Performance And Control Of Proof-Mass Actuators
Accounting For Stroke Saturation

Douglas K. Lindner
Bradley Department of Electrical Engineering
Virginia Tech
Blacksburg, VA 24061
(540) 231-4580
Fax (540) 231-3362
EMAIL: Lindner@vt.edu

Gregory A. Zvonar
Bradley Department of Electrical Engineering
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061

Dusan Borojevic
Bradley Department of Electrical Engineering
Virginia Polytechnic Institute and State University
Blacksburg, VA 24061
(540) 231-4381
Fax (540) 231-3362
EMAIL: borojevi@vt.edu

AIAA Journal of Guidance, Dynamics, and Control

Abstract

We investigate the relationship between the parameters of a proof-mass actuator and the performance of that actuator within a vibration suppression loop for a flexible structure. We parameterize the model of the actuator in terms of the length of the proof-mass, the mass of the proof-mass, and the saturation force of the electromagnetic subsystem. The performance of the actuator within a vibration suppression control loop is characterized in terms of the relationship between the parameters and the structure's natural frequency. These results can be used to size a proof-mass actuator to maximize the operating region of the actuator and to maximize the ratio of the mass of the proof-mass to the total mass of the actuator.
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>mass of the proof-mass, kg</td>
</tr>
<tr>
<td>d</td>
<td>stroke length, meters</td>
</tr>
<tr>
<td>F_{max}</td>
<td>maximum force available from the electromagnetic subsystem, N</td>
</tr>
<tr>
<td>v(t)</td>
<td>input signal to the power electronics, volts</td>
</tr>
<tr>
<td>f_{pm}(t)</td>
<td>force applied to the proof-mass by the electromagnetic subsystem, N</td>
</tr>
<tr>
<td>y_{pm}(t)</td>
<td>displacement of the proof-mass, meters</td>
</tr>
<tr>
<td>y_{r}(t)</td>
<td>relative displacement of the proof-mass from the structure, meters</td>
</tr>
<tr>
<td>r_{pm}(t)</td>
<td>reference input to the actuator control system, Volts</td>
</tr>
<tr>
<td>K_{pa}</td>
<td>forward path gain of the actuator control loop</td>
</tr>
<tr>
<td>K_{va}</td>
<td>gain of the velocity feedback loop of the actuator</td>
</tr>
<tr>
<td>H(s)</td>
<td>force-to-velocity transfer function of the structure</td>
</tr>
<tr>
<td>w_{st}, b, c</td>
<td>parameters of the structure, rad/s, N/m</td>
</tr>
<tr>
<td>f_{st}(t)</td>
<td>= -f_{pm}(t), reaction force applied to the structure, N</td>
</tr>
<tr>
<td>y_{st}(t)</td>
<td>displacement of the structure, meters</td>
</tr>
<tr>
<td>K_{ps}</td>
<td>gain of the position feedback vibration suppression control loop</td>
</tr>
<tr>
<td>K_{vs}</td>
<td>gain of the velocity feedback vibration suppression control loop</td>
</tr>
<tr>
<td>\alpha</td>
<td>scale factor for highwidth actuator</td>
</tr>
<tr>
<td>\omega_{h}</td>
<td>saturation break frequency, rad/s</td>
</tr>
<tr>
<td>G_{ry}(s)</td>
<td>transfer function from actuator command, r_{pm}(t), to position of the proof-mass, y_{pm}(t), structure constrained to be stationary</td>
</tr>
<tr>
<td>x(t)</td>
<td>state of the actuator/structure system, x(t) \in \mathbb{R}^4, meters</td>
</tr>
<tr>
<td>OR</td>
<td>operating region, a subset of the state space, meters</td>
</tr>
<tr>
<td>t_s(x_0)</td>
<td>1% settling time of the structure's position for the initial condition x_0</td>
</tr>
<tr>
<td>\zeta_{ed}(x_0)</td>
<td>equivalent damping of the initial condition x_0</td>
</tr>
<tr>
<td>\zeta_{PM}(x_0)</td>
<td>performance function evaluated for the initial condition x_0</td>
</tr>
</tbody>
</table>

Introduction

A proof-mass actuator is a motor that accelerates a mass in a linear direction rather than in rotational motion. When this actuator is attached to a flexible structure, it can be modeled as a mass connected to the structure with an equivalent spring and damper. In response to an external command, the motor will impart a force to the proof-mass which in turn imparts a reaction force to the structure. In this configuration proof-mass actuators have been proposed\textsuperscript{1-12} as actuators for the suppression of vibrations in flexible structures.

