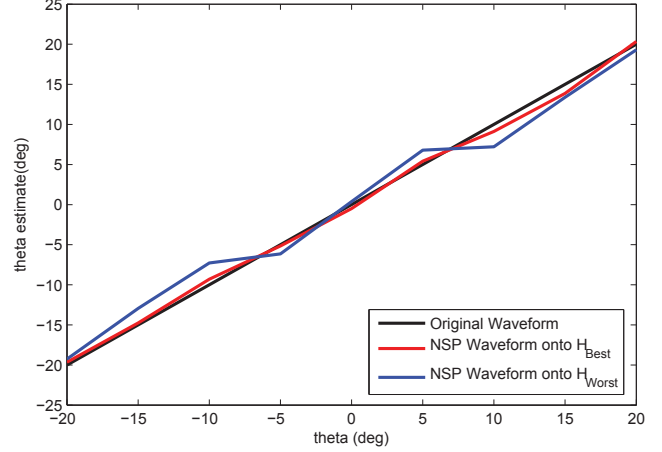


Fig. 4. ML on target direction estimation. \mathbf{H}_{Best} and $\mathbf{H}_{\text{Worst}}$ channels are selected using Algorithms (1) and (2).

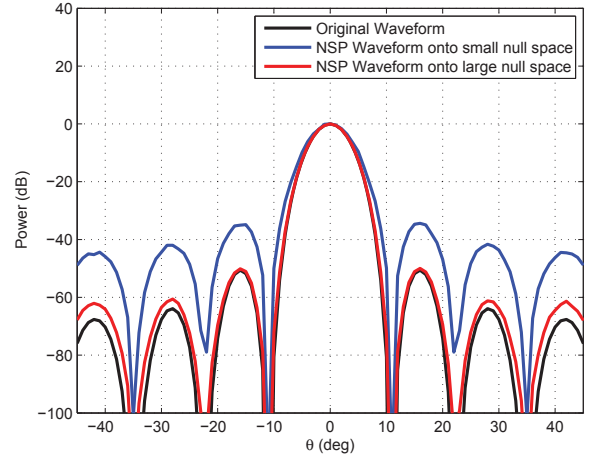


Fig. 5. Beam pattern of MIMO radar when different values of threshold are used to calculate the null space of interference channels in Algorithm (2).

calculate null space of interference channels. This is an important approach in the presence of rounding errors and fuzzy data. We select singular values below a certain threshold and take the corresponding columns of \mathbf{V}_i^H for our NSP equation. Thus, the value of threshold can be a limitation parameter in the projection algorithm, since, the bigger the value of threshold the bigger the null space and the better the performance of the NSP radar waveform. This can be easily noticed from Figure 5 where we compare the beam pattern of original radar waveform with the NSP waveform when we choose a larger and a smaller value of threshold. The larger value of threshold corresponds to the best channel and the smaller value corresponds to the worst channel, according to our definitions in Section III-B. Note that by increasing or decreasing the value of threshold we can manipulate the magnitude of sidelobes. Thus, for the best radar performance, it is desirable to select interference channel with the maximum

null space i.e. according to Algorithms (1) and (2).

V. CONCLUSION

Growing demand for spectrum, by commercial wireless operators, has resulted in willingness of the federal agencies to share their spectrum with commercial users. In this paper, we propose a spatial approach for interference mitigation of radar signals at an LTE cellular system. We focus on a radar-centric interference-mitigation approach where our goal is to manipulate radar signals such that they are not a source of interference to a chosen LTE BS. We extend the idea to project radar signals onto the null space of interference channel for a single system to a cellular system with many base stations. This extension results in many interference channels, instead of one, between the MIMO radar and LTE system. The candidate interference channel is selected using our interference-channel-selection algorithm i.e. Algorithm (1) which selects the interference-channel with the maximum null space among all available. The radar signal is then projected onto the null space of the candidate interference channel using our null space projection method i.e. Algorithm (2) in order to mitigate interference to the candidate BS. Our results demonstrate that by using Algorithms (1) and (2) the degradation in the radar performance is small. Thus, spatial approaches using null space projection are viable for MIMO radar interference mitigation to MIMO LTE systems as they don't incur heavy penalty on radar performance.

VI. FUTURE WORK

A spatial spectrum-sharing approach is presented in this paper to protect an LTE BS from MIMO radar interference. Using a combination of Algorithms (1) and (2) we protect the BS whose channel can guarantee the maximum null space and consequently least degradation in the radar performance, due to null space projection. This means the remaining $N_{BS} - 1$ LTE BSs are still subject to interference from the MIMO radar. Radar interference to these $N_{BS} - 1$ BSs can be mitigated by using carrier aggregation techniques to reallocate resources among different carriers, along the lines of [18]–[23]. Due to space limitations we do not cover this topic here. But, as an extension of this spatial approach, on the communication side, we have proposed a comprehensive model which uses time and frequency domain to allocate resources while at the MIMO radar we use spatial approach proposed in this paper. Together with spatial approach, at MIMO radar, and a time and frequency based approach, at LTE communication system, we develop a spectrum sharing model which guarantees zero interference from the MIMO radar to the LTE system without compromising the performance of both the systems [24].

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