Feedback and Delayed CSI Can Be as Good as Perfect CSI

Ravi Tandon*, Soheil Mohajer†, H. Vincent Poor*, and Shlomo Shamai (Shitz)‡

* Department of EE, Princeton University, (E-mail: {rtandon, poor}@princeton.edu)
† Department of EECS, UC Berkeley, (E-mail: mohajer@eecs.berkeley.edu)
‡ Department of EE, Technion-Israel Institute of Technology, (E-mail: sshlomo@ee.technion.ac.il)

***Abstract*—The degrees of freedom (DoF) region of the two-user MIMO interference channel (IC) is completely characterized in the presence of noiseless channel output feedback from each receiver to its respective transmitter and with the assumption of delayed channel state information (CSI) at the transmitters. It is shown that having output feedback and delayed CSI at the transmitters can strictly enlarge the DoF region when compared to the case in which only delayed CSI is available at the transmitters. Furthermore, cases are identified in which output feedback and delayed CSI alone are sufficient to achieve the DoF region achievable with perfect, instantaneous CSI.**

I. Introduction

In many wireless networks, multiple pairs of transmitters/receivers wish to communicate over a shared medium. In such situations, due to the broadcast and superposition nature of the wireless medium, the effect of interference is inevitable. Hence, management of interference is of extreme importance in such networks.

Various interference management techniques have been proposed over the past few decades. The more traditional approaches to deal with interference either treat it as noise (in the low interference regime) or decode and then remove it from the received signal (in the high interference regime). However, such techniques are not strong enough to achieve the optimal performance of the network even in the simple interference channel with two pairs of tranceivers equipped with multiple antennas. Recently, more sophisticated schemes, such as interference alignment [1], [2] and (aligned) interference neutralization [3]–[5] have been proposed for managing interference, which can significantly increase the achievable rate over the interference networks. However, these techniques are usually based on availability of instantaneous (perfect) channel state information (p-CSI) at the transmitters. Such an assumption is perhaps not very realistic in practical systems, at least when dealing with fast fading links.

Quite surprisingly, it is shown by Maddah-Ali and Tse that even delayed (stale) CSI is helpful to improve the achievable rate of wireless network with multiple flows, even if the channel realizations are independent over time. In a seminal paper [6], they showed that the sum Degrees of Freedom ($\mathbf{DoF}$) of $4/3$ is achievable for a broadcast multiple-input multiple-output (MIMO) network with two transmit antennas and one antenna at each receiver. This is in contrast to $\mathbf{DoF} = 1$, which is known to be optimal when no CSI is available. This technique is further studied in [7], where the authors showed that the delayed CSI can also improve the achievable $\mathbf{DoF}$ of the X-channel.

The $\mathbf{DoF}$ region of the two-user MIMO interference channel with delayed CSI is completely characterized by Vaze and Varanasi [8]. They have shown that, depending on the parameters of the channel (the number of antennas at each terminal), the $\mathbf{DoF}$ with delayed CSI can be strictly better than that of no CSI, and worse than that with instantaneous CSI. The effects of feedback on the capacity region of the interference channel have been studied in several recent papers (see [9] and references therein). It is known that under the perfect CSI assumption, feedback does not increase the $\mathbf{DoF}$ of MIMO BC and the MIMO IC [11]. One question that can be raised here is whether output feedback can be helpful with delayed CSI or not. For the case of the broadcast channel (BC), this question is answered in a negative way in [10]: having output feedback besides delayed CSI does not increase the $\mathbf{DoF}$ region of the MIMO BC.

In this work, we study this question for the two-user interference channel (IC), where each transmitter is provided with the past state information of the channel, as well as the received signal at its respective receiver. It turns out that, surprisingly, existence of output feedback can increase the $\mathbf{DoF}$ region of the interference channel. In the presence of output feedback with delayed CSI, transmitter 1 besides being able to reconstruct the interference it caused at receiver 2, can also reconstruct a part of the signal intended to receiver 2. This is in contrast to the case of the MIMO BC, where all information symbols are created at one transmitter, and hence output feedback in addition to delayed CSI does not increase the $\mathbf{DoF}$ region of the MIMO BC.