Proof-mass actuators are attractive for vibration suppression, in particular for space structures, because there is some indication that their weight-to-force ratio is superior to other actuators. To exploit this potential weight reduction, the actuator must be sized for a particular structure. This sizing is complicated, because the proof-mass may run against its stops. We call the contact of the proof-mass with its stops \textit{stroke saturation}. Stroke saturation imparts shocks to the structure, and it can result in damage to the actuator. In this paper we consider design of the proof-mass actuator control system to avoid stroke saturation. These results can also be viewed as a step in the development of a criteria for the sizing of a proof-mass actuator taking into account the stroke length of the actuator.

The capacity of the actuator is determined by the mass of the proof-mass, the stroke length, and the maximum force available from the electromechanical subsystem.\textsuperscript{8,9,10} Using a one mode model, we describe the relationship between these parameters, the natural frequency of the structure and the operating region of the actuator. The results here represent an alternative approach to determining the operating region through an the optimal control formulation.\textsuperscript{5}
The actuator model discussed in this paper focuses on the system level characterization of proof-mass actuators. Our purpose is to describe the operational envelope of proof-mass actuators with respect to those parameters that are common to all realizations of proof-mass actuators. This model does not include detailed descriptions of nonlinearities in the device itself because these nonlinearities are realization dependent. The results in this paper generally apply to proof-mass actuators that rely on a specific realization of a proof-mass actuator.1,2,4,6

The System Model

The Actuator Model

In this paper we are concerned with the performance of a proof-mass actuator within a vibration suppression control loop for a flexible structure. We define a proof-mass actuator as any device that accelerates a (proof-) mass in linear motion and so generates a reaction force on the structure. The actuator is composed of two primary components: the proof-mass and the base. The proof-mass has mass \( m \) (kg), and length \( 2d \) (meters) which defines the range of linear motion. We say that the actuator has a stroke of \( d \) (meters). The base contains the power electronics and other components in a configuration that can apply a force to the proof-mass through electromagnetic coupling. The maximum force that can be applied by the electromechanical subsystem to the proof-mass is \( F_{\text{max}} \). These parameters are common to any physical realization of a proof-mass actuator.

Consider a proof-mass actuator attached to a flexible structure. The structure is modeled by one flexible mode. The model of this system is shown in Fig. 1.

\[ y_{\text{st}}(t) \]
\[ y_{\text{pm}}(t) \]
\[ y_{\text{r}}(t) \]
\[ d \]
\[ \text{stroke length} \]
\[ \text{Base} \]
\[ \text{proof-mass} \]
\[ m \]
\[ \text{structure} \]

Fig. 1 Proof-Mass Actuator and a Single DOF Model.

A block diagram of the actuator and structure along with the control system components is shown in Fig. 2.

\[ y_{\text{st}}(t) \]
\[ y_{\text{pm}}(t) \]
\[ y_{\text{r}}(t) \]
\[ f_{\text{pm}}(t) \]
\[ f_{\text{st}}(t) \]
\[ v(t) \]
\[ K_{\text{ps}} \]
\[ K_{\text{pa}} \]
\[ K_{\text{va}} \]
\[ H(s) \]

Fig. 2 Block Diagram of a Single Mode Structure with a Proof-Mass Actuator.
To discuss the actuator model, we constrain the structure to be stationary. The input to the actuator is the voltage \( v(t) \). This electrical signal is proportional to the force, \( f_{pm}(t) \), on the proof-mass generated by the electromechanical subsystem. The resulting displacement and velocity of the proof-mass are denoted \( y_{pm}(t) \), and \( \dot{y}_{pm}(t) \), respectively, and they are identified in Fig. 2.

