Scanning holography and two-dimensional image processing
by acousto-optic two-pupil synthesis

Ting-Chung Poon

Department of Electrical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, Virginia

Received July 2, 1984; accepted November 27, 1984

An implementation of an acousto-optic heterodyning image processor was reported earlier [Opt. Lett. 4, 317
(1979)]. However, details of that report are limited only to the confirmation of the basic principle of operation. In
this paper, we present and emphasize the mathematical structure of the processor. We also compare this processor
with other kinds of scanning and nonscanning processors. The analysis is then extended to the defocused case,
where the optical transfer function is derived. A potential application of this in scanning holography is discussed.

1. INTRODUCTION
In a conventional incoherent scanning or imaging system, limitations exist on image processing that are due to the
resulting nonnegative intensity spread function [point-spread function (PSF)], which, in turn, imposes severe constraints
resulting nonnegative intensity spread function [point-spread function (OTF)]. Such limitations can be circumvented by
introducing a two-pupil system that is characterized by a great flexibility in pupil-function specification for a desired
synthesized PSF. Any bipolar impulse response can be synthesized by using two-pupil methods as long as the pupil
function can be arbitrarily specified. Two-pupil systems are usually implemented by separating the responses (i.e., separat-
ing the interactive term and the noninteractive term on the basis of either spatial or temporal carriers. The pupils
are created by either amplitude or wave-front division.

Two-pupil synthesis by means of acousto-optics has recently been reported. A simple pupil interaction processing
technique, in which both spatial-frequency offset (wave-front division) and temporal-frequency offset are brought about by
acousto-optic Bragg diffraction without complicated geometrical configurations, was demonstrated. However, details
of the report are pertinent only to the principle of operation. In this paper, we consider holographic implications of this
device in the context of two-pupil processing in general. We begin in Section 2 with a general review of OTF synthesis and
its equivalence in scanning applications. In Section 3, we present the principles of two-pupil synthesis by acousto-optics and
extend the analysis to the defocused case, in which a novel technique of optical-scanning holography is introduced that is
somewhat analogous to synthetic aperture radar. In the last section, we discuss some possible implementations of this idea.

2. GENERAL OUTLINE OF OPTICAL TRANSFER FUNCTION SYNTHESIS
A. One-Pupil Synthesis with Scanning Beam
Refer to the geometry of Fig. 1, where the system is under coherent illumination. Lens L₁ forms the Fourier transform

U'₁ of the optical field U₀ in its focal plane, plane 1, where a beam-shaping transparency Γ₁ multiplies U'₁ to create U₁. Len
L₂ forms the Fourier transform U₂ of U₁ in its back focal plane. The field U₂ in plane 2 is used to scan out a trans-
parency Γ₂(x, y). By scanning is meant that successive points (x, y) in transparency coordinates of Γ₂ are brought into co-
incidence with the center (x₂, y₂) = (0, 0) of the field in the x₂y₂ plane. This is most easily accomplished by keeping the beam
fixed while moving the transparency across it. The photodiode PD, which responds to the incident intensity of the
 optical wave field, accepts the entire field Γ₂U₂ and generates a direct current that is given by

\[ I(x, y) = ∫∫ U₂(x₂, y₂)U₂*(x₂, y₂) \times |Γ₂(x₂ + x, y₂ + y)|^2 dx₂dy₂ \] (1)

with an inessential constant left out.

Equation (1) may be readily recognized as the correlation of two functions U₂U₂* and |Γ₂|^2 if (x, y) is replaced by (−x, −y):

\[ I(−x, −y) = ∫∫ U₂(x₂, y₂)U₂*(x₂, y₂) \times |Γ₂(x₂ - x, y₂ - y)|^2 dx₂dy₂ = (U₂*U₂) ⊙ |Γ₂|^2; \] (2)

where ⊙ denotes the correlation operation. By taking the complex conjugate of Eq. (2), we have

\[ I*(−x, −y) = (U₂*U₂) ⊙ |Γ₂|^2; \] (3)

