MIMO Radar Waveform Design to support Spectrum Sharing

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Abstract—In this paper, we propose an information theoretic waveform design algorithm for multiple-input multiple-output radars to support co-existence with a communication system under additional constraints on interference reduction from the clutter, minimizing the peak-to-average-power ratio and minimizing correlation levels of the radar. Numerical results are presented which show the performance of the proposed waveform design algorithm in terms of mutual information, interference reduction and sidelobe levels of the radar waveform.

Index Terms- MIMO radar, waveform design, co-existence, spectrum sharing, error reduction algorithm

I. INTRODUCTION

Given the crowded RF spectrum, co-existence for spectrum sharing between disparate wireless systems has become an important research issue. Adaptive radar waveform design to satisfy spectral constraints is of great interest given the need for sharing spectrum between radar systems and communication systems such as Long Term Evolution (LTE). Waveform design algorithms for radars, in particular multiple-input multiple-output (MIMO) radars, have been studied extensively to enhance target detection and parameter estimation. MIMO radars, much like MIMO communication systems, offer significant gains in the detection and estimation of one or more targets by providing multiple degrees of freedom in the waveform design [1]. More specifically, MIMO radar presents a new paradigm for radar design that can be used for counteracting fading, interference reduction and jamming mitigation.

Interference reduction to communication systems such as LTE or WiMAX is necessary to enable spectral co-existence with the radar [2]. This can be practically difficult to achieve since both systems use the spectrum in very different ways. The high signal power used by the radars and the sidelobes (for example, of a beamforming-based radar) generated by an inadequate waveform design can saturate a communication system which operates at a much lower power level by comparison. In this paper, we address the waveform design of the MIMO radar from an information-theoretic perspective by maximizing the mutual-information between the target response (which mimics the channel of traditional MIMO systems) and the radar signal returns (received signal) while constraining the spectrum to avoid a co-existing communication system.

Mutual information (MI)-based waveform design algorithms have received significant attention over the past few years. An information theoretic radar receiver design was initially proposed by Woodward and Davies in [3]. Later Bell [4] showed that target detection capabilities can be enhanced by maximizing the mutual information between the radars’ received signal and the target impulse response (channel between the radar and target). This was further extended to MIMO radars by Yang and Blum in [5]. See [3]-[5] for more information on MI-based waveform design algorithms and its advantages. While most previous work has investigated information-theoretic waveform design for the purpose of improving target detection and estimation, there has been no significant research addressing the co-existence issue.

In this paper, we propose a MI-based MIMO radar waveform design algorithm in the frequency domain to reduce interference to co-existing communication systems while avoiding clutter and also satisfying the radar design constraints such as total power and low peak-to-average-power ratio (PAPR). A convex optimization algorithm is developed to design the power spectral density (PSD) of the transmit waveform of the MIMO radar to a) constrain interference to a communication system, b) avoid clutter (unwanted returns/ interference) and c) satisfy the radar design constraints such as maximum transmit power and PAPR. Furthermore, an iterative cyclic projection algorithm is formulated to design a unimodular time-domain waveform with good auto and cross-correlation properties to match the PSD.

Notation: In all analysis that follows, bold upper (lower) case letters X (x) denote matrices (vectors), $X^T$, $X^H$, $\text{tr}(X)$, $\det(X)$, $|X|_F$ represent the transpose, Hermitian transpose, trace, determinant and Frobenius norm of $X$, respectively. $I_N$ is the $N \times N$ identity matrix. blockdiag$(X, Y)$ is a block diagonal matrix with the matrices $X, Y$ on its leading diagonal. $X \odot Y$ is the Hadamard product between $X$ and $Y$. $X \succeq Y$ indicates that $X - Y$ is a positive semi-definite matrix.