The actuator contains two local loops to stabilize the proof-mass. The inner loop, labeled "Actuator Control Loop: Velocity Feedback," feeds back the velocity signal with a gain of \( K_{va} \). The outer loop is a negative unity feedback of the position of the proof-mass (when \( y_{st}(t) = 0 \)). This loop is labeled as the "Actuator Control Loop: Position Feedback." The two gains, \( K_{va} \) and \( K_{pa} \), determine the transfer function \( Y_{pm}(s)/R_{pm}(s) \) when the structure is constrained to be stationary. The outer loop represents the equivalent stiffness and the inner loop represents the equivalent damping of the proof-mass. The physical implementations of the actuator control loops may be mechanical or electronic or a combination of the two. The primary purpose of these loops is to manage the proof-mass, although they also effect the design of vibration suppression loops. From this point on, we assume that the actuator loops are closed around the proof-mass.

**The Model of the Total System**

From the full system in Fig. 2 we see that a force applied to the proof-mass, \( f_{pm}(t) \), results in a reaction force on the structure, \( f_{st}(t) \). At the location of the actuator attachment, the displacement of the structure is \( y_{st}(t) \). The relationship between the applied force and resulting velocity of the structure is given by

\[
\dot{y}_{st}(t) + 2\zeta\omega_{st}\dot{y}_{st}(t) + \omega_{st}^2 y_{st}(t) = b\dot{f}_{st}(t),
\]

\[
\ddot{y}_{st}(t) + c\dot{y}_{st}(t), \quad y_{st}(t) = c\dot{y}(t).
\]

The transfer function, \( H(s) \), in Fig. 2 is calculated from (1). In the numerical simulations below the parameter values of the structure in (1) are \( \omega_{st} = 8.2 \text{ rad/s}, b = c = 0.024 \), and \( \zeta = 0.003 \) (0.3% damping).

We consider two feedback loops to effect the vibration suppression control loops. The first loop, labeled "Vibration Suppression Loop: Position Feedback" in Fig. 2, feeds back the position of the structure with a gain of \( K_{ps} \) to the actuator reference input, \( r_{pm}(t) \). The second loop, labeled "Vibration Suppression Loop: Velocity Feedback" in Fig. 2, feeds back the velocity of the structure to the actuator with a gain of \( K_{vs} \). The primary purpose of the vibration suppression loops is to suppress vibrations in the structure. The gains in the vibration suppression loops are chosen after the gains of the actuator control loops are fixed.

**Nonlinearities**

In the analysis below we take into account two nonlinearities of the proof-mass actuator. The first nonlinearity models the maximum force available from the actuator electromagnetic subsystem, and it is described by a standard saturation nonlinearity

\[
f_{pm}(v) = \begin{cases} F_{\text{max}}, & v > F_{\text{max}} \\ v, & -F_{\text{max}} \leq v \leq F_{\text{max}} \\ -F_{\text{max}}, & v < -F_{\text{max}} \end{cases}
\]

This nonlinearity is identified in Fig. 2 as the "Force Limit."

The second nonlinearity associated with the actuator is the stroke saturation; i.e. if the relative displacement of the proof-mass is greater than stroke \( d \), \( |y_r(t)| \geq d \), then the proof-mass will run against its stops. This nonlinearity is not shown explicitly in the block diagram of the system in Fig. 2. In this paper the stroke saturation provides a boundary in the state space of the system over which the proof-mass may not cross. We do not consider the dynamics of the structure after the proof-mass contacts its stops.

**Parameterization of the Actuator**

In this paper we investigate the performance of a proof-mass actuator operating within a vibration suppression control loop. In particular we will relate the parameters of the actuator; i.e. the mass of the proof-mass,
the stroke length, and the maximum available force, to ability of the actuator to meet the mission requirements of the structure. The nonlinearities of the actuator place an upper bound on the performance of the actuator. The actuator control loops determine the performance of the actuator within the vibration suppression control system. This analysis revolves around the relationship between the nonlinearities of the actuator, the frequency response of the actuator control loops, and the natural frequency of the mode of vibration of the structure. First we assume that the actuator control loops are open.

