Estimation of Predetection SNR of LMR Analog FM Signals Using PL Tone Analysis

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1 Introduction

This report documents the estimation of predetection signal to noise ratio (SNR) of land mobile radio (LMR) analog FM signals using PL tone analysis. A particular application where this is desirable is the antenna system project [1], which requires on-the-air testing in TIA-603 analog system. In this antenna system project, it is desired to accurately determine the in situ RF SNR. To this end, a suitable approach is to first determine the audio SNR and use that to estimate the SNR at the input to the RF front end. Kumar and Ellingson (2012) [2] provide a method for “bottom line” assessment of analog FM SNR using PL tone analysis. Experimental techniques that accurately determine the audio SNR for predetection SNR range between 0 dB and 30 dB have been discussed in [2]. Jakes (1993) [3] presents a theoretical relationship between the audio SNR and the predetection SNR. In this report, we use this theoretical result to estimate the predetection SNR from the audio SNR.

The rest of the document is organized as follows. Section 2 details the theoretical approach to estimate the predetection SNR. Then in Section 3 we use simulated signals and perform some experiments to compare the theoretical estimate of the predetection SNR with the actual predetection SNR values. Finally in Section 4 we conclude the report with comments on the accuracy of the theoretical approach.

2 Theoretical estimation of predetection SNR

In this section, we present a theoretical result for the estimation of the predetection SNR from the audio SNR. Figure 1 shows the system model of the FM communication system used in the derivation of the result.

We make the following assumptions in our model:

1. The modulation index for the FM system is greater than unity. This holds true for the TIA-603 modulations [4, 5].
2. The IF filter has a Gaussian-shaped frequency response, \( G(f) \), given by:
   \[
   G(f) = e^{-4\pi(f-f_c)^2/9B^2},
   \]
   where \( f_c \) is the carrier frequency, and \( B \) is the 3-dB IF bandwidth in Hz. We lose little generality by making this assumption since the audio filter bandwidth is usually narrow relative to the IF bandwidth.

As the IF SNR falls and goes below the threshold, the signal modulation is suppressed. So the signal output from the FM detector, \( m_o(t) \), is given by [3, 6]:

\[
m_o(t) = m(t)(1 - e^{-\rho}),
\]
where \( m(t) \) is the input message signal. Hence the output baseband signal power, \( S_0 \), is related to the input modulation signal power, \( S \), as:

\[
S_0 = (1 - e^{-\rho})^2 S \tag{3}
\]

where \( \rho \) is the predetection SNR (at the output of the IF filter). Let \( c \) denote the transfer characteristic (gain) of the detector. Then from [2, Section 3.3], we have

\[
S = \frac{\pi^2 (B - 2W)^2}{10c^2}, \tag{4}
\]

where \( W \) is the cutoff frequency of the low pass audio filter. The total noise power, \( N \), out of a rectangular baseband audio filter is given by [2, Section 3.3]:

\[
N(\rho) = \frac{a(1 - e^{-\rho})^2}{c^2 \rho} + \frac{8\pi BW e^{-\rho}}{c^2 \sqrt{2(\rho + 2.35)}}, \tag{5}
\]
where $a$ is given by

$$a = \frac{4\pi^2 W^3}{3B} \left\{ 1 - \frac{4\pi}{15} \left( \frac{W}{B} \right)^2 + \frac{8\pi^2}{189} \left( \frac{W}{B} \right)^4 + \ldots \right\}. \quad (6)$$

Since we have assumed that the modulation index is greater than unity, $W$ is much smaller than $B$. Therefore, we will consider only first three terms in the equation (6) and still get a fairly precise value of $a$. Now from equations (3), (4) and (5), the audio SNR is

$$\text{SNR}_a = \frac{S_a}{N} = \frac{\pi^2 (B - 2W)^2}{10 \left( \frac{a}{\rho} + \frac{8\pi BW e^{-\rho}}{(1-e^{-\rho})^2 \sqrt{2(\rho+2.35)}} \right)}. \quad (7)$$

Given audio SNR, predetection SNR can be determined from equation (7). However it is not possible to find a closed form expression for predetection SNR.

## 3 Results

In this section, we evaluate the correctness of the theoretical predetection SNR by comparing it first with the simulated data and later with the experimental data.

### 3.1 Simulated Signal (stationary channel)

In this section, we evaluate the accuracy of the theoretical method through simulations. The channel is assumed to be additive white gaussian noise (AWGN) with no fading. The FM detector is implemented as a sequence of DSP operations described in [2, Section 2]. The audio SNR is determined using the PE/S method [2, Section 3.1]. Since for a stationary channel increasing the integration time beyond 100 ms does not have any significant impact on the performance of the PE/S method [2, Section 4.1.2], we set the integration time to 100 ms. $B$ and $W$ are set to 17 kHz and 3.5 kHz respectively.