II. System Model

We consider the (M_1, M_2, N_1, N_2) MIMO-IC with fast fading under the assumptions of (A-I) noiseless causal channel output feedback from each receiver to its respective transmitter and (A-II) the availability of delayed CSI at the transmitters (see Figure 1). We denote the transmitters by $\mathbf{Tx}_1$ and $\mathbf{Tx}_2$ and the receivers by $\mathbf{Rx}_1$ and $\mathbf{Rx}_2$. The channel outputs at the receivers are given as

$$Y_1(t) = \mathbf{H}_{11}(t)X_1(t) + \mathbf{H}_{12}(t)X_2(t) + Z_1(t)$$
$$Y_2(t) = \mathbf{H}_{21}(t)X_1(t) + \mathbf{H}_{22}(t)X_2(t) + Z_2(t),$$

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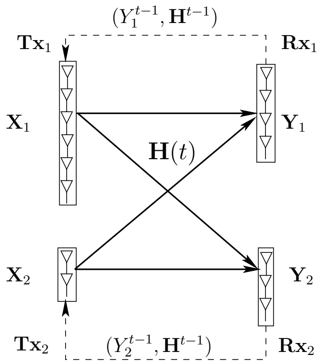

Fig. 1. MIMO-IC with output feedback and delayed CSI.

where $X_m(t)$ is the signal transmitted by mth transmitter $\mathbf{Tx}_m$; $\mathbf{H}_{nm}(t) \in \mathbb{C}^{N_n \times M_m}$ denotes the channel matrix between nth receiver and mth transmitter; and $Z_n(t) \sim \mathcal{CN}(0, I_{N_n})$, for $n = 1, 2$, is the additive noise at receiver n. The power constraints are $\mathbb{E}||X_m(t)||^2 \leq P$, for $\forall\ m, t$.

We denote $\mathbf{H}(t) = \{\mathbf{H}_{11}(t), \mathbf{H}_{12}(t), \mathbf{H}_{21}(t), \mathbf{H}_{22}(t)\}$ as the collection of all channel matrices at time t. Furthermore, $\mathbf{H}^{t-1} = \{\mathbf{H}(1), \mathbf{H}(2), \ldots, \mathbf{H}(t-1)\}$ denotes the set of all channel matrices up till time $(t-1)$. Similarly, we denote $Y_n^{t-1} = \{Y_n(1), \ldots, Y_n(t-1)\}$ as the set of all channel outputs at receiver n up till time $(t-1)$. A coding scheme with block length T for the MIMO-IC with feedback and delayed CSI consists of a sequence of encoding functions:

$$X_1(t) = f_{1,t}^T\left(W_1, \mathbf{H}^{t-1}, Y_1^{t-1}\right)$$
$$X_2(t) = f_{2,t}^T\left(W_2, \mathbf{H}^{t-1}, Y_2^{t-1}\right),$$

defined for $t = 1, \ldots, T$, and two decoding functions:

$$\hat{W}_1 = g_1^T(Y_1^n, \mathbf{H}^n), \quad \hat{W}_2 = g_2^T(Y_2^n, \mathbf{H}^n),$$

where W_1 (respectively W_2) denotes the message for $\mathbf{Rx}_1$ (respectively $\mathbf{Rx}_2$). A rate pair $(R_1(P), R_2(P))$ is achievable if there exists a sequence of coding schemes such that $\mathbb{P}(W_m \neq \hat{W}_m) \to 0$ as $T \to \infty$ for both $m = 1, 2$. The capacity region $\mathcal{C}(P)$ is defined as the set of all achievable rate pairs $(R_1(P), R_2(P))$ We define the DoF region as follows:

$$\mathbf{DoF}^{\mathrm{FB,d-CSI}} = \left\{(d_1, d_2) \middle| d_m \geq 0, \text{ and } \exists (R_1(P), R_2(P)) \in \mathcal{C}(P) \text{ s.t. } d_m = \lim_{P \to \infty} \frac{R_m(P)}{\log_2(P)}, m = 1, 2\right\}. \tag{1}$$

We denote the $\mathbf{DoF}$ regions corresponding to the cases of perfect-CSI, no-CSI, delayed-CSI, with output feedback, and with output feedback as well as delayed CSI as follows:

- No CSI: $\mathbf{DoF}^{\mathrm{No-CSI}}$
$$X_m(t) = f_{m,t}^T(W_m), m = 1, 2.$$
- Perfect CSI: $\mathbf{DoF}^{\mathrm{p-CSI}}$
$$X_m(t) = f_{m,t}^T(W_m, \mathbf{H}^T), m = 1, 2.$$
- Delayed CSI: $\mathbf{DoF}^{\mathrm{d-CSI}}$
$$X_m(t) = f_{m,t}^T(W_m, \mathbf{H}^{t-1}), m = 1, 2.$$
- Output feedback and delayed CSI: $\mathbf{DoF}^{\mathrm{FB,d-CSI}}$
$$X_m(t) = f_{m,t}^T(W_m, Y_m^{t-1}, \mathbf{H}^{t-1}), m = 1, 2.$$

