Another algorithm for phased array null formation.

L. Kogan
National Radio Astronomy Observatory, Socorro, NM 87801  

Abstract

Recently several algorithms for creating null at the phased array beam pattern were designed. See for example ATA memos of Geoffrey Bower ([1]) and G.R. Harp ([2]). The phased array beam pattern null towards the RFI source can mitigate effect of the RFI. Another algorithm for creating null at the phased array beam pattern is described at this memo. This algorithm is based on modification of my algorithm of minimizing side lobes of an array ([3]). We tested this algorithm for possible SKA antenna station as a phased array of 19 antennas and for VLA-D working as a phased array for VLBI experiments. The results are very similar to the simulation described by G. Bower and G.R Harp. The null can be created at one direction and at the square around the given direction. The phase and amplitude correction to create the null has to be given with the very high accuracy which may be not practical. Rounding phase correction to 1 degree aggravates the null from > 100db to ~ 50db. The narrow null (~ 0.1°) at the level > 100db can be easily created at the cost of losing signal to noise ratio at the main direction at the level of 3%. Widening the null to 0.5° leads to null level ~ 40db and losing 13% of signal to noise ratio. The depth of the null decreases dramatically when the required phase/weight correction is rounded to 1 degrees at phase and 0.01 at weight.

1 Description of the algorithm

The beam power pattern of a phased array can be described by the following expressions:

\[ BEAM(\vec{e}) = \sum_{n=1}^{N} w_n \exp(j(2\pi\vec{r}_n(\vec{e} - \vec{e}_0) + \phi_n) \right]^2 = \sum_{i=1}^{N} \sum_{k=1}^{N} w_i w_k \cos(2\pi(\vec{r}_i - \vec{r}_k)(\vec{e} - \vec{e}_0) + \phi_i - \phi_k) \]  

where \( \vec{e} \) is vector directed to the point at the sky;
\( \vec{e}_0 \) is vector towards the target source;
\( \vec{r}_n \) are vectors determining the position of antennas \( n \);
\( N \) is number of antennas in the array;
\( w_n, \phi_n \) are the weights and phases we want to add for each antenna to create the null at the vicinity of \( \vec{e} \).

It is supposed in the equation 1 that the array is ideally phased at the direction of vector \( \vec{e}_0 \). So the value of \( BEAM(\vec{e}) \) at the direction of the target source (\( \vec{e} = \vec{e}_0 \)) depends only on the added weights and phases \( w_n, \phi_n \). Positions of the antennas are given at portion of the array maximum baseline \( D \) and the component of vector \( \vec{e} \) at the array plane is given at \( \frac{\vec{e}}{D} \).

The derivatives of BEAM by phase \( \phi_k \) and by \( w_k \) are given by the following expressions:

\[ \frac{dBEAM}{d\phi_k} = \sum_{i=1}^{N} w_i w_k \sin(2\pi(\vec{r}_i - \vec{r}_k)(\vec{e} - \vec{e}_0) + \phi_i - \phi_k) \]  

\[ \frac{dBEAM}{dw_k} = \sum_{i=1}^{N} w_i \cos(2\pi(\vec{r}_i - \vec{r}_k)(\vec{e} - \vec{e}_0) + \phi_i - \phi_k) \]  

\(^1\)The National Radio Astronomy Observatory is a facility of the National Science Foundation, operated under cooperative agreement by Associated Universities, Inc.
If we want to minimize the value of BEAM at the direction \( \hat{e} \) we need to change the value of \( \phi_k \) and by \( w_k \) proportionally to the relevant derivatives. We used this statement for optimization of an array configuration minimizing side lobes ([3]). Now the same statement is used to find the correction the phase and weight of each antenna to create the null at the vicinity of the given direction in the sky. The creating of null is multi iteration process. At each iteration the algorithm changes the phase and/or weight on a small portion (GAIN controlled by user) of the recommended value. The solution requires minimum of time(seconds on my SUN station). The task (written in AIPS) can create two or one null but it is possible to increase the number of nulls.

It is clear that creating the nulls by adding phases and weights to each antenna decreases the signal to noise ratio observing the target source by the main lobe of the array (direction \( \hat{e}_0 \)). The reduction of the signal to noise ratio (SNR) can be calculated by the following expression:

\[
SNR = \left( \frac{\sum_{k=1}^{N} w_k \cos(\phi_k)}{\sqrt{N \sum_{k=1}^{N} w_k^2}} \right)^2
\]

If \( w_k = 1 \) for all \( k \) and \( \phi_k = 0 \) for all \( k \), then \( SNR = 1 \); else \( SNR < 1 \).

