Spatial filtering of interfering signals at the initial LOFAR phased array test station.

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1 Abstract

The Low Frequency Array (LOFAR) is a radio telescope currently being designed. Its targeted observational frequency window lies in the range 10-250 MHz. In frequency bands where there is interference, the sensitivity of LOFAR can be enhanced by interference mitigation techniques. In this paper we demonstrate spatial filtering capabilities at the LOFAR initial test station (ITS), and relate it to the LOFAR RFI mitigation strategy. We show that in frequency ranges which are occupied with moderate intensity man-made radio signals, the strongest observed astronomical sky sources can be recovered by spatial filtering. We also show that, under certain conditions, intermodulation products of point-like interfering sources remain point sources. This means that intermodulation product filtering can be done in the same was as for "direct" interference. We further discuss some of the ITS system properties such as cross talk and sky noise limited observations. Finally, we demonstrate the use of several beamformer types for ITS.

2 Introduction

The Low Frequency Array (LOFAR) is a next generation radio telescope which is currently being designed and which is planned to be located in the Netherlands. LOFAR [5] is an aperture array telescope [24, 20] and will consist of order 100 telescopes (stations), spread in spirals over an area of about 300 km, as well as in a more densely populated central core. The observational frequency window will lie in the 10-250 MHz range. Each of the stations will consist of order 100 phased array antennas. These antennas are sky noise limited, and are combined in such a way that station beams can be formed for each of the desired station look directions or pointings. The preliminary LOFAR design defines multiple beam capabilities, (non-contiguous) 4 MHz wide bands, and a frequency resolution of 1 kHz. The LOFAR initial operations phase is scheduled to start in 2006; the target date to have LOFAR fully operational is 2008.

For testing and demonstration purposes, several prototype stations are defined. One of these demonstrators is the initial test station (ITS). It is a full scale prototype of a LOFAR station, and it became operational in December 2003. ITS consists of 60 sky noise limited dipoles, configured in a five armed spiral, connected to a digital receiver backend. ITS operates in the frequency band 10-40 MHz, and the
observed signals are directly digitized without the use of mixers. The data can be stored either as time series or as covariance matrices.

In spectrum bands which are occupied with man-made radio signals with moderate signal powers, the unwanted man-made radio signals can be suppressed by applying filtering techniques. In this paper we demonstrate spatial filtering capabilities at the LOFAR ITS test station, and relate it to the LOFAR RFI mitigation strategy [2]. We show the effect of these spatial filters by applying them to antenna covariance matrices, and by applying different beamforming scenarios. We show that for moderate intensity interferers, the strongest observed astronomical sky sources can be recovered by spatial filtering. We also show that, under certain conditions, intermodulation products of point-like interfering sources remain point sources. This means that intermodulation product filtering can be done in the same way as for "direct" interference. We further discuss some of the ITS system properties such as cross talk and sky noise limited observations. Finally, we demonstrate the use of several beamformer types for ITS.

Notation

In this paper, scalars are denoted by non-bold lower case and upper case letters. Vectors are represented by bold lower case letters, and matrices by upper case bold letters. The hermitian conjugate transpose is denoted by $(\cdot)^H$, the transpose operator by $(\cdot)^T$, the expected value by $\mathcal{E}\{\cdot\}$, and the estimated values by $\hat{\cdot}$. The element wise multiplication (Hadamard) matrix operator is denoted by $\odot$. For a vector $\mathbf{a} = (a_1, \cdots, a_p)^T$, $e^{\mathbf{a}}$ is defined by $e^{\mathbf{a}} = (e^{a_1}, \cdots, e^{a_p})^T$. $\mathbf{I}$ represents the identity matrix, $\mathbf{A}^{-1}$ denotes the matrix inverse of $\mathbf{A}$, and $\mathbf{A}^{\frac{1}{2}}$ denotes the matrix $\mathbf{B}$ such that $\mathbf{B}^2 = \mathbf{A}$. Finally, $\mathbf{j} = \sqrt{-1}$, $\mathbf{0}$ is the null matrix, the complex conjugate is denoted by $\overline{\cdot}$, and $\text{diag}(\mathbf{a})$ converts the vector $\mathbf{a}$ to a diagonal matrix with $\mathbf{a}$ on the main diagonal.

3 LOFAR interference mitigation strategy

LOFAR will operate in bands where other spectrum users are active, and in which interference may occur. However, it is expected that the sensitivity of LOFAR can be enhanced by applying filtering and interference mitigation techniques. In this way, parts of the bands occupied with moderate intensity man-made radio signals, can be recovered for astronomical observations. A description and results of some of the interference mitigation techniques applied in radio astronomy can be found for example in [6, 7, 13, 15, 8, 1].

Spectral occupancy and LOFAR sensitivity

LOFAR will be one of the first radio telescopes in which RFI mitigation techniques will form an integral part of the system design. For several reasons, it was decided to equip LOFAR with relatively simple RFI mitigation techniques. In future phases of LOFAR, these techniques may be extended. A first constraint on complexity is that the computing power required for interference mitigation should be an order of magnitude less than what is required for the astronomical signal processing. Only in special cases is spending a major fraction of the computing resources on RFI mitigation acceptable. A second reason for relatively simple techniques is that the calibration of LOFAR [17, 18] requires stable station beams. Only slowly varying (sidelobe) gains are allowed, otherwise the calibration process will not converge. For this reason, at station level, only spatial filters with fixed or slowly varying nulls are considered as fast interference tracking would change the station beams too rapidly. A third reason is that for radio astronomy, interference mitigation is a relatively new field, and that the effects of interference mitigation related distortions are not in all cases quantified.