II. SYSTEM MODEL

Consider a $Q \times P$ MIMO radar system with $Q$ receive and $P$ transmit antennas. Let $X_{N \times P}$ represent the time domain transmit waveform with signal length $N$. A frequency domain MIMO signal model is developed here to design $X$ over a disjoint set of frequency bands in which the radar has to...
minimize interference to a communication system. Without loss of generality, we assume that the entire bandwidth of interest is represented using the normalized frequency range [0, 1] (normalized by half the sampling frequency) and \( f_{k_l}, f_{k_u} \) denote the lower and upper frequencies of the \( k \)th stop band.

The number of points in the discrete Fourier transform (DFT) \( N \) is chosen such that there is sufficient resolution to densely cover the frequency grid (more specifically, have enough points to allow waveform design over the stop bands) and \( N \geq N \). The target transfer function (target response) between the \( p \)th transmit and \( q \)th receive antennas is given by the \( \tilde{N} \times 1 \) vector \( \tilde{h}_{p,q} = [\hat{h}_{q,p}(0), \ldots, \hat{h}_{q,p}(\tilde{N}−1)]^T \). The target response for transmitter \( p \) can be collectively represented by the \( \tilde{N}Q \times 1 \) vector \( \tilde{h}_p = [\tilde{h}_{1,p}^T, \ldots, \tilde{h}_{P,p}^T]^T \). Thus, the transfer function for all the transmitters can be grouped together as the \( \tilde{N}QP \times 1 \) vector

\[
\tilde{h} = [\tilde{h}_1^T, \ldots, \tilde{h}_P^T]^T.
\] (1)

Along similar lines, the \( \tilde{N}QP \times 1 \) clutter transfer function is written as \( \tilde{c} = [c_1^T, \ldots, c_P^T]^T \).

Let \( \tilde{X} \) denote the frequency domain MIMO transmit signal given by \( \tilde{X} = \begin{bmatrix} X \\ D \end{bmatrix} \), where \( D \) is a \( \tilde{N} \times \tilde{N} \) unitary DFT matrix. Define a \( \tilde{N}QP \times 1 \) vector

\[
\tilde{x} = \begin{bmatrix} [\tilde{x}_1^T, \ldots, \tilde{x}_Q^T, \ldots, \tilde{x}_P^T]^T \\ Q \times \text{times} \\ Q \times \text{times} \end{bmatrix} (2)
\]

where \( \tilde{x}_p \) is the \( \tilde{N} \) point DFT of the signal from the \( p \)th transmit antenna, i.e., the \( p \)th column of \( \tilde{X} \). Using the above notations, the \( \tilde{N}QP \times 1 \) received signal vector \( y \) is

\[
y = \tilde{X}h + \tilde{X}c + w,
\] (3)

where \( \tilde{X} = \text{diag}(\tilde{x}) \) and \( w \) is the DFT of the additive Gaussian noise seen at the radar receiver. The transfer functions \( \hat{h}_{q,p}, c_{q,p} \) are assumed to be zero-mean i.i.d complex Gaussian random vectors with covariance matrices \( \Sigma_{h_{q,p}}, \Sigma_{c_{q,p}} \). The overall covariance matrix is given by

\[
\Sigma_h = \text{blkdiag}(\Sigma_{h_{1,p}}, \ldots, \Sigma_{h_{P,p}}), \quad \Sigma_c = \text{blkdiag}(\Sigma_{c_{1,p}}, \ldots, \Sigma_{c_{P,p}}),
\] (4)

which are assumed to be known a priori (a valid assumption in many radar waveform design problems, see [3]-[6]).