thus

\[ F[I] = F[I*] = F[U₂*U₂] ⊙ |Γ₂|^2; \] (4)

where F denotes the Fourier-transform operation, i.e.,

\[ F[I](f_x, f_y) = F[I(x, y)]exp(−j2πf_xx - j2πf_yy)dx dy, \] (5)

with f_x and f_y denoting the spatial frequencies. Since the fields U₁ and U₂ are in the front and back focal plane of lens
2, respectively, an exact Fourier-transform relation holds between these two fields. We have, with an inessential
constant and a phase factor left out,
with a coarse beam. Equation (8) is of course well known; we
is the reason why we cannot resolve high spatial frequencies
result in
to obtain the filter function, a coarse (i.e., broad) beam will
function. As we are Fourier transforming the beam profile
scanning beam
right-hand side of Eq. (7) is the Fourier transform of the
relation of the pupil function of the scanning field. The
expression is:
where in this derivation we have left out inessential factors.

\[ U_2(x_2, y_2) = \mathcal{F}[U_1]\left(\frac{x_2}{\lambda f_2}, \frac{y_2}{\lambda f_2}\right) \]
\[ = \int \int U_1(x_1, y_1) \exp \left[-j \frac{2\pi}{\lambda f_2} (x_1 x_2 + y_1 y_2)\right] dx_1 dy_1, \]  \hspace{1cm} (6)

where \( \lambda \) is the optical wavelength.

The OTF of the scanning system can be defined as
\[ \text{OTF}(f_x, f_y) = \mathcal{F}[\mathcal{F}[\Gamma_y^2]] = \mathcal{F}*[U_2^* U_2]. \]  \hspace{1cm} (7)

In terms of the pupil field \( U_1 \) and using Eq. (6), Eq. (7) becomes
\[ \text{OTF} = \mathcal{F}*[U_2^* U_2] \]
\[ = [\mathcal{F} U_2^*(x_2, y_2) U_2(x_2, y_2) \times \exp(-j 2\pi f_x x_2 - j 2\pi f_y y_2) dx_2 dy_2]^* \]
\[ = \left[ \int \int U_1^*(x_1, y_1) \times \exp \left[ \frac{2\pi}{\lambda f_2} (x_1 x_2 + y_1 y_2)\right] U_1(x_1', y_1') \times \exp \left[ -j \frac{2\pi}{\lambda f_2} (x_1 x_2 + y_1 y_2)\right] \times \exp(-j 2\pi f_x x_2 - j 2\pi f_y y_2) \times dx_1 dy_1 dx_1' dy_1 dx_2 dy_2 \right]^* \]
\[ = \left[ \int \int U_1^*(x_1, y_1) U_1(x_1 - \Delta f x_2, y_1 - \Delta f y_2) dx_1 dy_1 \right]^* \]
\[ = \left[ \int \int U_1^*(x_1, y_1) U_1^*(x_1 - \Delta f x_2, y_1 - \Delta f y_2) dx_1 dy_1 \right]^* \]
\[ = U_1 \otimes U_1, \]  \hspace{1cm} (8)

where in this derivation we have left out inessential factors.

We see that the OTF of the system is simply the autocorrelation of the pupil function of the scanning field. The
right-hand side of Eq. (7) is the Fourier transform of the scanning beam \( U_2 U_2^* \), and the left-hand side is the filter function. As we are Fourier transforming the beam profile
to obtain the filter function, a coarse (i.e., broad) beam will result in a narrow band in the spatial-frequency domain. This
is the reason why we cannot resolve high spatial frequencies
with a coarse beam. Equation (8) is of course well known; we

present it here only to establish the context for the acousto-
optic scanning system to be discussed later.