**Stroke/Force Saturation Curve**

Suppose that the motion of the proof-mass is

\[ y_{pm}(t) = d \sin(\omega t) \]  

(3)

Then the maximum instantaneous force imparted to the proof-mass is

\[ F_{\text{max stroke}} = m \omega^2. \]

(4)

If the actuator is attached to a structure, (4) is approximately true assuming the mass of the structure is much greater than the mass of the proof-mass. The frequency dependent curve (4) is plotted in Fig. 3 on a log-log scale, and it is labeled as the stroke saturation curve.

In particular, the stroke saturation curve relates the stroke length, d, to the dynamic response of the proof-mass.

The second force constraint is the maximum force that can be supplied by the electro-mechanical subsystem, \( F_{\text{max}} \). Assuming the signals are sinusoidal, the limitation on the signals of the actuator imposed by this nonlinearity is frequency independent. Hence, this constraint is a horizontal line shown in Fig. 3 as the force saturation curve.

The saturation break frequency, \( \omega_b \), is the frequency at which the maximum force output from the motion of the proof-mass in (4) is equal to \( F_{\text{max}} \).

\[ \omega_b = \sqrt{\frac{F_{\text{max}}}{md}}. \]  

(5)

The saturation break frequency is identified in Fig. 3. At frequencies below the saturation break frequency this maximum force output is determined by the product of the stroke length and the mass, md. The scalar, md, is called the mass/stroke constant. Above the saturation break frequency the maximum force output is determined by the maximum available force from the electromechanical subsystem, \( F_{\text{max}} \). These two force constraints can be combined together to form the stroke/force saturation curve, also shown in Fig. 3.
Actuator Bandwidth

The stroke/force saturation curve can be related to the parameters and control loops of the actuator by appealing to the concepts of steady state frequency response. During this discussion we assume that the actuator's control loops are closed. The transfer function from command input, \( r_{pm}(t) \), to position of the proof-mass, \( y_{pm}(t) \) is

\[
\frac{Y_{pm}(s)}{R_{pm}(s)} = \frac{\left( \frac{K_{pa}}{m} \right)}{s^2 + K_{va} \left( \frac{K_{pa}}{m} \right) s + \frac{K_{pa}}{m}} = G_{ry}(s). 
\] (6)

In the design of the actuator control loops, we are to select the two parameters \( K_{pa} \) and \( K_{va} \) in (6). For reasons that will be made clear below, we choose the poles of this transfer function to make this transfer function maximally flat. Then the design of the actuator control loops reduces to the selection of the bandwidth of (6) through the selection of \( K_{pa} \).

The straight line approximation of the magnitude Bode plot for the transfer function (6) is shown in Fig. 4.

The actuator break frequency, \( \omega_a \), for the transfer function in (6) is

\[
\omega_a = \sqrt{\frac{K_{pa}}{m}}. 
\] (7)

The transfer function from the command input signal, \( r_{pm}(t) \), to the force exerted on the proof-mass, \( f_{pm}(t) \), is

\[
\frac{F_{pm}(s)}{R_{pm}(s)} = \left( ms^2 \right) G_{ry}(s). 
\] (8)

The magnitude Bode plot of (8) is also shown in Fig. 4. Note that these two Bode plots are related by the dynamics of the proof-mass; a relationship that is unaltered by the control loops.

Let the input signal to the actuator is given by

\[
r_{pm}(t) = d \sin \omega t. 
\] (9)

The steady state displacement of the proof-mass is

\[
y_{pm}(t) = d \left| G_{ry}(j\omega) \right| \sin(\omega t + \theta_{ry}). \] (10)

The straight line approximation to the amplitude of the time response (10) as a function of frequency is plotted in Fig. 5 on a log - log scale.
For the command signal in (9) the force exerted on the proof-mass is given by

$$f_{pm}(t) = (d) \left[ m \omega^2 G_{ry}(j\omega) \sin(\omega t + \theta_{ry}) \right].$$  \hspace{1cm} (11)

The amplitude of this force applied to the proof-mass is also shown in Fig. 5.