### III. Mutual Information-based PSD Design

In this section, we consider an information-theoretic MIMO radar waveform design using the frequency domain system model developed in Section II. The goal of the waveform design is to maximize the MI, \( I(h; y|X) \), between the radar returns \( y \) and the target response \( h \) given the transmit signal \( X \). Using the system model in (3) and assuming additive white Gaussian noise with variance \( \sigma_w^2 \), the MI is given by

\[
I(h; y|X) = \log(\text{det}(\sigma_w^2 \Sigma_h^{-1} + \tilde{X}^H (I + \tilde{X} \Sigma_c \tilde{X}^H)^{-1} \tilde{X}))) = \theta(\tilde{X}, \Sigma_c, \Sigma_h, \sigma_w^2).
\]

(5)

MI-based waveform design is of specific interest due to the inherent relationship between MI \( I(h; y|X) \) and minimum mean square error (MMSE) in estimating \( h \) from \( X \) and \( y \) [5], [7]. It is known that conditioned on \( X \), the radar returns \( y \) and the target response \( h \) are jointly Gaussian [8].

\[
[\begin{array}{c} h \\ y \end{array}] \sim \mathcal{N} \left( \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \Sigma_h \Sigma_h^H & \Sigma_h \Sigma_c \Sigma_c^H + \sigma_w^2 I \\ \Sigma_c^H \Sigma_h & \Sigma_c^H \Sigma_c \Sigma_c^H + \sigma_w^2 I \end{bmatrix} \right),
\]

where \( \mathcal{N} \) indicates a complex normal distribution. The MMSE in estimating \( h \) from \( y \) is given by [8],

\[
\text{tr} \left( \sigma_w^2 \Sigma_h^{-1} + \tilde{X}^H (I + \tilde{X} \Sigma_c \tilde{X}^H)^{-1} \tilde{X} \right)^{-1}.
\]

(6)

From (5) and (6), it can be seen that the argument inside the expressions for MI and MMSE is the same. Following [5], it can be shown that the waveform \( X \) designed by maximizing MI is the same as the waveform obtained by minimizing MMSE. In this paper, we maximize (5) with a total power constraint and a stop band constraint to mitigate the interference to the communication system.

The transmit power constraint on the radar is given by \( \text{tr}(X^H X) \leq P_T \), where \( P_T \) is the total power constraint on the time domain transmit waveform \( X \). Define \( s \) as a \( \tilde{N} \times 1 \) vector, with 1’s at locations corresponding to the stop band frequencies and 0’s elsewhere. For example, if we have one stop band between 0.0Hz and 0.7Hz and \( \tilde{N} = 100 \), then the 60th to 70th elements of \( s \) are 1 while the other elements in the vector are all zeros. Let \( \tilde{s} \) be a \( \tilde{N}QP \times 1 \) vector given by \( [\tilde{s}_1, \ldots, \tilde{s}_P]^T \). Then, the stop band criterion (the total power in the stop band) is given by

\[
|\tilde{X}\tilde{s}|_F^2 \leq QP_{S} \overset{\Delta}{=} \text{tr}(X^H X s s^H) \leq QP_{S}.
\]

(7)

where \( P_S \) is the stop band power constraint on \( X \). From (5), (7), the optimization problem is

\[
\begin{array}{r}
\text{max} \log(\text{det}(\sigma_w^2 \Sigma_h^{-1} + \tilde{X}^H (I + \tilde{X} \Sigma_c \tilde{X}^H)^{-1} \tilde{X})) \\
\text{s.t.} \quad \text{tr}(X^H X s s^H) \leq QP_{S}, \\
\text{tr}(X^H X^H) \leq QP_{S}.
\end{array}
\]

(8)

As opposed to traditional MI-based waveform design problems [4],[5], it is difficult to see (8) as a water-filling solution. However, as will be explained in Section V, the solution to this optimization problem is indeed a water-filling solution and can be posed as a convex optimization problem. Define a \( \tilde{N}QP \times \tilde{N}QP \) auxiliary variable matrix \( T \) that satisfies

\[
0 \preceq T \preceq \sigma_w^2 \Sigma_h^{-1} + \tilde{X}^H (I + \tilde{X} \Sigma_c \tilde{X}^H)^{-1} \tilde{X}.
\]

(9)