B. Equivalence between the Scanning System and the Incoherent Imaging System
For coherent systems, we may find the complex field in the
image plane \( U(x_i, y_i) \) by convolving the field in the object
plane \( U_0(x_0, y_0) \) with the system impulse response \( h(x_i, y_i; x_0, y_0) \) (or the coherent spread function). For a spatially invariant system, we have
\[ U_i(x_i, y_i) = \mathcal{F} \int U_0(x_0, y_0) h(x_i - x_0, y_i - y_0) dx_0 dy_0 \]
\[ = U_0 \ast h, \]  \hspace{1cm} (9)

where \( h \) is the amplitude at image coordinates \( (x_i, y_i) \) in response
to a point-source object at \( (x_0, y_0) \) and \( \ast \) denotes
the convolution operation. The image intensity distribution is
then obtained by
\[ I_i(x_i, y_i) = \langle (U_i(x_i, y_i; t) U_i^*(x_i, y_i; t)) \rangle, \]  \hspace{1cm} (10)

where the angle bracket indicates a time average.

Fourier transforming Eq. (9), we have
\[ \mathcal{F}[U_i] = \mathcal{F}[U_0] \mathcal{F}[h]. \]  \hspace{1cm} (11)

The coherent transfer function of the imaging system is defined
on the basis of amplitudes:
\[ H(f_x, f_y) = \frac{\mathcal{F}[U_i]}{\mathcal{F}[U_0]} = \mathcal{F}[h]. \]  \hspace{1cm} (12)

However,
\[ h = \mathcal{F}[P] \]  \hspace{1cm} (13)

under the ideal condition (i.e., when the system is properly
focused), so that combining Eq. (12) with the definition of \( H \) gives
\[ H(f_x, f_y) = \mathcal{F}[h] = \mathcal{F}[\mathcal{F}[P]] = \mathcal{F}[F_x, F_y]. \]  \hspace{1cm} (14)

Strictly speaking, the coherent transfer function is the pupil
function of the system in a reflected frame of reference that
is due to the double Fourier-transform operation in Eq. (14).
The coherent transfer function \( H(f_x, f_y) \) characterizes
the performance of the coherent imaging system as it specifies the
passband of the spatial frequency.

In the case of an incoherent imaging system, the image intensity
distribution is
\[ I_i(x_i, y_i) = \int I_0(x_0, y_0) h(x_i - x_0, y_i - y_0) dx_0 dy_0 \]
\[ = U_0 \ast h, \]  \hspace{1cm} (15)

where \( |h|^2 \) is the intensity spread function (PSF).\(^9\) Expressing
the relationship in the frequency domain, we have
\[ \mathcal{F}[I_i] = \mathcal{F}[U_0] \mathcal{F}[|h|^2]. \]  \hspace{1cm} (16)

The OTF of the incoherent imaging system is defined on
the basis of intensities:
\[ \text{OTF} = \frac{\mathcal{F}[I_i]}{\mathcal{F}[I_0]} = \mathcal{F}[|h|^2]. \]  \hspace{1cm} (17)

In terms of the pupil function \( P \), we have, using Eqs. (13) and
(14) in Eq. (17),
Hence, for incoherent imaging systems, the OTF is simply the autocorrelation of the pupil function of the system. By comparing Eqs. (8) and (18), the equivalence between the scanning system and the incoherent system is established. Again, we claim no novelty in deriving this equivalence. Its derivation does, however, facilitate the explanations to follow.

C. Two-Pupil Synthesis

As we have seen from Eq. (15), the impulse response (PSF) of an incoherent optical system is a real and nonnegative function that imposes restrictions on both the magnitude and the phase of the OTF. Consequently, it limits the range of processing. For instance, suppose that we want to process a picture to obtain contrast enhancement in order that fine detail may be seen better. In that case, we could never create a filter to block off the dc background by autocorrelating the pupil function, because the autocorrelation of any function exhibits a central maximum. Another example is the decoding of information recorded on a spatial carrier on the transparency. For this, we have to use a bandpass filter to separate the carrier and its sidebands from the rest of the spectrum. One-pupil synthesis clearly cannot perform this task. In fact, many processing operations require a PSF's that are bipolar. In order to overcome the limitation of a positive PSF, the so-called two-pupil synthesis method has been introduced. There are basically two kinds of syntheses possible: nonpupil interaction synthesis and pupil interaction synthesis.