The shape of the curves in Fig. 5 is determined by the transfer functions in (6) and in (8). The break frequency of these curves is determined by the break frequency of the actuator, \( \omega_a \) in (7). In Fig. 5 we have shown the amplitude of the time responses in (10) and (11) for two different actuator break frequencies. In the first case we have chosen the actuator break frequency to be equal to the saturation break frequency,

$$\sqrt{\frac{K_{pa}}{m}} = \omega_a = \omega_b = \frac{F_{\text{max}}}{md}.$$  \hspace{1cm} (12)

These curves are labeled "low bandwidth." We have also shown these curves when the actuator break frequency is greater than the saturation break frequency, \( \omega_{ah} > \omega_b \). These curves are labeled "high bandwidth."

First we consider the low bandwidth curves. In the frequency band below the actuator break frequency, \( \omega < \omega_a \), the control loops of \( G_{ry}(s) \) in (6) are configured so that the Bode plots in Fig. 4 are flat at 0 dB. Here the position of the proof-mass tracks the command input signal. For the input signal in (9) the full stroke of the proof-mass is used. The amplitude of the displacement of the proof-mass is shown in Fig. 5. If the command signal's amplitude is allowed to exceed the stroke length, \( d \), the actuator will saturate in stroke.

For frequencies above the break frequency, \( \omega > \omega_a \), the proof-mass will generate a constant amplitude force on the structure as shown by the Bode plot in Fig. 4. For the input signal in (9) the amplitude of the resulting force signal is shown in Fig. 5 labeled as "low bandwidth." The amplitude of the force depends on the actuator control loops as shown by (8) and (11). This amplitude is exactly the maximum force capacity of the actuator. This relationship can be traced back to the selection of the actuator break frequency (choice of \( K_{pa} \)) to match the saturation break frequency of the actuator, \( \omega_b \), in (12).

We could attempt to improve the performance of the actuator by increasing the actuator break frequency, say

$$\omega_{ah} = \alpha \sqrt{\frac{F_{\text{max}}}{md}}, \quad \alpha > 1.$$  \hspace{1cm} (13)

The resulting curves in Fig. 4 and Fig. 5 are labeled "high bandwidth." At frequencies below \( \omega_b \) in (13), the actuator response is unchanged. At frequencies above this break frequency, however, the forces that are commanded are above the actuator capacity and the actuator will saturate in force. Since the actuator is driven into force saturation, it will behave similarly to other devices of its construction. In the worst case the proof-mass will saturate in stroke.
Summary
We have considered two different designs of the actuator control loops. First, we selected $K_{pa}$ such that the break frequency of the actuator is matched to the saturation break frequency of the actuator. For the full stroke command in (9) the amplitude of the force
\[ d \cdot (m \omega^2) G_{ry}(j\omega) \]
will match the stroke/force saturation curve in Fig. 3 over the entire frequency band. Second, we considered the selection of $K_{pa}$ such that the actuator break frequency was greater than the saturation break frequency. While the resulting amplitude of the position matched the stroke saturation curve, the actuator was driven into force saturation above the saturation break frequency. Hence, the saturation break frequency imposes a bound on the bandwidth of the actuator control loops. It also follows from this analysis that if the bandwidth of the actuator is selected to fall below the saturation break frequency, then the force capacity of the actuator will be under-utilized.

The correspondence between (14) and the stroke/force saturation curve establishes a relationship between the parameters of the actuator and its control loops, and the nonlinearities of the actuator. Below we will compare the closed loop performance of several actuators with different saturation break frequencies. Note that the actuator saturation break frequency (7) only changes when the ratio of the maximum force to the mass/stroke constant changes. Scaling both quantities by the same factor leaves the saturation break frequency constant.