Using (9), the matrix inversion lemma, Schur complement lemma and the linear matrix inequality, the max log-det problem in (8) can be simplified following [9] as

\[
\begin{array}{r}
\min_{\Sigma_T} -\log(\text{det}(T)) \\
\text{s.t.} \quad \begin{bmatrix} \sigma_w^2 \Sigma_h^{-1} + \eta - T & \Sigma_c^{-1} + \eta \\ \eta & \Sigma_c^{-1} + \eta \end{bmatrix} \succeq 0 \\
T \preceq 0, \quad \text{tr}(\eta) \leq QP_T, \quad \text{tr}(\eta V) \leq QP_S,
\end{array}
\]

(10)

where \( \eta = \tilde{X}^H \tilde{X} \) and \( V = \tilde{s} \tilde{s}^H \).
Since $\mathcal{X}$ is a diagonal matrix, $\eta$ is also diagonal and is the PSD of the required transmit waveform $X$. Thus, the PSD of $X$ is the solution to the optimization problem in (10). In Section IV, we present an iterative optimization algorithm to recover the time domain waveform from the PSD $\eta$.

IV. TIME DOMAIN WAVEFORM RECOVERY FROM PSD

The PSD $\eta$ obtained in Section III is subsequently used to design the time domain waveform that has the desired PSD (i.e., maximizes the MI while being constrained by coexistence requirements) and low PAPR. Signals with high PAPR would necessitate a higher dynamic range on the analog-to-digital converters and a large linear region of operation for the power amplifiers [10]. Hence, the PSD obtained in Section III is used to design a unimodular time domain signal that has PAPR = 1 and has good auto and cross-correlation properties to enhance target detection and thereby avoiding false alarms and missed detections.

The Error-Reduction Algorithm (ERA) proposed in [11] is used to design the time domain waveform $X$. The ERA is an extension to the Alternating Projection (AP) algorithm proposed in [12], for computing the point of intersection of two convex sets (in the current context, a set refers to a group of points which satisfy a certain property) via a series of alternating projections. The AP algorithm extended to the case when the sets do not intersect is known as the ERA, where the goal is to find points on these sets that are closest in distance and minimize the error between successive projections. For example, let $a, b$ denote points that belong to the sets $A, B$ on which the projections are performed. The operator $\mathcal{B}(a)$ indicates the projection of point $a$ onto the set $B$. The ERA algorithm is initialized with a certain property, here say $a_1 \in A$, and then proceeds as follows:

Step 1: for a given $a_1 \in A \ni \{b_i \in B \mid b_i = \mathcal{B}(a_i)\}$

Step 2: for this $b_i \in B \ni \{a_{i+1} \in A \mid a_{i+1} = \mathcal{A}(b_i)\}$.

The ERA ensures that $d(a_{i+1}, B) \leq d(a_i, B)$, where $d(a_i, B)$ is the distance between the point $a_i$ and the set $B$. This procedure is repeated until a desired stopping criterion is achieved. See [11], [13] for further details on the ERA.

Unfortunately, the sets under consideration in the context of this paper (for example, the set of signals with a given Fourier transform magnitude (FTM) are non-convex [11]. However, the ERA has been extended to multiple non-convex sets, where projections are done iteratively [14]-[16], using a projection operator that assigns the nearest point on a set to each projection (function being projected). This is known as the Cyclic Projection (CP) algorithm. A proof that the CP algorithm generates a sequence of points with non-increasing error is shown in the Appendix.

One of the main contributions of this paper is to design unimodular waveforms for MIMO radars using the CP algorithm by projecting onto three sets such that

1) $X$ is unimodular, i.e, $X = e^{j\phi}$, where $e^{j\phi}$ is a $N \times P$ matrix obtained by an element-by-element exponentiation of the $N \times P$ matrix $\phi$.

2) The FTM of $X$ is recovered from the PSD $\eta$, and

3) $X$ has good sidelobe levels i.e, good auto-correlation and cross-correlation properties.

Before we can describe the CP algorithm used in this paper, we need to develop a suitable projection operator to project $X$ onto a set of waveforms that have good correlation properties.