The use of the radio spectrum, in terms of signal power and time-frequency occupancy is roughly known from allocation tables and from monitoring observations. In order to estimated the required attenuation levels, the observed spectrum power needs to be related to the LOFAR sensitivity. One of the key parameters of the LOFAR aperture synthesis mode is that LOFAR will be sky noise dominated and its desired ultimate sensitivity will be a factor eight better than the thermal sky noise in a four hour full synthesis observation with order 100 stations. In [11] it is specified as a sensitivity of 2 mJy at 10 MHz down to 0.03 mJy at 240 MHz for 1 hour integration over 4 MHz bandwidth, which corresponds for a 1 kHz bandwidth to 127 mJy at 10 MHz down to 2.1 mJy at 240 MHz. In order to achieve this sensitivity and the required dynamic range, it is estimated the we need 4 MHz wide frequency bands for which we
can recover 80% bandwidth. Here is assumed, that 20% bandwidth loss due to RFI does not lead to a dramatic decrease in sensitivity of the instrument as a whole. It is also assumed that in the 80% cleanest frequency bins within a 4 MHz band, the RFI at station level is either at a 1-3 (station) sky noise sigma level, or can be reduced to this level by RFI mitigation techniques. The rationale of this is explained next.

**Power density flux levels**

The calibration capabilities of LOFAR [17] include removal of strong sky sources such as Cas.A from the observed (uvw) datasets and images. This suggests that, assuming that RFI sources can be suppressed to levels comparable to the level of Cas.A, the remaining RFI can be removed in the same way as astronomical sources removed. The LOFAR RFI strategy is based on this assumption which is illustrated in figure 1. The vertical scale represents radio wave flux levels and sensitivities in Jy, where $1 \text{ Jy} = 10^{-26}$ W m$^2$ Hz$^{-1}$. The curve "antenna sky noise" shows the sky noise flux $\Psi$ as a function of frequency, as would be observed with a single polarization LOFAR antenna dipole. It is based on the sky temperature, given by the approximate formula [5, 12]

$$T_{\text{sky}} \approx T_{s0} \times 2.55^\Delta (K)$$

and on a formula [12, 22] relating the sky temperature $T_{\text{sky}}$ to the flux density $\Psi$

$$\Psi = \frac{2k}{\chi^2} T_{\text{sky}} \Omega \ (Jy)$$

Here $\Omega$ is the antenna solid angle (assumed to be 4 Sr), $k$ the Boltzmann constant, and $T_{s0}$ is $60 \pm 20$ K for angular distances to the galactic plane larger than $10^\circ$. According to the ITU radio noise recommendation [10], the environmental RF noise floor in rural, residential and business areas lies typically 5 to 20 dB above the sky background noise, and for quiet rural areas, it lies $\approx 5$ to 10 dB below the background noise. In practical situations, these levels are strongly time and frequency dependent and also dependent on location and height. Note that for the LOFAR central core location in the Netherlands, the sky background noise was already detected for several frequencies, which means that in those cases the "rural curve" mentioned in [10] is too pessimistic.

The dash-dotted curve denotes a transmitter or interference flux level, corresponds to a free space field strength of 0 dBuVm$^{-1}$, assuming that the radio signal impinging on LOFAR is smeared out over a 1 kHz frequency channel width. At the LOFAR central core site site in the Netherlands (above 30
MHz and outside the FM bands) nearly all transmitters and interferers have observed powers less than 40 dB μV m$^{-1}$. This was measured in a monitoring campaign: a large fraction of these transmitters even have observed powers below 0 dB μV m$^{-1}$. An example of a monitoring measurement is shown in figure 2, where the median and 90 percentile curves are plotted for twelve observed spectra, uniformly distributed over a 24 hour period. In this particular spectrum monitoring example, the system noise is dominated by the monitor’s receiver noise, and it increases with frequency.

In a station, (order) 100 antennas are combined in a phased array to form one or more beam(s). This means that the rms noise level at the beam output is decreased by 20 dB and becomes approximately equal to the noise power of Cas.A, one of the strongest sky sources. The rms noise level at the beam output is represented by the curve ”station sky noise”. It lies a few dB’s below the ”Cas.A” flux curve.

In the lower part of the figure two ”LOFAR sensitivity” curves are drawn, both for 1 kHz bandwidth and for a synthesis array of (order) 100 stations. The two curves differ in integration time: 1ms for the upper curve, and 4h integration time for the lower curve. Between the two LOFAR sensitivity curves, the (‘mean’) ITU-R RA769 emission criterion [9] is given, which roughly states the RFI level at which the error in determining the signal power exceeds 10 % for an integration time of 2000 s.

**LOFAR RFI strategy**

The LOFAR RFI mitigation strategy is based on three steps:

- Select the cleanest (order 200 kHz) frequency subbands within the LOFAR bands.
- Reduce the RFI levels by RFI mitigation down to Cas.A power flux levels.
- Reduce the RFI levels further to levels close to the noise in the sky maps.

The first step in the strategy is to choose an optimum location of the LOFAR (order 200 kHz) subbands. Some of the 1 kHz bins in the 200 KHz bands may be affected by interference. In some cases, for example when a slight reduction of sensitivity is acceptable, these channels can be discarded, otherwise RFI mitigation measures need to be applied.