Performance Criteria

Mission Requirements of the Structure
We consider the selection of a proof-mass actuator for the vibration suppression control system in a flexible structure. We assume that the mission requirements for the structure are given as a minimum allowable damping (5%) in the structural modes of interest over the largest possible operating region. The actuator is to be selected so that it has the authority to meet the damping specifications, but it does not add excessive mass to the structure.

Damping
We consider nonlinear models of a proof-mass actuator attached to a flexible structure. If a set of initial conditions is close to the origin the system is completely linear so the usual definition of damping applies. As the norm of the initial condition increases the structure will enter into nonlinear behavior due to the force saturation of the actuator. In the nonlinear region we define the damping as follows. We let $t_s(x_0)$ be the 1% settling time of the structure's position, $y_{st}(t)$, corresponding to the initial condition $x_0$. Define the equivalent damping, $\zeta_{ed}(x_0)$, as
\[ \zeta_{ed}(x_0) = -\frac{\ln 0.01}{\omega_{st} t_s(x_0)}. \]
For a linear system this definition of damping will coincide with the usual definition of damping for all initial conditions.

Operating Region
The actuator's operating region is determined by the nonlinearities of the actuator. For a given set of initial conditions, we examine the time response of the relative position of the structure and the proof-mass. If the relative position is less than the stroke length of the actuator for $t \geq 0$, the actuator will not saturate in stroke and we say that the initial condition is in the operating region of the actuator. The states of our model are
\[ x = \begin{bmatrix} y_{pm} & \dot{y}_{pm} & y_{st} & \dot{y}_{st} \end{bmatrix}^T. \]
We assume that the mission requirements preclude stroke saturation of the actuator because shocks are imparted to the structure and damage may occur to the actuator itself. By insisting that the actuator not saturate in stroke, we have defined hard boundaries for the operating system.

On an absolute scale we assume that the actuator can meet the damping specification in the mission requirements of the structure for some choice of actuator parameters. For all actuators with a standard saturation characteristic (2), there is an inverse relationship between the vibration loop gain and the size of the operating region. As the loop gain increases, size of the operating region decreases. In the discussion here we normalize the vibration suppression loop gains so that near the origin the damping imparted to the structural mode is 5% for all closed loop systems. A suitable gain for the vibration loop, $K_{ps}$ or $K_{vs}$ in Fig. 2, can always be found since near the origin the system is completely linear. The size of the operating region will depend on the actuator's parameters and the actuator control loops. We will use the size of the actuator's operating region as a basis of comparison of actuator performance.

**Performance Function**

The equivalent damping captures the effect of the force saturation nonlinearity. The stroke saturation is captured by the operating region. To combine the two measures of equivalent damping and the operating region we define the performance function, $\zeta_{PM}$.

$$\zeta_{PM}(x_0) = \begin{cases} \zeta_{ed}(x_0), & x_0 \in \mathcal{OR} \\ 0, & x_0 \notin \mathcal{OR} \end{cases}$$

(18)

Given two actuators with approximately the same equivalent damping profile, the actuator with the largest operating region is considered more desirable.

**Mass-Efficiency**

This research was motivated by a desire to develop a sizing criteria for proof-mass actuators. Here we consider sizing an actuator with the minimum total mass, but with enough authority to meet the damping specifications. We assume that the structure's parameters in (1) are fixed. After the actuator is attached to the structure, we assume that 5% damping can be imparted to the structure using the vibration suppression control loops. This assumption implies that the actuator has a certain level of authority. One way to parameterize this authority is to assume that the maximum force available from the electro-mechanical subsystem of the actuator is fixed. Here we will use $F_{max} = 30$ N. This parameterization essentially determines a lower bound on the total mass of the actuator.

The distribution of the mass of each of the components of the actuator must be considered so that each component of the actuator is being used to full capacity most of the time. For a given total mass the authority of the actuator is divided between the electromagnetic force applied to the proof-mass and the mass and stroke length of the proof-mass. At high frequencies, the authority of the actuator is determined by maximum force available from the electromechanical subsystem, $F_{max}$. An increase in the maximum electromagnetic force applied to the proof-mass will increase the authority of the actuator. This increase in the authority will also increase the mass of the base since the base of the actuator contains the electromechanical subsystem. At low frequencies, the authority of the actuator is determined by the mass of the proof-mass and the stroke length. As mass of the proof-mass and its stroke length is increased, evidently the authority of the actuator is increased. We define the mass-efficiency of the actuator as the ratio of the mass of the proof-mass to the mass of the entire actuator. The mass-efficiency measures the relative mass contributions of the electromagnetic force and the proof-mass.