In the nonpupil interaction synthesis, we may obtain a bipolar PSF or eliminate the central maximum by subtracting the PSF's of two different filter functions OTF and OTF:

\[
\text{OTF}_{\text{tot}} = \mathcal{F}[|h_k|^2 - |h_0|^2]
\]

\[
= \text{OTF}_1 - \text{OTF}_2
\]

\[
= U_1 \odot U_1 - V_1 \odot V_1,
\]

where \(U_1\) and \(V_1\) are the different pupil functions. Because we cannot subtract intensities directly, this method requires special techniques.

In the pupil interaction synthesis, we add \(U_1\) to \(V_1\) in order to produce an effective pupil function, which is given by

\[
U_{\text{eff}} = U_1 + V_1.
\]

The PSF is given by [refer to Eq. (13)]

\[
\text{PSF} = |\mathcal{F}[U_{\text{eff}}]|^2
\]

\[
= |U_2 + V_2|^2
\]

\[
= |U_2|^2 + |V_2|^2 + U_2V_2^* + U_2^*V_2,
\]

where \(U_2 = \mathcal{F}[U_1]\) and \(V_2 = \mathcal{F}[V_1]\).

The corresponding OTF of the system is given by [see Eq. (17)]

\[
\text{OTF} = \mathcal{F}[\text{PSF}]
\]

\[
= \mathcal{F}[|U_2|^2 + |V_2|^2 + \mathcal{F}[U_2V_2^*] + \mathcal{F}[U_2^*V_2]
\]

\[
= U_1 \odot U_1 + V_1 \odot V_1 + U_1 \odot V_1 + V_1 \odot U_1.
\]

The last two terms of Eq. (22), which contain the bipolar information, are the cross correlations of \(U_1\) and \(V_1\) and \(U_1\), respectively. Interaction of the pupils actually occurs in this sense. The autocorrelation terms are considered as bias. To retrieve the bipolar information, it is necessary to separate the signal from the bias by a subtraction operation, unless carrier methods are used.

D. Implementation of Two-Pupil Synthesis with a Mach-Zehnder Interferometer

The system of Fig. 2 is called the Mach-Zehnder configuration and serves conveniently to illustrate both classes of synthesis methods. The configuration is basically the one illustrated in Fig. 1 (one-pupil synthesis). The only difference is the use of two paths instead of one. If one path is blocked off, the system behaves as a normal one-pupil system. For an ideal geometry, the optical path lengths should be the same. Two partially silvered mirrors \(P_1\) and \(P_2\) split the incoming beam into \(U_1^*\) and \(V_1^*\), which are modified by beam-shaping transparencies \(\Gamma_1\) and \(\Gamma_2\) to create \(U_1\) and \(V_1\), respectively. Lens \(L_2\) Fourier transforms \(U_1\) and \(V_1\) into \(U_2\) and \(V_2\) in its back focal plane. The effective pupil function \(U_2 + V_2\) scans out the transparency \(T_2\) in plane 2, and the OTF of the system is characterized by Eq. (22).

However, in order to attain nonpupil interaction synthesis, \(G_2\) is scanned alternately by \(U_1\) and \(V_1\). The outputs produced by the two filter functions \(U_1 \odot U_1\) and \(V_1 \odot V_1\) are then subtracted, for instance, by using an electronic storage device.

In order to use the pupil interaction method, we must separate the cross-correlation terms in Eq. (22) from the other correlation terms. Besides the methods of subtraction, there are two other ways to separate the cross-correlation components and the correlation components, namely, the use of a spatial-frequency offset or a spatial-carrier method or a temporal-frequency offset (or a temporal-carrier method).

Spatial-frequency offsetting can be obtained by introducing a wedge prism in one arm. The wedge prism simply deflects one of the wave fronts, for example \(V_2\), so that \(V_2\) is tilted at an angle with respect to plane 2 while \(U_2\) remains undeflected. Because of the angular offset, interference fringes now occur in plane 2. The spacing of the fringes is the period of the spatial-carrier offset. By electronically scanning the transparency \(T_2\) in plane 2, we can convert spatial frequencies into temporal frequencies. The desired carrier sidebands can then be separated and processed by conventional electronic means.