III. Main Results

The contribution of this paper is a complete characterization of $\mathbf{DoF}^{\mathrm{FB,d-CSI}}$, stated in the following theorem:

Theorem 1: The $\mathbf{DoF}$ region with channel output feedback and delayed CSI, $\mathbf{DoF}^{\mathrm{FB,d-CSI}}$ is given as the set of all non-negative pairs (d_1, d_2) that satisfy

$$d_1 \leq \min(M_1, N_1) \tag{2}$$
$$d_2 \leq \min(M_2, N_2) \tag{3}$$
$$d_1 + d_2 \leq \min\Big\{M_1 + M_2, N_1 + N_2, \max(M_1, N_2), \max(M_2, N_1)\Big\} \tag{4}$$
$$\frac{d_1}{\min(N_1 + N_2, M_1)} + \frac{d_2}{\min(N_2, M_1)} \leq \frac{\min(N_2, M_1 + M_2)}{\min(N_2, M_1)} \tag{5}$$
$$\frac{d_1}{\min(N_1, M_2)} + \frac{d_2}{\min(N_1 + N_2, M_2)} \leq \frac{\min(N_1, M_1 + M_2)}{\min(N_1, M_2)}. \tag{6}$$

For comparison, we recall the DoF region with perfect, instantaneous CSI at the transmitters $\mathbf{DoF}^{\mathrm{p-CSI}}$ [11]:

$$d_1 \leq \min(M_1, N_1) \tag{7}$$
$$d_2 \leq \min(M_2, N_2) \tag{8}$$
$$d_1 + d_2 \leq \min\Big\{M_1 + M_2, N_1 + N_2, \max(M_1, N_2), \max(M_2, N_1)\Big\}. \tag{9}$$

In addition, the $\mathbf{DoF}$ region with delayed CSI, $\mathbf{DoF}^{\mathrm{d-CSI}}$ was characterized in [8]. This region is given by the set of inequalities as in Theorem 1 along with two more inequalities (bounds L_4 and L_5 in Definitions 1, 2, [8] corresponding to two mutually exclusive technical conditions, which the authors call condition 1 and condition 2). Hence, from Theorem 1, we have the following relationship:

$$\mathbf{DoF}^{\mathrm{No-CSI}} \subseteq \mathbf{DoF}^{\mathrm{d-CSI}} \subseteq \mathbf{DoF}^{\mathrm{FB,d-CSI}} \subseteq \mathbf{DoF}^{\mathrm{p-CSI}}.$$

Sketch of the converse proof: the upper bounds (2)-(3) are straightforward from the point-to-point MIMO channel. The sum degrees of freedom bound in (4) can be proved in a manner similar to the proof of [11]. Hence, to show that $\mathbf{DoF}^{\mathrm{FB,d-CSI}}$ is contained in the region given by (2)-(4), we need only to prove the bounds (5) and (6) for the case of output feedback and delayed CSI. Since (5) and (6) are symmetric, we need to prove that if $(d_1, d_2) \in \mathbf{DoF}^{\mathrm{FB,d-CSI}}$, then (d_1, d_2) must satisfy the bound (5). The proof of bound (5) closely follows the converse proof in [8]. Due to space limitations, the detailed proof is deferred to [12].

IV. Coding Scheme with FB and Delayed CSI

In this section, we present coding schemes that achieve the **DoF** region stated in Theorem 1. We assume without loss of generality, that $N_1 \geq N_2$. We refer the reader to Table I in reference [8].

- If (M_1, M_2, N_1, N_2) are such that
$$\mathbf{DoF}^{\mathrm{FB,d-CSI}} = \mathbf{DoF}^{\mathrm{d-CSI}}, \quad (10)$$
coding schemes presented in [8] which use delayed CSI only suffice for our problem. The condition (10) corresponds to cases A.I, A.II, B.0, B.I, and B.II, as defined in [8].
- If (M_1, M_2, N_1, N_2) are such that
$$\mathbf{DoF}^{\mathrm{FB,d-CSI}} \supset \mathbf{DoF}^{\mathrm{d-CSI}}, \quad (11)$$
we present a novel coding scheme that achieves $\mathbf{DoF}^{\mathrm{FB,d-CSI}}$.