A. Projection operator for reducing sidelobes

Let the aperiodic cross-correlation between $x_i$ and $x_j$, the signals from the $i^{th}$ and $j^{th}$ transmit antennas, be denoted by $r_{i,j}(n)$ where $1 \leq i, j \leq P$ and $0 \leq n \leq N - 1$. Note that $r_{i,j}(n)$ represents the aperiodic auto-correlation when $i = j$.

The total energy in the sidelobes of $r_{i,j}(n)$ and $r_{i,j}(n)$, are to be minimized in order to reduce the sidelobe levels in the auto- and cross-correlation functions i.e, we wish to minimize

\[
\mathcal{R} = \sum_{i=1}^{P} \sum_{n \neq 0} |r_{i,i}(n)|^2 + \sum_{i=1}^{P} \sum_{j=1}^{P} \sum_{n \neq 0} |r_{i,j}(n)|^2.
\]

\[
\mathcal{R} = \sum_{n=-N}^{N-1} |R_n| - N|I_P|_F^2 + 2 \sum_{n=0}^{N-1} |R_n|_F^2, \tag{12}
\]

From [10], (11) can be re-written as

\[
\mathcal{R} = \sum_{n=-N}^{N-1} |R_n - N|I_P\delta_n|_F^2. \tag{13}
\]

Minimizing $\mathcal{R}$ is equivalent (see [10]) to minimizing

\[
\mathcal{R}(Z) = \|A^HZ - U\|^2_F, \tag{14}
\]

where $A^HA = I$, $A$ is a $2N \times 2N$ point DFT matrix,

\[
Z = \begin{bmatrix} X \\ 0_{N \times P} \end{bmatrix}_{2N \times P}, \tag{15}
\]

$U = [\alpha_1, \ldots, \alpha_{2N}]^T$ is an auxiliary variable matrix and \(\{\alpha_{i}\}_{i=1}^{2N}\) is a $P \times 1$ vector with $\|\alpha_i\|^2 = \frac{1}{2}$. Using the vectorized versions of $Z$ and $U$, $\mathcal{R}(Z)$ is written as

\[
\mathcal{R}(z) = \|\tilde{A}^Hz - u\|^2_F, \tag{16}
\]

where $\tilde{A}$ is $2NP \times 2NP$ matrix given by blkdiag($A, \ldots, A$). For a given $z$, it is seen that the optimal $u$ that minimizes (16) is $\tilde{A}^Hz$. Thus

\[
u = \frac{1}{\sqrt{2P}} \exp(j \angle \tilde{A}^HZ). \tag{17}
\]

Using this solution for $u$, we optimize (16) with respect to $z$. For a given $u$, the optimal $z$ minimizing (16) is given by $z = \tilde{A}u$. The unimodular transmit waveform $X$ is then given by $X = \exp(jZ^H)$, where $Z$ is the matrix reconstructed from
z (\( \angle z \) indicates an element by element phase vector of \( z \)).

### B. Cyclic Projection Algorithm

Let \( \tilde{\eta}_{N \times P} = \eta \tilde{x} \) be the Hermitian square root of the diagonal, positive semi-definite matrix \( \eta \). It represents the FTM of \( X \) recovered from the PSD \( \eta \) (recall, \( \eta = X^H X \), \( X = \text{diag}(\tilde{x}) \) where \( \tilde{x} \) is defined in Section II). The proposed CP-based procedure is shown in Algorithm 1.

**Algorithm 1 Cyclic Projection Algorithm**

1: Generate \( \phi \) uniform over \([0, 2\pi]\) and \( X = e^{j\phi} \)
2: \( \tilde{X} = [X; 0_{(N-N') \times P}]_{N \times P} \), \( X = \tilde{\eta} \odot e^{jD \tilde{x}} \)
3: \( X = [e^{jD^H \tilde{x}}]_{N \times P} \), i.e take the first \( N \) rows.
4: \( Z = [X; 0]_{2NP} \), \( z = \text{vec}(Z) \), \( u = \frac{1}{\sqrt{2P}} \exp(j \tilde{A} H z) \), \( z = \tilde{A} u \), \( X = [e^{jZ}]_{N \times P} \), i.e take the first \( N \) rows.
5: Go to step 2 and repeat until a fixed number of iterations.