The interference mitigation measures, mentioned in the second step, can be applied at station level before or after beamforming, or be applied at at a central level, before of after correlation. Where this is
done in the signal chain in an optimal way is determined by various factors: number of beam data bits, data transport load, number of correlator input bits, linearity of the RFI Mitigation methods, etcetera. For LOFAR it was decided to apply only fixed or very slowly time-varying spatial nulls at station level. Fast changing interferer nulling directions would lead to fast changes in the (sidelobe) gains and this would hamper the calibration.

Flagging or excising can best be done at a central level because interference often is localized. This means that interference may be present in one or a few stations, but not visible in an interferometer output. Flagging or excising at station level therefore often would remove too much data.

The reduction from 'Cas.A' level down to sky image noise levels, step three, should in principle not be too different from removing sources such as Cas.A. In addition to selfcal, other methods could be used as well [13, 18]. Long term and short term stationarity issues as well as estimation biases [14, 21, 25] are relevant here and need careful consideration.

Two additional effects help reduce the observed interference. The first is spectral dilution. Suppose that in a LOFAR band, a single narrowband RFI source is present. The energy of this RFI source is diluted by averaging all $N_f$ frequency channels in the band. This leads to a spectral dilution factor which scales with $\sqrt{N_f}$. For wideband signals such as CDMA, this obviously does not apply.

A second effect which reduces interference is spatial dilution. In the aperture synthesis mode, snapshot images are made, and are integrated to form the integrated map. Sky sources move with respect to the baseline vectors during an observation. The interferers on the other hand do not move, or move differently. This means that in the integration process the sky sources remain at the same sky positions and are 'added'. The RFI sources move with respect to the sky and are therefore 'diluted'. The dilution for an RFI point source is comparable to the dilution of a narrow band signal due to frequency averaging: the system noise decreases with the square root of number of snapshots whereas the RFI is reduced (as it moves around the map) by the number of snapshots. This means that if a point source is reduced to the station noise level, it will be reduced to below the integrated noise level by further integration (snapshot averaging). The spatial dilution is the two dimensional variant of fringe rotation, observed in interferometers.

Based on the analysis above and on initial observations with ITS, it seems feasible that, even in densely populated regions such as the Netherlands, the station sky noise level can be reached for certain frequency ranges. Using selfcal type approaches, the remaining RFI can be processed analogously to sources like Cas.A, and be further reduced to levels at or below the integrated sky noise.

4 Data model

4.1 Datamodel

In this section a single polarization point source telescope signal model is described [15]. This model includes a description of astronomical sources, additive interfering signals and noise. Assume that there are $p$ telescope antennas, and suppose that the antenna signals $x_i(t)$ are composed of $q_e$ astronomical source signals, $q_i$ interfering sources, and noise. Let the telescope output signals $x_i(t)$ be stacked in a vector $x(t)$

$$x(t) = (x_1(t), x_2(t), \ldots, x_p(t))^t$$

(3)

Further let $x_{a\ell}(t)$ be the telescope array output signal corresponding to the $\ell^{th}$ astronomical source in the direction $s_{a\ell}$, let $x_{i\ell}(t)$ be the telescope array output signal corresponding to the $i^{th}$ interfering source in the direction $s_{i\ell}$, and let $x^{n}(t)$ be the noise vector. The resulting array output signal then can be expressed by

$$x(t) = \sum_{\ell=1}^{q_e} x_{a\ell}(t) + \sum_{k=1}^{q_i} x_{i\ell}(t) + x^{n}(t)$$

(4)

The noise $x^{n}(t)$ is independent identically distributed (i.i.d.) Gaussian noise, so it is uncorrelated between the array elements, or in other words spatially white at the aperture plane. The astronomical source
signals also are assumed to be identically distributed Gaussian noise signals. The sources are assumed to be independent, or in other words spatially white at the celestial sphere. Finally, the interfering sources are assumed to be quasi stationary, Gaussian noise signals.

For the LOFAR ITS telescope, the narrow band condition

$$\Delta f \ll \frac{1}{2\pi \tau} \quad (5)$$

holds, which means that geometric delays can be represented by phase shifts. Here \( \Delta f \) is the frequency resolution, which is 10 kHz for the observations discussed in this paper. The maximum geometric time delay across the array \( \tau \), is determined by the array size (200 m) and look direction.

Assume that there is an interferer with index \( k \), with signal \( y_k^\ell(t) \). Because the narrow band condition holds, the telescope output signal \( x_k^\ell(t) \) can be written in terms of the array response vector \( a_k^\ell \). Let the array response vector \( a_k^\ell \) be defined by

$$a_k^\ell \equiv \begin{pmatrix} a_{r_1}^\ell e^{2\pi j \frac{\lambda}{\lambda_s} (b_{10} s_k^\ell)} \\ \vdots \\ a_{r_p}^\ell e^{2\pi j \frac{\lambda}{\lambda_s} (b_{p0} s_k^\ell)} \end{pmatrix} \quad (6)$$

where \( b_{i0} \) is the location of the \( i^{th} \) antenna with respect to an arbitrary reference location, \( \lambda \) the wavelength of the impinging signal, and \( a_r^\ell \) are the antenna gains in the direction \( s_k^\ell \). This yields

$$x_k^\ell(t) = a_k^\ell y_k^\ell(t) \quad (7)$$

Let the antenna directional gain vector \( a_k^\ell \) be defined by \( a_k^\ell = (a_{r_1}^\ell, \ldots, a_{r_p}^\ell)^T \), and define \( \mathcal{R} = (b_{10}, \ldots, b_{p0})^T \), then \( a_k^\ell \) can be compactly written as

$$a_k^\ell = a_k^\ell \odot e^{2\pi j \mathcal{R} s_k^\ell} \quad (8)$$

Here the vector \( e^{2\pi j \mathcal{R} s_k^\ell} \) represents the geometrical delay (phase) vector for the telescope antenna locations \( \mathcal{R} \) and the source direction \( s_k^\ell \).

