Ionospheric Tomography and Sounding

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1. Tomography

In LWA Memorandum No. 128, Aaron Cohen and Nagini Paravastu proposed a method for generating a dynamic, all-sky model of the ionosphere that in turn could be used to correct observations of astronomical objects. The method is based upon rapid cycling observations of about 100 bright, isolated radio sources, at any one time, which could be analyzed using standard methods (i.e., self-calibration) to provide measurements of differential total electron content (ΔTEC) along various lines of sights between the radio sources and the stations of the LWA. The time required for a complete cycle of observations would be less than 10 seconds. For an LWA of 52 stations, such a scheme could generate ΔTECs for ≥5000 lines-of-sight.

Although not addressed in the Memorandum, such a scheme could be expanded to include measurements of TEC along lines-of-sight between GPS receivers located at LWA stations and GPS satellites. Currently, that might provide 6-10 lines-of-sight per GPS receiver, but interoperability between GPS, GALILEO, and GLONASS at some future time may greatly expand those numbers, as could adding receivers at additional non-LWA locations. One must remember to specify GPS receivers that dump TEC measurements every second, which should be an adequate match to the time scales of the LWA measurements.

I have surveyed some of the literature on ionospheric tomography. I did not find any work comparable to the LWA problem in either spatial or temporal resolution. The spatial and temporal scales were typically ~100 km and 10 minutes, respectively. These works assumed that the lines-of-sight between the GPS receivers and satellites were straight lines; whether this assumption is adequate at LWA spatial and temporal resolutions should be confirmed. For the LWA measurements themselves ray-tracing calculations will be necessary for every line-of-sight and possibly every time stamp. And I think that Chris Watts was using the New Mexico supercomputer to do such calculations.

2. Sounding

Ionosondes are a basic instrument for the study of the global structure of the ionosphere. An ionosonde is a high-frequency (HF) radar that records the time of flight of a transmitted HF signal as a measure of its ionospheric reflection height. By sweeping in frequency, perhaps from 0.5 to 30 MHz, an ionosonde obtains a measurement of the ionospheric reflection height as a function of frequency. The plasma frequency at a height h is given by

$$f_p = (e/2\pi)(n_e(h)/m\varepsilon_0)^{1/2}, \quad (1)$$

where n_e is the electron density, e and m are the charge and mass of an electron, and ε_0 is the permitivitty of free space. Radio waves with frequencies $f \leq f_p$ will be reflected.

Knowledge of the behavior of the global electron-density profile as a function of height is the essential first step in understanding and modeling the behavior and structure of the ionosphere. Unfortunately, ionosondes are few and far between; the nearest one to the LWA in full-time operation may be in Boulder, CO.

Fortunately, analysis of the pointing errors caused by ionospheric refraction for radio sources distributed over the whole visible sky may provide an alternate approach to determining the electron-density profile of the ionosphere.

One of the earliest and largest study of such pointing errors was published by Slee & Lee (1974). They used the Culgoora radioheliograph to obtain about 2000 measurements of the positions of bright radio sources at 80 MHz between 1968 and 1971. All measurements were obtained at night, within an hour of transit, and covered the declination range $[-45^{\circ},35^{\circ}]$; zenith angles were $\leq 50^{\circ}$. The long time interval allowed the study of dependences on zenith angle, local time, and season. Since the observations were close to transit, the observed refraction in declination consisted of two well-defined components: the wedge component due to the north-south gradient in TEC and a spherical component caused by the radial gradient in ionization. The observed refraction in hour angle is due almost entirely to the east-west gradient in TEC. The systematic effects are of order ± 1 arc minute and the variations with zenith angle and season are the most significant. On the other hand, the random effects as measured by the standard deviations depend strongly on zenith angle and to a lesser extent on season; they increase from about 1 arc minute at the zenith to as much as 2-3 arc minutes at a zenith angle of 50°.

An astrometric accuracy of about 10 arc seconds or better will be necessary for such observations to be useful. Furthermore, such observations will likely be feasible only in the Compact Core and to the extent that all the stations are in the same coherent patch on the ground.

Slee & Lee (1974) include some theoretical discussion of ionospheric refraction, as does Spoelstra (1983). One of the earliest theoretical papers is Komesaroff (1960) but I found the most useful discussion to be that by Hagfors (1976). The spherical term is the counterpart of the standard atmospheric refraction correction applied at microwave and optical wavelengths, for example. It arises because, to first order, the ionosphere is horizontally stratified around a spherical earth. If the true zenith angle is Z_0 (i.e., in the absence of the ionosphere) and $n_e(h)$ is the electron density as a function of height h above the ground, the total bending in radians (positive toward the zenith) is given by

$$\delta = \sin Z_0\, e^2/(8\pi^2\, r_0 m \varepsilon_0 f^2) \int_0^{\infty} dh (1 + h/r_0) n_e(h) [(1 + h/r_0)^2 - \sin^2 Z_0]^{-3/2}, \qquad (2)$$

where f is the frequency and r_0 is the radius of the earth.

If the electron-density profile is available from another source (i.e., an ionosonde), the refractive errors can be calculated and corrections applied to the observations. Hagfors presents an example of such a calculation at 100 MHz: The refraction correction at a zenith angle of 80° is about 15 arc minutes, which corresponds to 25° at a frequency of 10 MHz. The plasma frequency is ≤10 MHz. Errors of such magnitude obviously affect the geometric delays as well.

However, it seems possible that the LWA could determine the electron-density profile itself by measuring the apparent positions of known radio sources across the whole sky. This could be done by cross correlating the signals from the individual dipoles in a coherent part of the Central Core and imaging the whole visible sky. The apparent positions would be compared to the catalog positions to determine the refraction errors. Since the spherical errors are symmetric about the zenith and the wedge errors are not, fit for the symmetric terms and invert the results to determine the electron-density profile. The integral appears to be well behaved; however, if inversion is not simple, fitting with a genetic algorithm, for example, might work.