Due to space limitations, we highlight the contribution of our coding scheme through two examples which captures its essential features and leads to valuable insights for the case of arbitrary (M_1, M_2, N_1, N_2).

V. (6, 2, 4, 3)-IC with Feedback and Delayed CSI

We first focus on the case of the $(6, 2, 4, 3)$-MIMO IC. For comparison purposes, we note here the **DoF** regions with no-CSI, perfect CSI, delayed CSI, output feedback and delayed CSI. For all these four regions, we have the following bounds:

$$d_1 \leq 4; \quad d_2 \leq 2.$$

Besides these, we have the following additional bounds:

- No-CSI:
$$\frac{d_1}{4} + \frac{d_2}{2} \leq 1.$$
- Perfect CSI: $d_1 + d_2 \leq 4$.
- Delayed CSI (case B-III, [8]):
$$d_1 + d_2 \leq 4; \quad d_1 + \frac{8d_2}{3} \leq 7.$$
- Output feedback and delayed CSI (Theorem 1):
$$d_1 + d_2 \leq 4; \quad \frac{d_1}{6} + \frac{d_2}{3} \leq 1.$$

It can be verified that this region is the same as the DoF region with perfect CSI, since the bound $\frac{d_1}{6} + \frac{d_2}{3} \leq 1$ is redundant as $d_1 \geq 2 \geq d_2$ is a valid choice (see Figure 2).

The main contribution of the coding scheme is to show the achievability of the point $(2, 2)$ under the assumption of output feedback and delayed CSI. To show the achievability of point $(2, 2)$, we will show that in three uses of the channel, we can reliably transmit 6 information symbols to receiver 1, and 6 information symbols to receiver 2.

Encoding at transmitter 2: transmitter 2 sends fresh information symbols on both its antennas for $t = 1, 2, 3$, i.e., the channel input of transmitter 2, denoted as $X_2(t)$ for $t = 1, 2, 3$

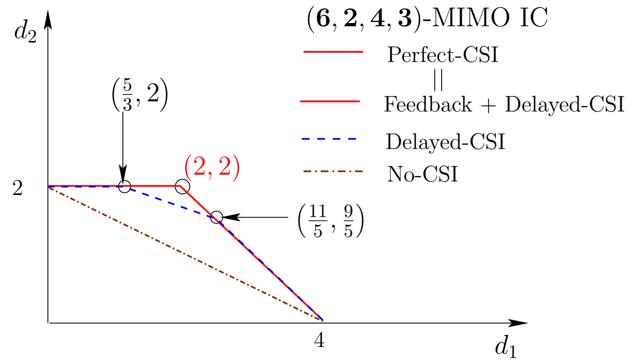

Fig. 2. **DoF** region for $(6, 2, 4, 3)$-MIMO-IC with various assumptions.

can be written as

$$X_2(1) = [v_1 \;\; v_2]^T, X_2(2) = [v_3 \;\; v_4]^T, X_2(3) = [v_5 \;\; v_6]^T.$$

At $t = 1$, transmitter 1 sends 6 information symbols on its 6 antennas, i.e., it sends $X_1(1) = [u_1 \; u_2 \; \ldots \; u_6]^T$. Let us denote by $\mathbf{u} = (u_1, \ldots, u_6)$ the vector of information symbols intended for receiver 1. The outputs at receivers 1 and 2 at $t = 1$ (ignoring noise) are given as

$$Y_1(1) = \begin{bmatrix} A_1(\mathbf{u}) + B_1(v_1, v_2) \\ A_2(\mathbf{u}) + B_2(v_1, v_2) \\ A_3(\mathbf{u}) + B_3(v_1, v_2) \\ A_4(\mathbf{u}) + B_4(v_1, v_2) \end{bmatrix}, \quad (12)$$

$$Y_2(1) = \begin{bmatrix} P_1(\mathbf{u}) + Q_1(v_1, v_2) \\ P_2(\mathbf{u}) + Q_2(v_1, v_2) \\ P_3(\mathbf{u}) + Q_3(v_1, v_2) \end{bmatrix}. \quad (13)$$