Assume there are \( q_r \) interferers, and define \( x^r(t) \) by \( x^r(t) = \sum_k x_k^r(t) \), which also can be written as

$$x^r(t) = \sum_{k=1}^{q_r} a_k^r y_k^r(t) \quad (9)$$

The interferer signal power \( \sigma_k^2 \) is given by \( \mathbb{E} \{ y_k^r(t) y_k^r(t) \} = \sigma_k^2 \), which lead to the following expression for the interference array covariance matrix \( \mathbf{R}_r \), dropping the time index \( t \) for \( \mathbf{R} \):

$$\mathbf{R}_r = \mathbb{E} \{ x^r(t) x^r(t)^H \} = \sum_{k=1}^{q_r} \sigma_k^2 a_k^r (a_k^r)^H \quad (10)$$

The array response vector and the covariance matrix \( \mathbf{R}_r \) for astronomical sources can be expressed in a similar way. Concerning the system noise, it can be represented by a diagonal noise matrix \( \mathbf{D} \), and is given by

$$\mathbf{D} = \mathbb{E} \{ x^n(t) x^n(t)^H \} = \text{diag}(\sigma_1^2, \ldots, \sigma_p^2) \quad (11)$$

where the \( \sigma_i^2 \) is the noise power of the \( i^{th} \) antenna without source or interferer contributions. The covariance matrix

$$\mathbf{R} = \mathbb{E} \{ x(t) x(t)^H \} \quad (12)$$

can be expressed as

$$\mathbf{R} = \mathbf{R}_r + \mathbf{R}_n + \mathbf{D} = \sum_{k=1}^{q_r} \sigma_k^2 a_k^r (a_k^r)^H + \sum_{k=1}^{q_r} \sigma_k^2 a_k^n (a_k^n)^H + \mathbf{D} \quad (13)$$
Let $\mathbf{A}_r$ be defined by stacking the array response vectors for the interferers in a matrix

$$\mathbf{A}_r = [\mathbf{a}_1^\top, \cdots, \mathbf{a}_p^\top]$$

and stack the interfering source powers in a diagonal matrix $\mathbf{B}_r$. For the astronomical sources the same definitions for $\mathbf{A}_s$ and $\mathbf{B}_s$ can be made. Using these definitions, the covariance matrix $\mathbf{R}$ can be expressed in a more compact form:

$$\mathbf{R} = \mathbf{A}_r \mathbf{B}_r \mathbf{A}_r^H + \mathbf{A}_s \mathbf{B}_s \mathbf{A}_s^H + \mathbf{D}$$

(15)

### 4.2 Imaging and beamforming

Traditionally [19], the synthesized sky images are generated by fourier transforming the correlation data, here represented by the covariance matrix $\mathbf{R}$. For the ITS station, the observed snapshots contain only a fairly limited number of spatial sample points. This implies that making sky images with the ITS station using beamforming is more practical than using spatial fourier transforms. Therefore the beamforming approach, used for ITS imaging, is discussed next.

Assume that the complex gain of the array antenna elements can be adjusted by a multiplicative complex weight number $w_i$ for each of the antenna elements $i$. Given the array output signal vector $\mathbf{x}(t)$ and a weight vector for the array $\mathbf{w} = (w_1, \cdots, w_p)^T$, then the weighted-summed array output signal $y(t)$ is given by

$$y(t) = \mathbf{w}^H \mathbf{x}(t)$$

(16)

The beamformer output power $P$ then is given by

$$P = \mathbb{E} \{ y(t) y(t)^H \} = \mathbf{w}^H \mathbb{E} \{ x(t) x(t)^H \} \mathbf{w} = \mathbf{w}^H \mathbf{R} \mathbf{w}$$

(17)

For a classical or Capon beamformer we can define the weight vector, in terms of sky direction cosine coordinates $(l, m)$:

$$\mathbf{w}^H(l, m) = \mathbf{a}^H(l, m)$$

(18)

where $\mathbf{a}(l, m)$ is the array response to signals from direction $(l, m)$. The classical beamformer is equivalent to direct fourier transform or taking the fourier transform of all $u,v$ data points without weighting. The sidelobe pattern of this beamformer is shown in figure 3.

The weights of the classical beamformer are independent of the data. The image quality can be improved by using a data dependent beamformer. MVDR beamforming gives a significant suppression of the sidelobes compared to classical beamforming. The MVDR beamformer minimizes the output power under the constraint that the gain in the desired direction remains unity.

$$\mathbf{w}_{\text{MVDR}}(l, m) = \arg \min_{\mathbf{w}} \mathbf{w}(l, m)^H \mathbf{R} \mathbf{w}(l, m)$$

(19)

with the constraint

$$\mathbf{w}_{\text{MVDR}}(l, m)^H \mathbf{a}(l, m) = 1$$

(20)

The solution to this equation can be found using Lagrange multipliers, and is given by

$$\mathbf{w}_{\text{MVDR}}(l, m) = \frac{\mathbf{R}^{-1} \mathbf{a}(l, m)}{\mathbf{a}(l, m)^H \mathbf{R}^{-1} \mathbf{a}(l, m)}$$

(21)

The measured intensity is given by

$$I(l, m) = \frac{1}{\mathbf{a}(l, m)^H \mathbf{R}^{-1} \mathbf{a}(l, m)}$$

(22)