Position of the null and its width is determined (at this algorithm) by the number of the synthesize beams - ratios \( \frac{\Delta \lambda}{D} \), where \( D \) is the array size (maximum baseline). So if the position of the null and its width is determined as \( \theta = \alpha \frac{\Delta \lambda}{D}, \Delta \theta = \Delta \alpha \frac{\Delta \lambda}{D} \), then the width of the null at the frequency axis is determined as \( \Delta \theta = \alpha \frac{\Delta \lambda}{D} \). Therefore the bandwidth of the null can be found as:

\[
\frac{\Delta \alpha}{\alpha} = \frac{\Delta \lambda}{\lambda} = \frac{\Delta f}{f}
\]

It is clear from the equation (5) that the far null area locates of the main beam the lower bandwidth is.

2 SKA antenna station

We tested this algorithm for possible SKA antenna station as a phased array of 19 antennas, the array size \( \sim 100m \) (Figure 2). We found that the depth of the null decreases dramatically when the implemented phase/weight correction is rounded to 1 degrees at phase and 0.01 at weight. The synthesized beam of the SKA antenna station is 410" (\( \lambda = 20cm, D=100m \)). Consider the null is located at 10 degrees from the main lobe (\( \alpha = 90 \)) and its width at some level is equal 200" (\( \Delta \alpha = 0.5 \)). Then bandwidth at the central frequency 1440 MHz should be equal (equation 5): \( \Delta f = f \cdot \Delta \alpha \alpha = 1440 \cdot 0.5/90 = 9 MHz \). The table 2 shows the parameters of the array pattern at the created null area.

<table>
<thead>
<tr>
<th>null depth</th>
<th>null width</th>
<th>SNR</th>
<th>( \Delta f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>narrow null</td>
<td>124</td>
<td>65</td>
<td>0.984</td>
</tr>
<tr>
<td>phase, weight</td>
<td>47</td>
<td>70</td>
<td>0.985</td>
</tr>
<tr>
<td>wide null</td>
<td>45</td>
<td>200</td>
<td>0.915</td>
</tr>
</tbody>
</table>

Phase/weight correction is rounded to 1 degrees at phase and 0.01 at weight for the second row of the table. \( f = 1440 MHz; \frac{\Delta \alpha}{\alpha}=400" \)

Figure 1 shows the effect of rounding of weights and phase on the beam pattern near the null centered at 40 degrees apart of the main lobe. Phases/weights are rounded to 1 degrees and 0.01 respectively. The depth of the null drops down from 123db to 49db when rounding. The width of the null at the level of 30db stays the same \( \sim 40\text{arcsec} \).
Figure 1: Cross section of the two dimensional narrow null area. The null center is 40 degrees apart of the main lobe. The blue plot corresponds to the weight and phase solution with six digits after the decimal point. Phases/weights are rounded to 1 degrees and 0.01 respectively for the red curve. Calculations were performed at $\lambda = 20cm$

3 VLA-D phased array

There is a potential source of the RFI for VLA located at the top of Sandia crest at Albuquerque (130 miles of VLA). The group of several communication antennas are located there. We tested the current algorithm for VLA-D used as element of a VLBI experiment, when all 27 VLA antenna are phased together. The array size $\sim 1000m$ (Figure 3). The table 3 shows the parameter of the array pattern at the created null area. The synthesized beam of the VLA-D as an antenna station is 41" ($\lambda = 20cm$, D=1000m). Consider the Sandia crest is located at 10 degrees from the observed source ($\alpha = 900$) and its width at level 30db is equal 8" ($\Delta \alpha = 0.2$). Then bandwidth at the central frequency 1440 MHz should be equal (equation 5): $\Delta f = f \cdot \frac{\Delta \alpha}{\alpha} = 1440 \cdot 0.2/900 \sim 0.3MHz$.

4 Conclusion

1. Too high accuracy of the required phase/weight to get the deep null.

2. The difference in atmosphere delay for different array antennas can exceed the descret of the adding phase and as the result destroy the null.

3. The null deeper than 100db may not be required because of the individual antenna receiver saturation.
Table 2: Parameters of the null at 10 degrees of the main lobe (VLA-D example).

<table>
<thead>
<tr>
<th></th>
<th>null depth db</th>
<th>null width asec(30db)</th>
<th>SNR</th>
<th>Δf MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>narrow null</td>
<td>132</td>
<td>8</td>
<td>0.99</td>
<td>0.3</td>
</tr>
<tr>
<td>phase, weight rounded</td>
<td>56</td>
<td>8</td>
<td>0.97</td>
<td>0.3</td>
</tr>
<tr>
<td>wide null</td>
<td>59</td>
<td>20</td>
<td>0.91</td>
<td>0.8</td>
</tr>
<tr>
<td>wide null</td>
<td>43</td>
<td>40</td>
<td>0.90</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Phase correction is rounded to 1 degree and weight ≡ 1 for the second row of the table. \( \frac{λ}{D} = 400'' \).

References

[1] Bower, Geoffrey C., Simulations of Narrow-Band Phased-Array Null Formation for the ATA, ATA memo #37, 2001


[3] L. Kogan, Mathematic basis of an array configuration optimization minimizing side lobes, EVLA memo #71, 2004
Figure 2: A possible SKA antenna station with 19 antennas.

Figure 3: VLA-D configuration with 27 antennas.