Upon receiving $Y_1(t)$ (channel output feedback) from receiver 1 and $H(1)$ (delayed CSI), transmitter 1 can use $(u_1, \ldots, u_6, Y_1(1), H(1))$ to solve for (v_1, v_2). Consequently, it can reconstruct $Q_1(v_1, v_2)$ and $Q_2(v_1, v_2)$ which constitute a part of the received signal, $Y_2(1)$, at receiver 2. In addition, having access to delayed CSI, $H(1)$, it can also compute $P_1(\mathbf{u})$ and $P_2(\mathbf{u})$, a part of the interference it caused at receiver 2. In the next two time instants, i.e., at $t = 2$ and 3, transmitter 1 sends

$$X_1(2) = [P_1(\mathbf{u}) \;\; Q_1(v_1, v_2) \;\; \phi \;\; \phi \;\; \phi \;\; \phi]^T \quad (14)$$
$$X_1(3) = [P_2(\mathbf{u}) \;\; Q_2(v_1, v_2) \;\; \phi \;\; \phi \;\; \phi \;\; \phi]^T, \quad (15)$$

where ϕ denotes a constant symbol known to all terminals.

The channel outputs at receiver 1 at $t = 2, 3$ are given as follows:

$$Y_1(2) = \begin{bmatrix} C_1(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4) \\ C_2(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4) \\ C_3(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4) \\ C_4(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4) \end{bmatrix}, \quad (16)$$

$$Y_1(3) = \begin{bmatrix} D_1(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6) \\ D_2(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6) \\ D_3(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6) \\ D_4(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6) \end{bmatrix}. \quad (17)$$

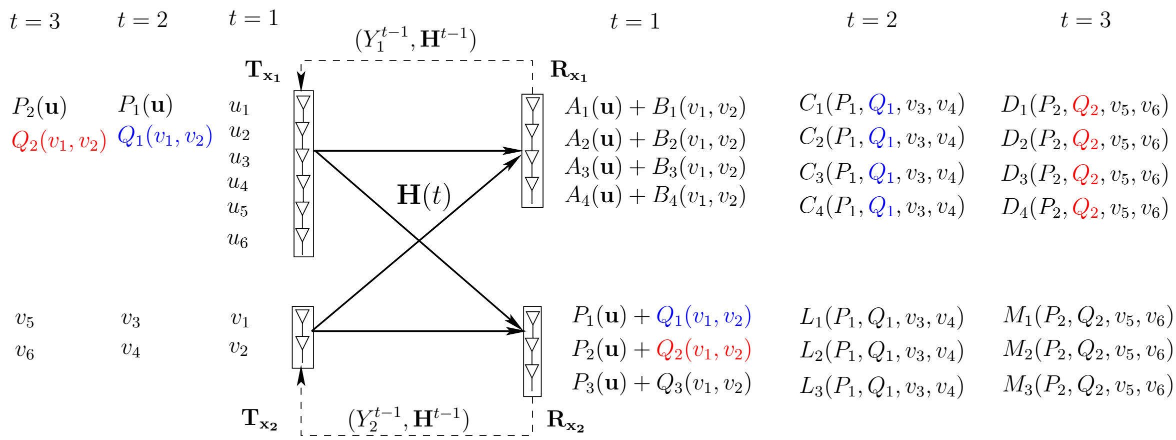

Fig. 3. Coding scheme with FB, delayed CSI: $(6, 2, 4, 3)$-MIMO-IC.

Decoding at receiver 1: Note that $Y_1(2)$ is comprised of 4 linearly independent equations in 4 variables $(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4)$ and similarly, $Y_1(3)$ is comprised of 4 linearly independent equations in 4 variables $(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6)$. Hence receiver 1 can decode $(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4)$ from $Y_1(2)$ and it can decode $(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6)$ from $Y_1(3)$. Having decoded $(Q_1(v_1, v_2), Q_2(v_1, v_2))$, it can solve for (v_1, v_2) and compute $\{B_j(v_1, v_2)\}_{j=1}^4$, the interference caused at $t = 1$. Upon subtracting $B_j(v_1, v_2)$ from the output at the jth antenna corresponding to $Y_1(1)$, receiver 1 obtains $(A_1(\mathbf{u}), A_2(\mathbf{u}), A_3(\mathbf{u}), A_4(\mathbf{u}), P_1(\mathbf{u}), P_2(\mathbf{u}))$, i.e., it has 6 linearly independent equations in 6 variables $(u_1, \ldots, u_6)$. Hence all 6 information symbols $(u_1, \ldots, u_6)$ can be decoded by receiver 1 in three uses of the channel.