The MVDR beamformer is known to be sensitive to array calibration errors, leading to errors in the beam gain. More robust versions of MVDR exist such as robust Capon beamforming [23], but these are not discussed in detail here.
5 Measurement results

5.1 ITS test station

The LOFAR ITS test station is located in the north-east of the Netherlands. It is a sky noise limited antenna array station consisting of \( p = 60 \) linearly polarized antennas which are grouped in five spiral arms. Each of the arms contain 12 single polarization inverted-v dipoles with a resonance frequency of 34 MHz as can be seen in figure 4. The dipoles are oriented in an east-west direction. The shortest antenna distance is 5 m, which corresponds to \( \frac{1}{2} \lambda \) at 30 MHz. The diameter of the station is 200 m. The geometrical layout of the antenna locations is given in figure 3 (left figure). A snapshot correlation measurement combines each of the antennas with all the others, yielding \( p^2 \) interferometer products. Each interferometer product corresponds to a certain telescope distance and direction, called baseline. The middle figure shows the snapshot baseline configuration in a righthanded coordinate (u,v,w) system. This figure shows the way in which the aperture is spatially sampled. Combining snapshot images will gradually fill the open spaces because the earth rotation changes the relative antenna positions with respect to the sky. The right figure shows the phased array beamshape at 30 MHz for the zenith direction. The beamwidth at this frequency is 5.5°. The coordinates are direction cosines (1,m). The north is defined in the positive m-direction, the east in the positive l-direction.

![ITS, antenna locations](image1)

![ITS, snapshot uv configuration](image2)

![ITS beamshape](image3)

Figure 3: LOFAR Station ITS antenna configuration (left), station snapshot uv coverage (middle), and station beamshape (right).

The antenna outputs are connected to low noise amplifiers, filtered by a 10-35 MHz filter, and digitized with a sampling frequency of 80 MHz. For the experiments described in this paper, a 8192 sample length, hanning tapered, FFT was used. This yields a frequency resolution of 9.77 kHz. The spectra were correlated and integrated to 6.7 seconds, yielding a 60 \( \times \) 60 covariance matrix for each of the 4096 frequency bins. Figure 4 shows 12 observed interferometer spectra, of the inner antenna of one of the arms, correlated with all antennas in the same arm. Disconnecting an antenna and placing a matched load instead, reduces the observed autocorrelation power \( r_{11} \) by \( \approx 75 \% \), indicating that the sky noise is the largest contributor to the system noise. Further proof is given in [26], where it is shown that the noise in the observed snapshot images is dominated by the sky noise. The baseline \( \mathbf{b}_{10} - \mathbf{b}_{20} \) is short, it is \( \frac{1}{2} \lambda \) at 30 MHz.

The figure also shows that the magnitudes of \( r_{12}, r_{13}, \) and \( r_{14} \) are higher than those of the longer baselines. This is caused by the astronomical extended sources in the sky, a well known effect in aperture synthesis [12, 22, 19]. As the antennas closest to the centre of the ITS station (\( \mathbf{b}_{10} \) and \( \mathbf{b}_{20} \)) are only at a \( \frac{1}{2} \lambda \) distance at 30 MHz, there will be mutual coupling. This means that part of the sky and receiver noise current in an antenna is coupled to the other elements. In addition there could be electronic coupling between receiver boxes, cabling etcetera. Crosstalk would be best visible on the shortest antenna spacings, making it difficult to distinguish it from extended astronomical sources. Method of moment antenna simulations show however, that the crosstalk fraction of the dipole at 40 MHz is -20 dB, and decreases to -43 dB at 30 MHz and -65 dB at 20 MHz. This implies that for most data processing applications, the crosstalk can be ignored.

The observed spectrum shows that at nighttime, a large fraction of the 15-35 MHz band is sky noise
limited so LOFAR observations could be carried out in those frequency slots. In daytime, the spectrum is more densely occupied, a inventory of the occupancy statistics at ITS is currently being carried out. The spectrum shows harmonics and intermodulation products of strong transmitters at 12 MHz. These signals appear at 24 and 36 MHz. As will be shown in the next sections, these harmonics and intermodulation products can be suppressed in the same way as "ordinary" transmitters.

5.2 Spatial filtering

Spatial filtering is demonstrated by applying projection filters and subtraction filters to the observed covariance matrices \( \hat{R} \). These matrices \( \hat{R} \) are sample estimates of \( R \) and are constructed by

\[
\hat{R} = \frac{1}{N} \sum_{n=1}^{N} x_n x_n^H
\]  

(23)

Here \( x_n \) is the array signal output vector at time index \( n \); this index replaces the time variable \( t \) in equation (4). We assume that model (15) is valid. As we are investigating the suppression of relatively strong interferes, we assume that the interferer power \( \sigma_i^2 \) is much larger than the astronomical sources power \( \sigma_s^2 \). We further assume that the noise power matrix \( D \) contains the spatially white sky noise which is dominant in strength. Now \( D \) is initially unknown but it can be estimated for example with factor analysis approaches [16, 4, 3]. Once \( D \) is estimated, the matrix \( \hat{R} \) can be whitened, for example by pre and post multiplying it with \( D^{-\frac{1}{2}} \). The whitening process also affects \( \hat{R}_s \), but this can be corrected for after the filtering. Consider now the following simplified whitened model

\[
R = A_s B_s A_s^H + R_s + \sigma_n^2 I
\]  

(24)

where we assume that the interferer power is dominant. This model will be used further to explain the working of the spatial filters.