In the temporal-frequency offset, a time-varying phase element, such as a constant-velocity mirror or a sinusoidal vi-
The phase and the amplitude of the photodiode current, as we find, from Eq. (24), that with an inessential constant left out.

\[ \text{beat frequency } A = \text{the optical convention for the phasor } A = \sqrt{U_2} \exp(-j0), \]

where the two scanning fields in the \( x_2y_2 \) plane are given by

\[ U_2(x_2, y_2) \exp\left(-\frac{2\pi \Delta \nu t}{2}\right), \quad V_2(x_2, y_2) \exp\left(+\frac{2\pi \Delta \nu t}{2}\right), \]

and where the optical convention for the phasor \( \tilde{A} = |\tilde{A}| \exp(-j\phi), \)

\[ |\tilde{A}| \cos(2\pi \nu t + \phi) = \text{Re}[\tilde{A} \exp(-j2\pi \nu t)], \]

is adopted.

The time-varying part of Eq. (23) is

\[ \tilde{i}(x, y, t) = \int T(x_2, y_2) V_2^*(x_2, y_2) U_2(x_2, y_2) \exp(-j2\pi \Delta \nu t) + \int V_2(x_2, y_2) U_2(x_2, y_2) \exp(j2\pi \Delta \nu t) \times |\Gamma_2(x + x_2, y + y_2)|^2 \text{d}x_2 \text{d}y_2 \]

\[ = \text{Re}[\int U_2(x_2, y_2) V_2^*(x_2, y_2) \times |\Gamma_2(x + x_2, y + y_2)|^2 \text{d}x_2 \text{d}y_2 \times \exp(-j2\pi \Delta \nu t)] \]

with an inessential constant left out.

In terms of a phasor \( \tilde{I}(x, y), \) such that

\[ \tilde{i}(x, y, t) = \text{Re}[\tilde{I}(x, y) \exp(-j2\pi \Delta \nu t)], \]

we find, from Eq. (24), that

\[ \tilde{I}(x, y) = \int U_2(x_2, y_2) V_2^*(x_2, y_2) \times |\Gamma_2(x + x_2, y + y_2)|^2 \text{d}x_2 \text{d}y_2, \]

The phase and the amplitude of the photodiode current, as a function of \( x \) and \( y, \) constitute the scanned and processed version of the transparency \( \Gamma_2(x, y). \) From the analysis of one-pupil synthesis [i.e., from Eqs. (1)–(8)], we can immediately write down the OTF of the two-pupil systems:

\[ \text{OTF} = \mathcal{F}[\tilde{I}(x, y)]/\mathcal{F}[\Gamma_2(x, y)], \]

or, equivalently,

\[ \text{OTF} = \mathcal{F}[U_2 V_2] = U_1 \odot V_1, \]

by which is meant

\[ \text{OTF}(f_x, f_y) = \int U_1(x_1, y_1) \times V_1^*(x_1 - \lambda f_{x_2}, y_1 - \lambda f_{y_2}) \text{d}x_1 \text{d}y_1. \]

At this point, it is clear that the system is incoherent in the sense that its operation is described by the same formalism as that used for incoherent pupil interaction methods [see Eq. (22)], even though the system uses coherent light and its output (a heterodyne current) is characterized by amplitude and phase, which is generally characteristic of coherent systems.

It has been demonstrated \(^7\) that the acousto-optical technique is relatively easy to implement and leads to the desired result of more general OTF synthesis. Specifically, a band-pass response has been generated through the practical implementation of the idealized version (Fig. 3). However, the resulting OTF is essentially one dimensional.

4. OPTICAL TRANSFER FUNCTION SYNTHESIS IN DEFOCUSED CASE

In this section, we show that more interesting OTF's can be obtained by drastically modifying \( U_1 \) relative to \( V_1. \) In this context, a chirp-type impulse response in an out-of-focus plane near plane 2 is derived.