The channel outputs at receiver 2 at $t = 2, 3$ are given as follows:

$$Y_2(2) = \begin{bmatrix} L_1(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4) \\ L_2(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4) \\ L_3(P_1(\mathbf{u}), Q_1(v_1, v_2), v_3, v_4) \end{bmatrix}, \tag{18}$$

$$Y_2(3) = \begin{bmatrix} M_1(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6) \\ M_2(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6) \\ M_3(P_2(\mathbf{u}), Q_2(v_1, v_2), v_5, v_6) \end{bmatrix}. \tag{19}$$

Decoding at receiver 2: at receiver 2, we have 9 linearly independent equations (from $Y_2(1), Y_2(2), Y_2(3)$) in 9 variables $(v_1, \ldots, v_6, P_1(\mathbf{u}), \ldots, P_3(\mathbf{u}))$, where $(P_1(\mathbf{u}), \ldots, P_3(\mathbf{u}))$ is the additive interference at $t = 1$. Hence it can decode 6 information symbols $(v_1, v_2, v_3, v_4, v_5, v_6)$ in three uses of the channel. Hence, we have shown the achievability of the point $(2, 2)$ with channel output feedback and delayed CSI.

Remark 1: It is instructive to compare this coding scheme to the case of delayed CSI. In particular, the point $(5/3, 2)$ lies on the boundary of $\mathbf{DoF}^{\text{d}-\text{CSI}}$. In the coding scheme that achieves this point, it suffices to transmit 5 symbols to receiver 1 and 6 symbols to receiver 2 in three channel uses. Under the delayed CSI assumption, transmitter 1 can at best reconstruct the interference it caused at receiver 2. In the terminology of the coding scheme described above, transmitter 1 can construct $(P_1(\mathbf{u}), P_2(\mathbf{u}), P_3(\mathbf{u}))$. In subsequent channel uses, $t = 2$, and $t = 3$, transmitter 1 sends $P_1(\mathbf{u})$, and $P_2(\mathbf{u})$ respectively. At the end of transmission, note that receiver 2 still has 9 equations in 9 variables, $(v_1, \ldots, v_6, P_1(\mathbf{u}), \ldots, P_3(\mathbf{u}))$, therefore it can reliably decode $(v_1, \ldots, v_6)$.

The difference between the optimal coding schemes for these two models is highlighted by the decoding capability of receiver 1. For instance, in the scheme with delayed CSI alone, receiver 1 has 10 linearly independent equations in 11 variables $(u_1, \ldots, u_6, v_1, \ldots, v_6)$, hence at best it can decode any 5 of the 6 information symbols $(u_1, \ldots, u_6)$. On the other hand, in our scheme, which allows for output feedback along with delayed CSI, transmitter 1 can exactly separate the interference and signal component of receiver 2, i.e., besides knowing $(P_1(\mathbf{u}), \ldots, P_3(\mathbf{u}))$, it can also exactly reconstruct $(Q_1(v_1, v_2), Q_2(v_1, v_2))$ (see Figure 3, which also highlights the difference of the coding schemes). This additional knowledge of $(Q_1(v_1, v_2), Q_2(v_1, v_2))$ is useful in transmission of one additional symbol to receiver 1 in three channel uses.

Remark 2: We note here that for this particular example, we can achieve the **DoF** region with perfect instantaneous CSI. Recall from [11] that the point $(2, 2)$ can be achieved with perfect CSI in one shot, i.e., in one channel use. As we have shown, output feedback and delayed CSI can also achieve the point $(2, 2)$, albeit, we pay the price of a larger delay, i.e., we can only achieve this rate in three channel uses. This observation also highlights the delay penalty incurred by the *causal* knowledge of output feedback and delayed CSI.

From this example it is clear that $\mathbf{DoF}^{\text{FB},\text{d}-\text{CSI}} = \mathbf{DoF}^{\text{p}-\text{CSI}}$. However, this equality does not hold in general. In the next section, we illustrate by an example for which $\mathbf{DoF}^{\text{d}-\text{CSI}} \subset \mathbf{DoF}^{\text{FB},\text{d}-\text{CSI}} \subset \mathbf{DoF}^{\text{p}-\text{CSI}}$, i.e., having feedback and delayed CSI is strictly worse than having perfect CSI and strictly better than only having delayed CSI.