Projection filtering

The covariance matrix \( R \) can be filtered using projection matrices [15]. As before, let \( A_{k_s}^\prime \) be a matrix containing the interferer array response vectors \( a_{k_s}^\prime \), and assume that \( a_{k_s}^\prime \) is known. Define the projection

Figure 4: Night time ITS test station spectra of interferometers \( r_{ij} \), with \( j = 1 \cdots 12 \), and with \( \Delta f = 9.77 \text{ kHz} \).
matrix $\mathbf{P}$ by

$$
\mathbf{P} = \mathbf{I} - \mathbf{A}_r (\mathbf{A}_r^H \mathbf{A}_r)^{-1} \mathbf{A}_r^H
$$

(25)

Because $\mathbf{P} \mathbf{A}_r = \mathbf{0}$, pre and post multiplying $\mathbf{R}$ with $\mathbf{P}$ yields for the filtered covariance matrix $\mathbf{R}_\perp$:

$$
\begin{align*}
\mathbf{R}_\perp &= \mathbf{P} \hat{\mathbf{R}} \mathbf{P} \\
\mathcal{E}\{\mathbf{R}_\perp\} &= \mathbf{P} (\mathbf{R}_* + \sigma_n^2 \mathbf{I}) \mathbf{P}
\end{align*}
$$

(26)

(27)

The interference is removed, but the astronomical sources are distorted by the filter. This distortion can be removed by approaches such as described in [15, 21, 25]. These approaches depend (a.o.) on the telescope interferometer "fringe rate" and stationarity properties, which are required to be constant in short time intervals and varying over long time intervals. The equivalent requirement for ITS is that when snapshots are averaged, the interference spatial sidelobes vary over long timescales with respect to the (moving) sky sources.

**Subtraction filtering**

An alternative filtering method is interference subtraction. With known $\sigma_n^2$, $\mathbf{B}_r$, and $\mathbf{A}_r$, or their estimates, the contribution of the interferer can be reduced by subtracting it from the observed covariance matrix. Let $\mathbf{R}_-$ be the filtered covariance matrix, i.e. the observed covariance matrix with the estimated interference removed by subtraction, then:

$$
\begin{align*}
\mathbf{R}_- &= \hat{\mathbf{R}} - \mathbf{A}_r \mathbf{B}_r \mathbf{A}_r^H \\
\mathcal{E}\{\mathbf{R}_-\} &= \mathbf{R}_* + \sigma_n^2 \mathbf{I}
\end{align*}
$$

(28)

(29)

**Attenuation limits and subspace analysis**

When the spatial signature of the interferers and its power is unknown, it can be estimated by an eigenanalysis of the sample covariance matrix $\hat{\mathbf{R}}$. Due to limits in the estimation accuracies, both filter types will have estimation errors and also may be biased [13]. In some case the bias can be corrected for [25]. These estimation errors will not be discussed in detail here. Now we will briefly describe how to estimate the interferer parameters using a subspace analysis. The covariance matrix $\mathbf{R}$ can be written in terms of eigenvalues and eigenvectors as [15]

$$
\mathbf{R} = \mathbf{U} \Lambda \mathbf{U}^H
$$

(30)

where $\mathbf{U}$ is a unitary matrices containing the eigenvectors, and $\Lambda$ is a diagonal matrices containing the eigenvalues. Assuming that the astronomical contribution (apart from the extended, spatially white, emission) is small so that it can be ignored, the eigenvalue decomposition can be expressed as

$$
\mathbf{R} = \begin{bmatrix} \mathbf{U}_r & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \Lambda_r + \sigma_n^2 \mathbf{I}_{q_r} & 0 \\ 0 & \sigma_n^2 \mathbf{I}_{p-q_r} \end{bmatrix} \begin{bmatrix} \mathbf{U}_r^H \\ \mathbf{U}_n^H \end{bmatrix}
$$

where $\mathbf{U}_r$ is a $p \times q_r$ matrix, containing the eigenvectors corresponding to the $q_r$ eigenvalues in $\Lambda_r$. $\mathbf{U}_n$ is a $p \times p$ matrix containing the eigenvectors, corresponding to the noise subspace. Given a matrix $\mathbf{R}$, the signal subspace can be found by applying a singular value decomposition to $\mathbf{R}$. Note that the signal subspace and the noise subspace span the entire space, $\mathbf{U} = [\mathbf{U}_r \mathbf{U}_n]$, and $\mathbf{U}$ is unitary: $\mathbf{UU}^H = \mathbf{U}_r \mathbf{U}_r^H + \mathbf{U}_n \mathbf{U}_n^H = \mathbf{I}_p$. Without further knowledge, the best estimate of $\mathbf{A}_r$ is the dominant eigenspace $\mathbf{U}_r$ of $\hat{\mathbf{R}}$, and likewise the best estimate of the interferer powers $\mathbf{B}_r$ is $\Lambda_r - \sigma_n^2 \mathbf{I}$.

**Experimental results**

Figure 5 shows the eigenvalue structure of $\hat{\mathbf{R}}$ from the nighttime observation discussed earlier. The eigenvalue decomposition was applied after a whitening step. The largest eigenvalue of the observed transmitter at 25.752 MHz lies 20 dB above the remaining eigenvalues. This means that the observed transmitter power lies 20 dB above the sky noise level, and that the transmitter occupies (for at least 99% of its power) only one dimension of the subspace of $\hat{\mathbf{R}}$. Therefore we have chosen to base the spatial filters on one direction vector only, namely the one corresponding to the largest eigenvalue.
Figure 5: Eigenvalue distribution, for $N_{sam} = 131000$, $\Delta t = 6.7$ s, $\Delta f = 9.77$, kHz, measured on February 26, 2004.