Such a result would be an even more global measurement than that provided by ionosonde, which probably has a field of view comparable to or larger that of an LWA station – I found descriptions of receiving antennas 100 m – but smaller than that of an individual LWA dipole. So the output of an ionosonde is an average over a conical volume with an opening angle of $\sim 10^{\circ}$ or greater; consequently, the resulting ionograms are very smooth and show no fine details; in one example I found the vertical resolution was 6 km. The electron-density profile determined by fitting the symmetric pointing errors as described above is probably comparable. On the other hand, since the LWA measurements cover the whole sky at once, the asymmetric residuals potentially may provide additional information on the three-dimensional structure of the ionosphere. The level of detail will depend upon the angular resolution and the density and distribution of pointing calibrators.

As example, if the ionosphere can be modeled as multiple, possibly overlapping, layers with parabolic cross sections:

$$n(h) = \begin{cases} n_{max}\,[1 - (h - h_m)^2/d^2 & \text{for } |\,h - h_m\,| < d, \\ 0 & \text{otherwise,} \end{cases} \qquad (3)$$

where n_{max} is the maximum electron density and h_m is the corresponding altitude. The bending introduced by each layer is given by

$$\delta = \sin Z_0\, e^2 (4\pi\, m r_0 f^2)^{-1} N (1 + h_m/r_0)(\cos^2 Z_0 + 2h_m/r_0)^{-3/2}, \qquad (4)$$

where N (=4/3 n_{max}d) is the column density at the zenith. If we define a critical zenith angle Z_c,

$Z_c = \cos^{-1} [(2h_m/r_0)^{1/2}],$ (5)

the behavior of δ can be divided into two regimes: For $Z_0 < Z_c$,

$\delta \approx \sin Z_0\, e^2(4\pi\, mr_0 f^2)^{-1} N \cos^{-3} Z_0,$ (6)

and for $Z_0 > Z_c$,

$\delta \approx \sin Z_0\, e^2(4\pi\, mr_0 f^2)^{-1} N (2h_m/r_0)^{-3/2}.$ (7)

In the first case, if $Z_0 \ll Z_c$,

$\delta \approx \sin Z_0\, e^2(4\pi\, mr_0 f^2)^{-1} N.$ (8)

The transition between the two regimes is broad; and if layers are present at multiple altitudes, multiple transition regions will be superposed. For example, if four layers are present (Rich 1985) – D (70-90 km, $\sim 10^{3.2}$ cm^{-3}), E (95-140 km, $\sim 10^{5.2}$ cm^{-3}) F1 (140-200 km, $\sim 10^6$ cm^{-3}), and F2 (200-400 km, $\sim 10^{6.5}$ cm^{-3}), the critical angles will span 68°-82° and the transition region will be even broader.

Over most of the sky, for $Z_0 < 50°$, the dominant factor determining the bending is the column density of electrons at the zenith, which is dominated by the contributions of the F1 and F2 layers. At large zenith angles, the contributions of the D and E layers are somewhat more important because of the $h_m^{-3/2}$ weighting of electron density in Equation (6), but the contributions of the F1 and F2 layers still dominate.

The model calculation in Hagfors (1976) illustrates the difficulty in properly registering at high zenith angles the sky surveys proposed for the LWA-1.

Any asymmetric errors will indicate the presence and orientation of ionospheric wedges. And potentially might be useful for developing some information about the three-dimensional structure of the ionosphere.

3. Questions and Comments

How much detail in the electron-density profile is needed for accurate ray tracing?

Is the smooth profile provided by an ionosonde or the LWA adequate for determining the paths followed by the “rays”?

Are the ray paths affected significantly by the fluctuations and inhomogeneities that the LWA will study?

How frequently should the electron-density profile be updated?

Is it possible to reduce the number of supercomputer calculations? For example, by calculating a family of ray paths for a given profile – with $\Delta Z \sim 0.1^o$ – and use them until the profile is updated several minutes later?

The TBN data should be suitable for measuring the pointing errors caused by ionospheric refraction. For example, thousands of sources from the VLSS (Cohen et al. 2007) should be suitable: for an SEFD of 4370 Jy at 80 MHz per station (Pihlström 2009) and using 16 stations, 10 seconds of TBN data (100 kHz) would provide a sensitivity of ~¼ mJy at the zenith.

In the absence of a nearby ionosonde operating 24/7, inverting the refractive pointing errors may provide a local profile of the electron density of the ionosphere – with high time resolution.

In fact, given the large number of suitable sources, it may be possible to obtain a two-dimensional set of electron-density profiles surrounding the Compact Core – for example, solving for 360 wedges each 10^o wide. Asymmetric results will provide clues to the three-dimensional structure of the ionosphere and how it might be changing. But, given that the pierce points will be at distances of hundreds of kilometers and the lines of sight are correspondingly long, the meaning may be difficult to disentangle.

Without a locally determined electron-density profile, predictions of ionospheric refraction may not be reliable for $Z_0 > 50^o$. But for smaller zenith angles, knowledge of the electron column density at the zenith may be adequate, and perhaps that knowledge may be obtained with sufficient accuracy from a seasonal model or from a distant ionosonde.

It may be impractical to do sky surveys using a single LWA station – e.g., LWA-1 – at large zenith angles.

At low frequencies (~10 MHz) it may be possible to observe over the horizon by 25-30 degrees.

A uniform, spherically distributed ionosphere will introduce differential pointing errors in zenith angle that decrease with zenith angle. Radio sources will tend to crowd toward the zenith, with sources at higher zenith angles moving farther.

4. References

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