** The actuator placement, not considered in this paper, also effects the ability of the actuator to achieve specified damping levels.
Summary

Given the three actuators parameters m, d, and $F_{\text{max}}$, the stroke/force saturation curve is completely determined. Then the actuator loop gains are chosen such that the frequency response of the transfer function (6) matches the stroke/force saturation curve. Finally, the vibration suppression loops are chosen such that the structure's mode has 5% damping in the linear region around the origin.

Given the controlled structure, the actuator's performance can be evaluated through simulation using the performance function (18). The larger the operating region the better the performance of the actuator. In this way the actuator can be sized by the three parameters m, d, and $F_{\text{max}}$. These parameters are to be chosen such that the operating region is as large as possible, the actuator is being used to its full capacity most of the time over the largest possible operating region, the total mass is minimized, and the mass-efficiency is as close to unity as possible.

Actuator Parameter Selection For The Vibration Loops

Actuator Performance Functions

In this section we investigate the performance of a proof-mass actuator within a vibration suppression loop for a flexible structure. Since $\omega_b$ is determined by the parameters of the actuator we can investigate the performance of actuators of different sizes for a given structure by studying the performance of the actuators as $\omega_b$ varies with respect to $\omega_{\text{st}}$. We consider three values of $\omega_b$ shown in Table 1. The gains for the actuator and vibration suppression loops are chosen to impart 5% damping to the structure's flexible mode in the linear region of the actuator.

Table 1 Actuator and control loop parameters

<table>
<thead>
<tr>
<th>Case</th>
<th>$\omega_b$, rad/s</th>
<th>$K_{va}$</th>
<th>$K_{pa}$</th>
<th>$K_{ps}$</th>
<th>$K_{vs}$</th>
<th>md, kg-m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>14.142</td>
<td>0.09810</td>
<td>200</td>
<td>0</td>
<td>26</td>
<td>0.15</td>
</tr>
<tr>
<td>Case 2</td>
<td>8.165</td>
<td>0.17146</td>
<td>200</td>
<td>-70</td>
<td>0</td>
<td>0.45</td>
</tr>
<tr>
<td>Case 3</td>
<td>4.472</td>
<td>0.31305</td>
<td>200</td>
<td>0</td>
<td>-11.5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

The stroke/force saturation curve is shown in Fig. 6 for each set of parameters.

Case 1: $\omega_{b1} = 14.142$ rad/s $> \omega_{\text{st}}$.

For this choice of parameters, the structure's natural frequency is well below the actuator's saturation break frequency. The performance function is shown in Fig. 7.
The performance function is defined on a 4-dimensional domain. In Fig. 7 and in the figures that follow we have plotted this function on the subspace of initial conditions where

\[ y_{st}(0) = y_{pm}(0), \quad \text{or}, \quad y_{r}(0) = 0, \quad \text{and} \quad \dot{y}_{st}(0) = \dot{y}_{pm}(0). \]  

(19)

This subspace serves as a basis of comparison for different actuators with widely varying parameters. Similar results are obtained for other subspaces.

Fig. 7 shows that if the initial condition occurs where the performance function is zero, then the corresponding relative position response of the structure and the proof-mass, \( y_{r}(t) \), violates the stroke limit for some value of \( t > 0 \). If the initial condition corresponds to a relative position response that does not violate the stroke limit for all \( t \), then the equivalent damping is 5%.

**Case 2:** \( \omega_{b2} = 8.165 \text{ rad/s} = \omega_{st} \).

The performance function for this choice of parameters is similar to the performance function shown in Fig. 7 except the operating region is increased.