Figure 6: Spatial filtering at LOFAR ITS test station. Snapshot image without interference at 26.89 MHz (upper left), snapshot image with a transmitter at 26.75 MHz (upper right), image with transmission removed by spatial filtering using a projection filter (lower left) and subtraction filter (lower right).
Figure 6, upper left, shows the results of the beamformer scan over the entire sky of the dataset of February 26, 2004, at 03:50 MET. Shown is the sky at a single frequency bin at 26.89 MHz; no interference or transmitters were detected. Clearly visible are the astronomical sources Cas.A (right), and Cyg.A (upper). An extended structure, the ”north galactic spur” is just visible as a band from \((m,l)=(-0.6,-0.2)\) to \((m,l)=(-0.3,0.9)\). The figure to the right is a sky image from the same data set, but now at 26.75 MHz in which a transmitter was detected. It is visible at the horizon at \((l,m)=(0.45,0.9)\). It is 20 dB above the noise, cf. figure 5, and its sidelobes spread around the map and obscures the astronomical sources.

In the lower left figure, the same map is shown, but now it is improved by applying a projection filter. In this experiment, the distortion correction, discussed earlier, was not applied. Clearly, sidelobe structure residuals of the uncorrected projection filter distorts the map more than the subtraction filter which is shown in the lower right figure. This however does not imply that subtraction filters are better than projection filters (which to are large extent are similar mathematical operations [15]), as the projection filter was uncorrected. The point here is that with spatial filtering, we can attenuate a transmitter 20 dB above the sky noise to levels below the Cas.A flux level. At 18.92 MHz we showed (not displayed here), the same for a transmitter 30 dB above the sky noise level. The residual transmitter sidelobes were also suppressed to levels below Cas.A.

**Detection of RFI using eigenvalue decomposition**

As a further illustration of the relation between the eigenstructure of the observed covariance matrices and the number of detectable interfering sources, we show two examples in figure 7. The upper figures show an eigenvalue distribution with one dominant largest eigenvalue and one dominant source in accompanying the map. The lower figures show three dominant eigenvalues and three interfering sources in the map.

![Eigenvalue decomposition](image)

Figure 7: Eigenvalue decomposition of covariance matrices and celestial maps from the LOFAR initial tests station. The upper row shows observations at 27.800 MHz, the lower row corresponds to 27.096 MHz. There is a clear correlation between the number of observed strong sources and the number of large eigenvalues.

### 5.3 Intermodulation Products

The purpose of the following analysis is to show that intermodulation products appear as additional point sources in the map. The consequence of this is that these sources can be mitigated just like ordinary
sources and that the spatial dilution is also applicable for these sources.

Intermodulation products are caused by nonlinearities in the receiver, often caused by high power transmitter signals distorting the low noise amplifier linearity. Assume that the transmitter signals are semi stationary. For a simple second order model of the amplifier and given input signal \( x(t) \), the output \( y(t) \) is given by

\[
y(t) = \beta_1 x(t) + \beta_2 x^2(t)
\]  

(31)

where \( \beta_1 \) and \( \beta_2 \) are two real parameters describing the (non)linearity behaviour. Let us consider the scenario where the input consists of the sum of two cosines with different amplitude \( (\alpha_1, \alpha_2) \), frequency \( (f_1, f_2) \) and phase \( (\theta_1, \theta_2) \)

\[
x(t) = \alpha_1 \cos(2\pi f_1 t + \theta_1) + \alpha_2 \cos(2\pi f_2 t + \theta_2),
\]  

(32)

the output of the amplifier is then given by

\[
y(t) = \beta_1 \alpha_1 \cos(2\pi f_1 t + \theta_1) + \beta_1 \alpha_2 \cos(2\pi f_2 t + \theta_2) + \\
+ \beta_2 \alpha_1^2/2(1 + \cos(2\pi f_1 t + 2\theta_1)) + \\
+ \beta_2 \alpha_2^2/2(1 + \cos(2\pi f_2 t + 2\theta_2)) + \\
+ \beta_2 \alpha_1 \alpha_2 \cos(2\pi(f_1 + f_2) + \theta_1 + \theta_2) + \\
+ \beta_2 \alpha_1 \alpha_2 \cos(2\pi(f_1 - f_2) + \theta_1 - \theta_2)
\]

The first two terms are wanted, the last four are intermodulation products.

Now consider two cosine signals impinging on an array of antennas. The sum of these two cosines can be modeled as

\[
x(t) = \alpha_1 \cos(2\pi f_1 t + \theta_1) + \alpha_2 \cos(2\pi f_2 t + \theta_2)
\]  

(33)

where \( \alpha_k \) is the vector containing the real signal amplitudes, and \( \theta_k \) is the antenna phase vector of the \( k^{th} \) transmitter. The phase vector \( \theta_k \) can be expressed in terms of geometric telescope positions \( \mathcal{R} = (\mathbf{b}_{10}, \cdots, \mathbf{b}_{10})^t \), wavelength \( \lambda_k \) and the source direction \( \mathbf{s}_k \) of the \( k^{th} \) transmitter

\[
\theta_k = \frac{2\pi}{\lambda_k} \mathcal{R} \mathbf{s}_k
\]  

(34)

The transmitter source direction vector \( \mathbf{s}_k \) is a unit norm vector.

\[
\mathbf{s}_k \equiv \begin{bmatrix} l \\ m \\ n \end{bmatrix}
\]  

(35)

To specify a location only two coordinates \( (l, m) \) are necessary, the third coordinate is chosen such that the vector unit norm. For a planar array in the x,y-plane the z coordinate of the antenna positions is zero. All items of the third column of \( \mathcal{R} \) are zero, which means that the phase \( \theta \) is independent of the third component of \( \mathbf{s}_k \).