A random realization of \( X \) initializes the algorithm. Steps 2-4 constitute the crux of the algorithm, and correspond to the three steps mentioned earlier. In step 2, we project the waveform onto the set of signals that have the desired FTM. A unimodular sequence is constructed in step 3 by taking the phase of the inverse DFT of the waveform obtained in step 2 (that has the required FTM). In step 4, the sidelobes of the required unimodular waveform are reduced using the projection described in Section IV-A. This algorithm is repeated until a desired stop criterion is achieved (here, it can be the stop band criterion or the sidelobe criterion) or for a fixed number of iterations. The performance of the proposed algorithm is shown in Section V.

### V. Numerical Results

In all the results presented below, the Matlab toolbox CVX is used for solving the convex optimization problem in (10). Consider a \( 4 \times 2 \) MIMO radar system whose transmit waveform is to be designed. The receive SNR is assumed to be 10dB, the stop band power \( P_S \) is taken to be 0 and \( N = 100 \), \( N' = 100 \).

The results of the MI-based PSD design are shown in Figs. 1 and 2. It is seen that the resultant PSD \( \eta \) indeed represents a water-filling solution. The power is allotted to those frequencies where the target response is stronger than the clutter response and less power is allotted to those frequencies where the clutter is stronger. It is also seen that the power allotted in the stop band (between 20th to 40th frequency bins) is close to zero (\( \approx -120dB \)) indicating that the stop band constraint is indeed satisfied. Fig. 2, shows that the MI increases as the SNR increases and that more information about the target is available when the additional stop band constraint is not imposed on the radar. Also, the MI obtained is higher when a waterfilling-based design is used instead of equal power allocation (total power is equally allotted to all frequencies excluding the stopband i.e, the diagonal matrix \( \eta \) has constant entries at all locations except at the stopband frequencies). These results demonstrate that the radar must sacrifice target detection performance in the stopband frequencies in order to reduce interference to the communication system.

We use the merit factor introduced in [10] to compare the sidelobe levels of the time domain waveform \( X \). Define a shift matrix \( J_n \) where \( n \) is the lag at which the correlation level is evaluated.

\[
J_n = J^T_{-n} = \begin{pmatrix}
0_{(N-n) \times n} & I_{N-n} \\
0_{n \times N} & 0_{n \times (N-n)}
\end{pmatrix}.
\]

Then we have \( R_n = X^H J_n X \), where \( R_n \) is defined in Section IV. The merit factor defined at all lags \( n = -N + 1, \ldots, 0, \ldots, N - 1 \) is given by

\[
20 \log_{10} \frac{||R_n||_F}{||R_0||_F}.
\]

The performance of the CP-based time domain waveform recovery algorithm is shown in Figs. 3-6. The convergence of the normalized stopband PSD is shown in Fig. 3 for a single initialization of the CP algorithm. The CP-based waveform design algorithm converges in about 30 iterations both with and without the sidelobe constraint. As a conservative estimate, the algorithm is allowed to run for 100 iterations. Also, it is seen that the stopband suppression achieved is better (i.e, lower power in the stopband frequencies) without the stopband constraint than with the stopband constraint. Further,
the suppression achieved is dependent on the signal length used for the waveform recovery as shown in Fig. 3. It is seen in Fig. 4 that the reconstructed time-domain waveform (with one stopband constraint between 20th to 30th frequency bins, \( N = 100 \)) follows the PSD \( \eta \) at all frequencies.