A. Derivation of the Optical Transfer Function

Refer to Fig. 3. \( \Gamma_2 \) is now placed in an out-of-focus plane, plane 2', near plane 2. We shall derive an expression of the OTF in this case. Following the procedures employed to derive the OTF in Section 3, we have

\[ \text{OTF}(f_x, f_y) = \mathcal{F}[\tilde{I}(x, y)]/\mathcal{F}[\Gamma_2^2] \]

\[ = \mathcal{F}[U_2' V_2' V_2' U_2'], \]

where \( U_2' \) and \( V_2' \) are obtained through the Fresnel diffraction of \( U_2 \) and \( V_2, \) respectively, and \( U_2 \) and \( V_2 \) are related to \( U_1 \) and \( V_1, \) respectively, through the action of the lens \( L_2. \)

The explicit relations are given as follows:
Ting-Chung Poon

The expression for

$$U_2(x_2, y_2) = \frac{\exp(jk z)}{j \lambda_2} \iint U_1(x_1, y_1) \times \exp\left[-j2\pi \frac{x_1 x_2 + y_1 y_2}{\lambda_2}\right] \, dx_1 \, dy_1,$$

(30)

$$U_2'(x_2', y_2') = \frac{\exp(jk z)}{j \lambda_2} \exp\left[j \frac{k}{2x} (x_2'^2 + y_2'^2)\right] \times \iint U_2(x_2, y_2) \exp\left[j \frac{k}{2x} (x_2'^2 + y_2'^2)\right] \times \exp\left[-j \frac{k}{z} (x_2 x_2' + y_2 y_2')\right] \, dx_2 \, dy_2,$$

(31)

where $$k = 2\pi/\lambda.$$ Substituting Eq. (30) into Eq. (31), we have

$$U_2'(x_2', y_2') = (-\exp(jk z_2)) \exp(jk z) \times \exp\left[j \frac{k}{2z} (x_2'^2 + y_2'^2)\right] \iint U_1(x_1, y_1) \times \exp\left[-j \frac{2\pi}{\lambda_2} (x_1 x_2 + y_1 y_2)\right] \times \exp\left[j \frac{k}{2z} (x_2'^2 + y_2'^2)\right] \times \exp\left[-j \frac{k}{z} (x_2 x_2' + y_2 y_2')\right] \, dx_1 \, dy_1.$$ (32)

The expression for $$V_2'(x_2', y_2')$$ can be found identically, with $$U_2'$$ and $$U_1$$ replaced by $$V_2'$$ and $$V_1,$$ respectively, in the above expression.

Using the expressions of $$U_2'$$ [as shown in Eq. (32)] and $$V_2',$$ it may be shown [see Appendix A] that

$$\text{OTF}(f_x, f_y) = \exp[j\pi \lambda z (f_x^2 + f_y^2)] \times \iint U_1(x_1, y_1) V_1^*(x_1 - \lambda f_x, y_1 - \lambda f_y) \times \exp\left[-j \frac{2\pi}{\lambda_2} (x_1 f_x + y_1 f_y)\right] \, dx_1 \, dy_1.$$ (33)

Equation (33) is the OTF of the defocused system, where $$\Gamma_2$$ is placed in plane 2'. As a check, Eq. (28) is readily recovered by putting $$z = 0$$ into Eq. (33).

B. Application: Scanning Holography

It has been pointed out that there exists a possibility of creating a chirp-type impulse response in an out-of-focus plane near plane 2 (see Fig. 3) by making $$V_1$$ a delta function and $$U_1$$ a plane-wave function. The situation is further illustrated and explained in Fig. 4. $$U_1$$ and $$V_1$$ are first made to be plane waves by narrowing the incoming beam in the front focal plane of lens $$L_1$$ (see Fig. 3). Intercepting $$V_1$$ in plane 1 by a narrow slit will result in a delta function. The overlapping of two beams creates a chirp-type fringe pattern along plane 2'. The fringe wavelength is increasing from point A to point B because the two wave fronts intersect at a larger angle at point A than at point B (i.e., $$\alpha_A > \alpha_B$$).