VI. $(8, 4, 6, 5)$-MIMO IC

We now focus on the $(8, 4, 6, 5)$-MIMO IC (see Figure 4). The main contribution is to show the achievability of the point $(8/5, 4)$ under the assumption of output feedback and

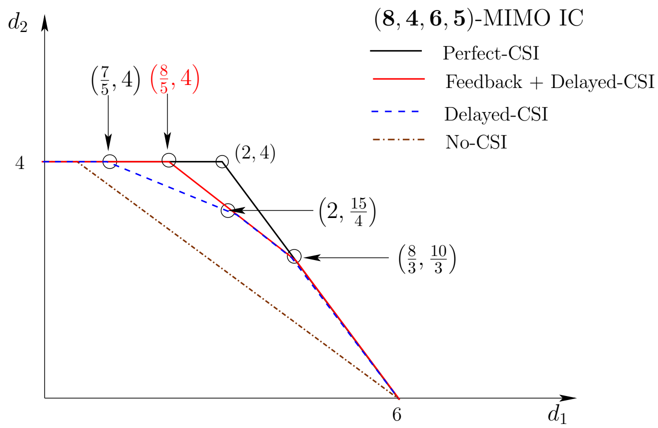

Fig. 4. **DoF** region for $(8,4,6,5)$-MIMO-IC with various assumptions.

delayed CSI. To this end, we will show that in 5 channel uses, transmitter 1 can send 8 symbols to receiver 1 and transmitter 2 can send 20 symbols to receiver 2.

For all 5 channel uses, transmitter 2 sends fresh information symbols, i.e., it sends

$$X_2(1) = [v_1 \dots v_4]^T, \dots, X_2(5) = [v_{17} \dots v_{20}]^T. \tag{20}$$

In the first channel use, transmitter 1 sends 8 fresh information symbols, i.e.,

$$X_1(1) = [u_1 \quad u_2 \dots u_8]^T. \tag{21}$$

Let us denote $\mathbf{u} = (u_1, u_2, \dots, u_8)$. The outputs at receivers 1 and 2 at $t = 1$ (ignoring noise) are given as:

$$Y_1(1) = \begin{bmatrix} A_1(\mathbf{u}) + B_1(v_1,v_2,v_3,v_4) \\ A_2(\mathbf{u}) + B_2(v_1,v_2,v_3,v_4) \\ A_3(\mathbf{u}) + B_3(v_1,v_2,v_3,v_4) \\ A_4(\mathbf{u}) + B_4(v_1,v_2,v_3,v_4) \\ A_5(\mathbf{u}) + B_5(v_1,v_2,v_3,v_4) \\ A_6(\mathbf{u}) + B_6(v_1,v_2,v_3,v_4) \end{bmatrix}, \tag{22}$$

$$Y_2(1) = \begin{bmatrix} P_1(\mathbf{u}) + Q_1(v_1,v_2,v_3,v_4) \\ P_2(\mathbf{u}) + Q_2(v_1,v_2,v_3,v_4) \\ P_3(\mathbf{u}) + Q_3(v_1,v_2,v_3,v_4) \\ P_4(\mathbf{u}) + Q_4(v_1,v_2,v_3,v_4) \\ P_5(\mathbf{u}) + Q_5(v_1,v_2,v_3,v_4) \end{bmatrix}. \tag{23}$$

Upon receiving feedback $Y_1(1)$, and CSI $H(1)$, having access to $\mathbf{u}$, transmitter 1 can reconstruct $(P_1(\mathbf{u}), \dots, P_4(\mathbf{u}))$ and $(Q_1(v_1,v_2,v_3,v_4), \dots, Q_4(v_1,v_2,v_3,v_4))$. In the subsequent channel uses, $2 \le t \le 5$ transmitter 1 sends

$$X_1(t) = [P_{t-1}(\mathbf{u}) \quad Q_{t-1}(v_1,v_2,v_3,v_4) \quad \phi \quad \phi \quad \phi \quad \phi \quad \phi \quad \phi]^T,$$