Then both the theoretical estimate (from equation (7)) and the measured predetection SNR values are plotted against the audio SNR as shown in Figure 2. We observe that the results are consistent but not in close agreement. The theoretical result follows the measured predetection SNR with a 1–10 dB offset, for audio SNR range from $-15$ dB to $35$ dB. Some offset should be expected because there are some approximations and assumptions in the theoretical result. However it is not possible to comment on the correctness of the theoretical result based just on the simulations. Hence we perform an experiment as described in the next section to determine the validity of the results.
Figure 2: Relationship between predetection SNR and audio SNR (Non-fading channel).
3.2 Control Experiment 1

We now compare the theoretical method with measurements from a control experiment, where all the system parameters are known and under our control. The experimental setup is shown in Figure 3. We generated an FM signal using Agilent E4438C ESG vector signal generator by modulating a sinusoidal carrier at 453.2 MHz using a 123 Hz PL tone (no voice audio). The maximum frequency deviation is set to 4 kHz. The signal generator is directly connected to Tektronix RSA6114A spectrum analyzer. The spectrum analyzer has a “RF IQ vs. Time” measurement option. We used this option to record the RF I and Q baseband data to a PC. On the PC, we passed the recorded RF data through a Chebyshev FIR low-pass filter with 3-dB bandwidth ($B/2$) set to 7.5 kHz. The order of the filter is 850 taps. We then measured the predetection SNR by measuring the total (signal+noise) power when the signal generator is on and the noise power when there is no transmission. The filtered RF signal is then demodulated using a FM detector which is same as in Section 3.1. The output of the detector is passed through an audio filter which is a Chebyshev FIR low-pass filter of order 803 taps and with 3-dB bandwidth ($W$) set to 3.5 kHz. The audio SNR is then measured using the PE/S method. The integration time for the PE/S method is set to 100 ms. The experiment is then repeated for different values of carrier power, to get a predetection SNR range from $-7$ dB to $35$ dB.

Figure 4 shows predetection SNR as a function of audio SNR, using the experimental result, the simulation, and the theoretical result. We observe that the experimental result is in close agreement with the simulation result. This is not surprising since the control experiment is different from the simulation only in that the simulation modulator is replaced by a hardware modulator. Thus agreement between the simulation and the control experiment indicates only that the modulator is not the source of disagreement between simulation and control experiment. Hence, in the next section, we perform another control experiment which replaces the simulation receiver with a hardware receiver.

3.3 Control Experiment 2

We now compare the theoretical method with measurements from an another control experiment. The experimental setup is shown in Figure 5. We generated an FM signal as described in Section 3.2. The signal generator output is split two ways using a Mini-Circuits ZFSC-2-2500+ power splitter. One of the outputs goes to a Tektronix RSA6114A spectrum analyzer while the other goes to a ICOM IC-PCR 1500 receiver. The output from the spectrum analyzer is then used to measure the predetection SNR as described in Section 3.2. The audio output from the ICOM receiver is recorded to a PC. On the PC, we filter the recorded audio through a Chebyshev FIR low-pass filter of order 1003 taps and with 3-dB bandwidth ($W$) set to 3.5 kHz. The audio SNR is then measured using the PE/S method. The integration time for the PE/S method is 100 ms. The experiment is then repeated for different values of carrier power to get a predetection SNR range from $-12$ dB to $+25$ dB.
Figure 3: Control Experiment 1 setup.
Figure 4: Control Experiment 1 - Relationship between predetection SNR and audio SNR.
Signal Generator (Agilent E4338C)

Power splitter (Mini-Circuits ZFSC-2-2500+)

Spectrum Analyzer (Tektronix RSA6114A)

RF I&Q baseband output

Low-pass filter (3-dB BW: 7.5 kHz)

Filtered RF signal (Measure predetection SNR)

ICOM Receiver (ICOM IC-PCR 1500)

Audio filter (LPF) (3-dB BW: 3.5 kHz)

Audio Output (Measure audio SNR - PE/S method)

Figure 5: Control Experiment 2 setup.
Figure 6 shows predetection SNR as a function of audio SNR, using the experimental result, the simulation, and the theoretical result. We observe that the experimental result is in close agreement with the simulation result for audio SNR between 0 dB and 20 dB. For audio SNR < 0 dB, there is a 1–3 dB difference between the simulation and experimental result. For audio SNR > 20 dB, the difference between the simulation and experimental results increases and becomes asymptotically large at audio SNR of 25 dB. This is an expected behavior. Due to the non-linear nature of FM modulation, the frequency estimation in receiver doesn’t get any better by increasing the predetection SNR beyond a particular value. Hence the audio SNR is upper bounded.

So in order to accurately estimate the predetection SNR, we would first need to determine the audio SNR corresponding to the theoretical predetection SNR using Figure 6. Then determine the simulation predetection SNR corresponding to the measured audio SNR using Figure 6 again. We would use this value of simulation predetection SNR as our final estimate for predetection SNR.

4 Conclusion

The predetection SNR can be estimated using the following procedure:

1. Record the audio output from a receiver (such as ICOM IC-PCR 1500) onto a PC.
2. On the PC, filter the recorded audio through a Chebyshev FIR low-pass filter of order 1003 taps and with 3-dB bandwidth set to 3.5 kHz.
3. Measure the audio SNR for the filtered audio signal using the PE/S method (set the integration time to 100 ms).
4. Use the theoretical result described in Section 2 to estimate the predetection SNR from the audio SNR.
5. Using Figure 6 determine the audio SNR value corresponding to the estimated predetection SNR in step 4.
6. Again using Figure 6 determine the simulation predetection SNR corresponding to the audio SNR determined in step 5. We would use this value of simulation predetection SNR as our final estimate for predetection SNR.

The estimated predetection SNR is quite accurate (within 0.5 dB of actual result) for audio SNR between 0–20 dB. For audio SNR between -18–0 dB, there is a 1–3 dB difference between the estimated and actual value of predetection SNR. For audio SNR > 20 dB, the difference between the estimated and actual predetection SNR increases and becomes asymptotically large at audio SNR of 25 dB (for reasons provided in Section 3.3). Hence we conclude that our proposed method works with ±3 dB accuracy for predetection SNR between -18–22 dB.
Figure 6: Control Experiment 2 - Relationship between predetection SNR and audio SNR.
References