If we let $$U_1(x_1, y_1) = \delta(x_1, y_1)$$ and $$V_1(x_1, y_1) = 1,$$ where we assume the delta function to be located at the center of the $$x_1 y_1$$ plane, we obtain, using Eq. (33),

$$\text{OTF}(f_x, f_y) = \exp[j\pi \lambda z (f_x^2 + f_y^2)].$$ (34)

By using the above expression for the OTF, the impulse response function $$\tilde{I}(x, y; z)$$ may be expressed as

$$\tilde{I}(x, y; z) = \lim_{z \to 0} \frac{\exp[j\pi z (x^2 + y^2)]}{\lambda z}.$$

(35)

which is obviously a chirp-type impulse response. In fact, by moving a slit along plane 2' and measuring $$\tilde{I}(x, y; z),$$ we should be able to observe this. By investigating the chirp rate, we can deduce how far plane 2' is out of focus from plane 2. This scanning system is thus able to encode position ($$x, y$$ coordinate) and depth of focus ($$z$$ coordinate), and thus it carries obvious implication to holographic recording. In this context, the notion of scanning holography becomes clear in that we scan the transparency located along plane 2' with a combination of two light frequencies ($$U_1$$ and $$V_1$$). The heterodyned rf signal is detected by a photodiode. In order to extract both the amplitude and the phase of the diode current, the rf signal is further coherently detected. The final demodulated signal may then be fed in to modulate the intensity of the electron beam. If this modulated cathode-ray display is imaged on a strip of photographic film, then the successive scans will be recorded side by side, thereby producing a two-dimensional (2-D) recording. This 2-D recording of the transmitted signals is effectively a hologram, recorded in a manner somewhat reminiscent of side-looking radar. Scanning here is equivalent to the motion of the radar platform. However, whereas in side-looking radar the reference signal is generated electronically on board, here it is provided by the plane-wave beam, i.e., provided locally at the scatterer, the only practical way in optics at visible wavelengths.

Finally, as a check, for $$z = 0,$$ Eq. (35) becomes

$$\tilde{I}(x, y; z) = \delta(x, y),$$

which is consistent with physical intuition, since the overlapping of two beams occurs only at one point in plane 2.

5. SUMMARY AND OTHER IMPLICATIONS OF THE ACOUSTO-OPTIC TECHNIQUES

To establish the context of the new technique, we have reviewed the equivalence between scanning systems and the incoherent imaging systems in general. A technique for two-pupil synthesis by means of acousto-optics has been analyzed, emphasizing the similarities in the mathematical treatment. Finally, OTF synthesis in the defocused case has been carried out, and its applications to scanning holography have been discussed.

By utilizing the practical implementation of the existing processor operated in the rf domain (Fig. 3), a bandpass filter
was synthesized earlier. However, the resulting OTF was essentially one dimensional because of the actual experimental setup, in which the optical configuration was confined to one dimension, i.e., a cylinder lens was used to tailor the beam to fit and focus into the sound cell. In the above situation, cross correlation of two separated one-dimensional coplanar fields resulted effectively in a one-dimensional response. In order to realize true 2-D image processing, we point out the possibility of synthesizing a 2-D bandpass filter by modifying the existing processor. For a radially symmetric filter, this can be achieved by concentrically aligning radially symmetric fields \( U_1 \) and \( V_1 \) such that they overlap in plane 1. Cross correlation of the two overlapping fields will then result in a true 2-D response. (Although, in general, cross correlation of two separated 2-D coplanar fields may also result in a 2-D response, we could never synthesize a direction-independent filter.) Two-dimensional responses have been reported recently \(^5,10,11,12\), however, a simpler system using processing in the rf domain is not available to date.