where ϕ denotes a constant symbol known to all terminals. It is straightforward to verify that receiver 2 has 25 linearly independent equations in 25 variables, $(v_1, \dots, v_{20})$ and $(P_1(\mathbf{u}), \dots, P_5(\mathbf{u}))$. Hence, it can decode all 20 information symbols $(v_1, \dots, v_{20})$. On the other hand, using $Y_2(t)$, receiver 1 can decode $P_t(\mathbf{u})$, and $Q_t(v_1,v_2,v_3,v_4)$, where $2 \le t \le 5$. Therefore, from $\{Y_2(t)\}_{t=2}^5$, it has $(P_2(\mathbf{u}), \dots, P_5(\mathbf{u}))$ and (v_1,v_2,v_3,v_4). Using (v_1,v_2,v_3,v_4), receiver 1 can construct the interference signals $B_1(v_1,\dots,v_4), \dots, B_6(v_1,\dots,v_4)$ for the first channel use. Subsequently, it can subtract these and obtain $(A_1(\mathbf{u}), \dots, A_6(\mathbf{u}))$. To summarize, receiver 1 can obtain 10 equations $(A_1(\mathbf{u}), \dots, A_6(\mathbf{u}), P_1(\mathbf{u}), \dots, P_4(\mathbf{u}))$ in 8 variables and it can reliably decode $(u_1, \dots, u_8)$.

Remark 3: From Figure 4, note that with perfect CSI, the pair $(2,4)$ is achievable; in other words, in 5 channel uses, one can send 10 symbols to receiver 1 and 20 symbols to receiver 2. However, with output feedback and delayed CSI, to guarantee the decodability of 20 symbols at receiver 2 necessitates transmitter 1 to repeat the interference component $(P_1, \dots, P_4)$ and a part of the signal component $(Q_1, \dots, Q_4)$. This coding scheme fills up all the dimensions (for this example, there are 25) at receiver 2. However, this leaves 2 dimensions redundant at receiver 1, which is the reason why feedback and delayed CSI cannot achieve the point $(2,4)$.

VII. Conclusions

In this paper, the **DoF** region of the MIMO-IC has been characterized under the assumption of output feedback and delayed CSI. It has been shown that output feedback and delayed CSI always outperform delayed CSI and can sometimes be as good as perfect CSI. We are currently investigating the generalization of this model by accounting for the cost of feedback, i.e., when feedback is available in a rate-limited manner to the transmitters.

References

[1] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani. Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis. *IEEE Transactions on Information Theory*, 54(8):3457–3470, 2008.

[2] V. R. Cadambe and S. A. Jafar. Interference alignment and degrees of freedom of the K-user interference channel. *IEEE Transactions on Information Theory*, 54(8):3425–3441, 2008.

[3] S. Mohajer, S. Diggavi, C. Fragouli, and D. Tse. Approximate capacity characterization for a class of Gaussian relay-interference wireless networks. *IEEE Transactions on Information Theory*, 57(5):2837–2864, May 2011.

[4] T. Gou, S. A. Jafar, S. W. Jeon, and S. Y. Chung. Aligned interference neutralization and the degrees of freedom of the $2 \times 2 \times 2$ interference channel. *Arxiv preprint arXiv:1012.2350*, 2010.

[5] S. A. Jafar. Interference alignment: A new look at signal dimensions in a communication network. *Foundations and Trends in Communications and Information Theory*, 7(1):1–134, 2010.

[6] M. A. Maddah-Ali and D. Tse. Completely stale transmitter channel state information is still very useful. In *48th Annual Allerton Conference on Communication, Control, and Computing, 2010*, 2010.

[7] H. Maleki, S. A. Jafar, and S. Shamai. Retrospective interference alignment. *Arxiv preprint arXiv:1009.3593*, 2010.

[8] C. Vaze and M. K. Varanasi. The degrees of freedom region and interference alignment for the MIMO interference channel with delayed CSI. *IEEE Trans. on Information Theory,* submitted, January 2011.

[9] C. Suh and D. Tse. Feedback capacity of the Gaussian interference channel to within two bits. *IEEE Trans. on Information Theory*, 57(5):2667–2685, May 2011.

[10] C. S. Vaze and M. K. Varanasi. The degrees of freedom region of the two-user MIMO broadcast channel with delayed CSI. *Arxiv preprint arXiv:1101.0306*, 2010.

[11] C. Huang and S. A. Jafar. Degrees of freedom of the MIMO interference channel with cooperation and cognition. *IEEE Trans. on Information Theory*, 55(9):4211–4220, Sep. 2009.

[12] R. Tandon, S. Mohajer, H. V. Poor, and S. Shamai (Shitz). On interference networks with feedback and delayed CSI. Submitted to *IEEE Transactions on Information Theory* arXiv:1109.5373, September 2011.