Figs. 5-6 show the PSD and the sidelobe levels (here, the merit factor) of the time domain waveform with and without the sidelobe constraint in Algorithm 1. The suppression achieved without the sidelobe constraint is less (stopband power is higher) than what is achieved in the MI-based PSD design because the waveform is constrained to be unimodular. Although the reconstructed waveform follows the required PSD accurately at all frequencies, the stopband (two stopbands between 20th to 30th and 70th to 80th frequency bins are considered) suppression achieved in the case when the sidelobe constraint is active, is worse (less suppression) than the suppression without the sidelobe constraint. Also seen in Fig. 6 is the performance of the sidelobe levels of the waveform with and without the sidelobe constraint.

The power spectrum is the Fourier transform of the correlation function and thus has the information regarding the correlation levels (or sidelobe levels) of the signal. The results in Figs. 5-6 reiterate the inherent Fourier transform relationship between PSD and correlation function that is, an impulse in the time domain manifests itself as a flat response in the frequency domain. Hence, a better suppression in the PSD can be achieved only at the expense of the sidelobe level performance.

Let \( X_1 \) denote the time domain waveform reconstructed from the PSD alone, specifically \( X_1 = X \) where \( X \) is the waveform generated in step 3 of Algorithm 1 and \( X_2 \) denote the waveform obtained after reducing the sidelobes, i.e., the waveform obtained in step 4 of the algorithm. We define a new waveform \( X_3 \) as

\[
X_3 = (1 - \alpha) X_1 + \alpha X_2, \tag{19}
\]

where \( \alpha \in [0, 1] \) is a weight factor to control the relative importance of conflicting PSD and sidelobe requirements. Figs. 7-8 show the performance of the proposed algorithm as a function of the weight factor \( \alpha \). Thus, based on the target detection and interference reduction requirements, the proposed algorithm is flexible in choosing the necessary constraints for the radar waveform design.

VI. CONCLUSION

In this paper, we have proposed an information theoretic waveform design algorithm for MIMO radars to support spectral co-existence with a communication system. The designed PSD represents a water-filling solution that maximizes the target detection capabilities of the radar. An iterative cyclic projection algorithm was proposed for the time domain signal recovery from the designed PSD. Since correlation and PSD are related through the Fourier transform, a fundamental trade off exists between the two requirements. Our results show that a better PSD performance is obtained at the expense of the sidelobe suppression and vice versa.
REFERENCES


VII. APPENDIX

We show that the CP algorithm presented in Section IV generates a sequence of points with non-increasing error. Define the distance between a vector x and a set F as

\[ d(x, F) = \inf_{f \in F} d(x, f), \]

where \( d(x, f) \) is the distance between the vectors x and f. The distance operator d is symmetric, \( d(x, f) = d(f, x) \). Using these notations, we work through the CP algorithm.

Define three sets \( \mathcal{U}, \mathcal{F}, \mathcal{S} \) as the three sets onto which the signal is projected. According to Algorithm 1 they correspond to the projection on to the unimodular sequence set, set of signals with the given FTM and the set of signals with the required correlation properties. Let the algorithm be initialized with a unimodular sequence \( u_k \in \mathcal{U} \). Define \( f_k = F(u_k) \) the projection onto the set F and \( s_k = S(f_k) \). We define \( d(u_k, s_k) = d(u_k, f_k) + d(f_k, s_k) \) and \( u_{k+1} = u'(s_k) \). Then by the properties of the projection algorithm [13], we have

\[
\begin{align*}
\text{d}(u_{k+1}, \mathcal{S}) & \leq \text{d}(u_{k+1}, s_k) = \text{d}(s_k, u_{k+1}) \\
& = \text{d}(s_k, \mathcal{U}) + \text{d}(s_k, f_k) + \text{d}(f_k, u_k) \\
& \leq \text{d}(s_k, f_k) + \text{d}(f_k, u_k) \\
& = \text{d}(f_k, s_k) + \text{d}(u_k, f_k) = \text{d}(u_k, s_k).
\end{align*}
\]

Thus we have \( \text{d}(u_{k+1}, \mathcal{S}) \leq \text{d}(u_k, \mathcal{S}) \). This shows that the algorithm generates sequences with non increasing error.