**APPENDIX A: DERIVATION OF EQUATION (34)**

From Eq. (29), we have

\[
\text{OTF}(f_x, f_y) = [\int \text{U}^*_d(x', y') V_d(x', y') \times \exp{-j2\pi (f_x x' + f_y y')} dx'dy']^*. \tag{A1}
\]

When we substitute Eq. (32), and a similar expression for \( V_d(x', y') \) into the above expression, its explicit form is

\[
\text{OTF}(f_x, f_y) = \left(\frac{1}{\lambda \nu_2^2}\right)^2 \int^6 \int^4 U_1^*(x_1, y_1) \times \exp\left[j \frac{2\pi}{\lambda \nu_2} (x_1 x + y_1 y)\right] 
\]

\[
\times \exp\left[-j \frac{h_2}{z} (x_2^2 + y_2^2)\right] 
\times \exp\left[-j \frac{2\pi}{\lambda \nu_2} (x_2 x_1 + y_2 y_1)\right] 
\times d\rho_1 d\gamma_1 
\times \exp\left[-j \frac{2\pi}{\lambda \nu_2} (x_2 x - y_2 y)\right] dx'dy'
\]

\[
\times \exp\left[-j \frac{2\pi f_x x' - j2\pi f_y y'}{h} dx'dy'\right]^*, \tag{A2}
\]

where \( \int^n \) represents an n-fold integration, n being a positive integer. In this case, \( n = 2 \) represents the Fourier transform in two dimensions, and \( n = 4 \) represents the effect of diffraction.

We first evaluate the integral over \( x_2' \) and \( y_2' \) in the above expression; we have

\[
\int^2 \exp\left[-j \frac{h_2}{z} (x_2 x' + y_2 y')\right] = \left(2\pi \right)^2 \delta(x_2 + x - 2\pi f_x, y_2 + y - 2\pi f_y) 
= \left(2\pi \right)^2 \delta(x - x_2, y - y_2 - 2\pi f_y).
\]

Substituting Eq. (A3) into Eq. (A2) and using the property of a \( \delta \) function [i.e., replacing \( x_2 \) by \( x_2 + (2\pi /k_f) x \) and \( y_2 \) by \( y_2 + (2\pi /k_f) y \) in Eq. (A2)], we have

\[
\text{OTF}(f_x, f_y) = \left(\frac{1}{\lambda \nu_2}\right)^2 \int^6 \int^4 U_1^*(x_1, y_1) 
\times \exp\left[j \frac{2\pi}{\lambda \nu_2} (x_1 x + y_1 y)\right] 
\times \exp\left[-j \frac{h_2}{z} (x_2 + x_1 + \frac{2\pi}{k_f} x + y_1 y + \frac{2\pi}{k_f} y)\right] dx'dy
\times \exp\left[-j \frac{2\pi}{\lambda \nu_2} (x_2 x_1 + y_2 y_1)\right] 
\times \exp\left[-j \frac{2\pi}{\lambda \nu_2} (x_2 x + y_2 y)\right] dx'dy
\times \exp\left[-j \frac{2\pi}{\lambda \nu_2} (x_2 x_1 + y_2 y_1)\right] 
\times \exp\left[-j \frac{2\pi}{\lambda \nu_2} (x_2 x + y_2 y)\right] dx'dy.
\]

Now for the \( \delta \) and \( \gamma \) integration, we have

\[
\int^2 \exp\left[-j \frac{h_2}{z} (x_2 x' + y_2 y')\right] = \left(2\pi \right)^2 \delta(x_2 + x_2 - 2\pi f_x, y_2 + y_2 - 2\pi f_y).
\]

Substituting the above result into Eq. (A5) leads us to the final form of Eq. (33):

\[
\text{OTF}(f_x, f_y) = \exp\left[j \frac{2\pi}{f_2} (x_1 x + y_1 y)\right] 
\times \int^2 \int^4 U_1^*(x_1, y_1) V_1^*(x - \lambda \nu_2 f_x, y + \lambda \nu_2 f_y).
\]

**ACKNOWLEDGMENTS**

I would like to thank A. Karpel for his continuous support and encouragement. Thanks are also extended to P. P. Banerjee and G. Indebetouw for their helpful suggestions.

This research was supported by the National Science Foundation under grant no. ECS-8400636.

**REFERENCES**