The sum of two cosines with frequencies \( f_1 \) and \( f_2 \) at the input gives the sum of six cosines with frequencies \( f_1, f_2, 2f_1, 2f_2, f_1 + f_2 \) and \( f_1 - f_2 \). Let us consider the intermodulation response \( y_i(t) \) at \( f_{12} = f_1 + f_2 \) in more detail

\[
y_i(t) = \beta_2 \alpha_1 \alpha_2 \cos(2\pi (f_1 + f_2) t 1 + \theta_1 + \theta_2)
\]  

(36)

where \( \beta_2 \) is the vector containing the second order nonlinearity parameters for each of the antennas. The sum of the phases \( \theta_1 + \theta_2 \) can be expressed by

\[
\theta_1 + \theta_2 = \frac{2\pi}{\lambda_1} \mathcal{R} \mathbf{s}_1 + \frac{2\pi}{\lambda_2} \mathcal{R} \mathbf{s}_2 = 2\pi \mathcal{R} \left( \frac{\mathbf{s}_1}{\lambda_1} + \frac{\mathbf{s}_2}{\lambda_2} \right)
\]  

(37)
Table 1: Predicted directions in \((l,m)\) coordinates of transmitters and their intermodulation products as they will appear in celestial maps.

Suppose there exists a real source (i.e., not an intermodulation product) at frequency \(f_{12} = f_1 + f_2\), or wavelength \(\lambda_{12}\)

\[
\lambda_{12} = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}
\]  

and suppose this source has a direction given by

\[
s_{12} = \frac{\lambda_2 s_1 + \lambda_1 s_2}{\lambda_1 + \lambda_1}
\]

Then this source will have the following phases

\[
\theta = \frac{2\pi}{\lambda_{12}} \mathcal{R}s_{12} = 2\pi \mathcal{R} \frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} \frac{\lambda_2 s_1 + \lambda_1 s_2}{\lambda_1 + \lambda_1} = 2\pi \mathcal{R} \left( \frac{s_1}{\lambda_1} + \frac{s_2}{\lambda_2} \right)
\]

These phases are equal to the phases of the intermodulation product described earlier, which means that the intermodulation product will appear as a point source in the map in the weighted direction \(s_{12}\). The direction vector \(s_{12}\) is not unit norm, but there does exist a vector with the same \((l, m)\) and a different \(n\) coordinate which is unit norm. Since for a planar array the phases do not depend on the \(n\) coordinate, a signal from this direction has the same phases as the intermodulation product. The absolute value of the spatial signature is different.

So we can conclude that intermodulation products appear as additional sources in the image at predictable positions as given in Table 1. As the nonlinearity variation over the array differs from the antenna (sidelobe) gain variation over the array, in principle we can distinguish intermodulation products from real sources. Figure 8 shows an ITS observation with strong interferers at 11.77 MHz and 11.86 MHz. The intermodulation product consisting of the sum of the two signals appears exactly at the predicted location, indicated by the white cross.
5.4 MVDR and robust Capon beamforming (RCB)

Figure 9: Celestial daytime maps obtained with the LOFAR test station using the MVDR beamformer and the Classical beamformer. The upper row shows Cas.A and a transmitter at the horizon at 11.86 MHz and 9.77 kHz bandwidth. The lower figure shows the same results, but at a nearby frequency with a more dominant transmitter.

Correction of spatially filtered data in the sky maps can be implemented using space varying beams [14]. MVDR and robust Capon beamformers can be extended to include such operations. Figure 9 shows illustrations of the use of a classical beamformer and an MVDR beamformer to produce "dirty" images. The upper figures show that an MVDR beamformer has a much sharper beam compared to the classical beamformer, and a much smoother sidelobe structure. In this daytime observation, the classical beamformer does not reveal the strong source Cas.A; the MVDR beamformer does show the source. A drawback of MVDR is that calibration errors can cause the MVDR beamformer to underestimate the power. Especially the higher peaks can be strongly diminished by this effect, resulting in a lower dynamic range. A clear example of this effect is shown in the lower two figures. The difference in dynamic range is more than 20 dB. In literature methods have been proposed to improve the performance of the MVDR beamformer for arrays with imperfect calibration. We have chosen a the Robust Capon Beamforming method of Li, Stoica and Wang as proposed in [23]. The result of this method is shown in figure 10. The intensity scaling (array/beam gain) is restored. The observed beamwidth is very small which suggests that robust capon beamforming could enhance some of the calibration approaches in the astronomical imaging process. This however needs further study.
6 Conclusions

In this paper we have demonstrated spatial filtering capabilities at the LOFAR initial test station (ITS), and have related it to the LOFAR RFI mitigation strategy. We have shown that with ITS, in frequency ranges which are occupied with moderate intensity interfering signals, the strongest astronomical sky sources can be recovered by spatial filtering. The same selfcal and CLEAN approaches which remove the sidelobe structures of the strongest sources such as Cas.A, can in principle also be used to mitigate the interference further. The spatial dilution effect helps reducing the interference. We also have shown that, under certain conditions, intermodulation products originating from point sources, remain point sources and can be attenuated with the same spatial filtering techniques as non-intermodulation interference. We have shown and verified experimentally that even the direction of an intermodulation product can be predicted. Finally, we have demonstrate the use of several beamformer types for ITS.

References