**Case 3:** \( \omega_{b3} = 4.472 \text{ rad/s} < \omega_{st} \).

In this case the saturation break frequency is well below the structure's natural frequency. Fig. 8 shows the performance function for this system.
Note that the equivalent damping has been reduced near the boundary of the operating region. Furthermore, the operating region is skewed near the boundary. In these regions the actuator shows pronounced nonlinear behavior due to the force saturation. This performance function is qualitatively different than the performance function in Fig. 7.

**Comparison of Operating Regions**

Fig. 9 shows the operating region for the three cases discussed above.

![Fig 9. Comparison of Operating Regions for Different Values of the Saturation Break Frequency.](image)

When the actuator's saturation break frequency is greater than the structure's natural frequency, \( \omega_b > \omega_{st} \), as in Case 1, the force available from the actuator is limited by the stroke length of the actuator. Here the stroke nonlinearity defines the operating region. The actuator exhibits linear behavior within the operating region because the force saturation nonlinearity is not affecting the dynamics of the actuator in the operating region. The operating region increases as the saturation break frequency increases, as shown in Fig. 9, because more of the available force is used.

When the actuator's saturation break frequency is less than the structure's natural frequency, \( \omega_b < \omega_{st} \), as in Case 3, the operating region takes on an irregular shape. Close to the origin the performance function matches the previous performance functions. For larger excitations, however, the force saturation nonlinearity dominates the actuator dynamics. This nonlinear response corresponds to a reduced equivalent damping at the boundaries of the operating region and the irregular shape of the operating region. In this regime limit cycles can be also be observed for some sets of parameters.\(^3\)

For concreteness, suppose that the stroke length is \( d = 0.15 \text{ m} \). If we chose the mass of the proof-mass to be \( m = 1 \text{ kg} \), then the parameters of the actuator match the parameters of Case 1. If we chose the mass of the proof-mass to be \( 10 \text{ kg} \), then the parameters of the actuator match the parameters of Case 3. From Fig. 9 we see that as the mass of the proof-mass increases, the operating region of the actuator increases, and the saturation break frequency decreases. When the saturation break frequency goes below the natural frequency of the structure, the operating region, while expanding, takes on an irregular shape. If a disturbance results in an initial condition into the far reaches of the operating region denoted as point A in Fig. 9, the actuator will not saturate in stroke. Such a disturbance, however, could easily result in an initial condition at point B in Fig. 9. Here the actuator will saturate in stroke. This observation suggests that for practical purposes, the operating region for Case 3 is much smaller than the computed operating region suggests. If all other parameters are held constant, increasing the mass of the proof-mass to increase the operating region leads to diminishing returns.

If the saturation break frequency is above the structure's frequency as in Case 1, the available force is under-utilized as shown in Fig. 10.
If the saturation break frequency is below the structure's frequency as in Case 3, the full stroke of the proof-mass is not utilized as shown in Fig. 11.

The implication of this analysis is that for Case 3, the mass of the proof-mass could be decreased without affecting the performance of the actuator. So the total mass of the actuator is reduced. Similarly, for Case 1 the gain of the electromechanical subsystem of the actuator can be reduced with an attendant reduction in mass of the actuator. These observations suggest that, all other things being equal, the parameters of the proof-mass should be chosen such that the saturation break frequency is matched to the natural frequency of the structure. These results also provide guidelines for designing an actuator that is mass-efficient.

These results can be used to design nonlinear control laws to prevent stroke saturation. These results will be presented elsewhere.

**Conclusions**

We have examined the relationship between the parameters of a proof-mass actuator and the performance of that actuator in a vibration suppression control loop for a single mode structure. The proof-mass actuator was parameterized by the mass and stroke length of the proof-mass and the maximum force available from the electromechanical subsystem. These parameters determine a stroke/force saturation curve that is characterized by the saturation break frequency. When the actuator is sized such that the saturation break frequency matches the natural frequency of the structure, the actuator is optimally utilized.

**Acknowledgment**

Mr. Gregory A. Zvonar was partially supported by a Fellowship from the Bradley Endowment.
References